Math exam, nationwide, Baccalaureate june 2003, "real" track
(science-oriented high schools)

All problems are required. Exam time is 3 hours. You have 10 points from the start - for taking the exam- (total: 100 pnts). Each question is 3 points.

Consider $(a_n)_{n\in\mathbf{N}^*}$ and $(b_n)_{n\in\mathbf{N}^*}$,

$$a_n = \frac{1}{2^{1^2}} + \frac{1}{2^{2^2}} + \frac{1}{2^{3^2}} + ... + \frac{1}{2^{n^2}}; \quad b_n = a_n + \frac{1}{2n \cdot 2^{n^2}}, \ (\forall) n \in \mathbf{N}^* \tag{1}$$

1. The set $\{n \in \mathbf{N}^* | a_n < a_{n+1}\}$ is:
a) Made up of a single element;
b) $\mathbf{N}^*$
c) $\emptyset$
d) Finite, having at least two elements.
2. The set $\{n \in \mathbf{N}^* | b_n > b_{n+1}\}$ is:
a) $\mathbf{N}^*$
b) Finite, having at least two elements;
c) Made up of a single element
d) $\emptyset$.
3. Knowing that $(a_n)_{n\in\mathbf{N}^*}$ and $(b_n)_{n\in\mathbf{N}^*}$ are convergent, denote by $a = \lim_{n\to\infty} a_n$ and $b = \lim_{n\to\infty} b_n$. Then $a - b$ is:
a) 0.5 b) 0 c) 0.25 1
4. The number $a = \lim_{n\to\infty} a_n$ belongs to the set:

$$a)\ \mathbf{Q} \setminus \mathbf{Z} \qquad b)\ \mathbf{Z} \setminus \mathbf{N} \qquad c)\ \mathbf{R} \setminus \mathbf{Q} \qquad d)\ \mathbf{N} \tag{2}$$

In the cartesion system of coordinates xOy consider the points $A_n(n, n^2),\ n \in \mathbf{N}$.
5. The slope of the straight line A_0A_1 is:
a) 2 b) 1 c) -2 -1
6. The equation of the straight line A_0A_1 is:

$$a)\ y = x^2 \qquad b)\ y = x \qquad c)\ x^2 + y = 0 \qquad d)\ x + y = 0 \tag{3}$$

7. The length of the segment A_1A_2 is:
a) 10 b) $\sqrt{10}$ c) 3 d) 4
8. The area of the triangle $A_nA_{n+1}A_{n+2}, n \in \mathbf{N}$ is:
a) 2 b) n c) 1 d) n+1
9. The number of straight lines that pass through 2 points in the set $\{A_1, A_2, A_3, A_4, A_5\}$ is:
a) 9 b) 8 c) 10 d) 20
10. How many triangles have the vertices in the set $\{A_1, A_2, A_3, A_4, A_5\}$?
a) 15 b) 5 c) 20 d) 10

Consider the function $f : \mathbf{R} \to \mathbf{R},\ f(x) = \sin x$. Denote by $f^{(n)}(x)$ the n-th order derivative of the function f, taken at point x.

11. Which of the following numbers is the period of the function f?

$$a)\ \pi \qquad b)\ \pi/2 \qquad c)\ 2\pi \qquad d)\ 3\pi \tag{4}$$

12. How many points of local maximum does the function f have in the interval $[0, 11\pi]$?
a) 11 b) 5 c) 6 10

13. The area of the plane surface between the graph of the function f, the Ox axis and the straight lines of equations $x = 0$ and $x = 2\pi$ is:
a) 0 b) 2 c) 4 d) 3

14. The value of

$$\lim_{x\to\infty} \frac{\int_0^x |f(t)|dt}{x} \tag{5}$$

is:

$$a)\ 0 \qquad b)\ \frac{2}{\pi} \qquad c)1 \qquad d)\ \infty \tag{6}$$

15. The maximum length of an interval included in $[0, 2\pi]$ on which the function f is convex, is:

$$a)\ \frac{\pi}{2} \qquad b)\ \pi \qquad c)\ 2\pi \qquad d)\ \frac{3\pi}{2} \tag{7}$$

16. The value $f^{(2004)}(0)$ is:
a) 1 b) -1 c) 0 d) 0.5

Consider the polynomial $f = x^4 - 14x^2 + 9$, with roots $x_1, x_2, x_3, x_4 \in \mathbf{C}$, the number $a = \sqrt{2} + \sqrt{5}$ and the sets $A = \{g(a)|g \in \mathbf{Z}[X]\},\ B = \{g(a)|g \in \mathbf{Z}[X], rank(g) \leq 3\}$.

17. Which of the following numbers is not a root of the polynomial f?

$$a)\ -\sqrt{2}+\sqrt{5} \qquad b)\ \sqrt{2}+\sqrt{5} \qquad c)\ \sqrt{2}-\sqrt{5} \qquad d)\ \sqrt{2}+\sqrt{3} \tag{8}$$

18. The sum $x_1 + x_2 + x_3 + x_4$ equals:
a) 4 b) 14 c) 0 d) -14

19. The product $x_1x_2x_3x_4$ equals:
a) 0 b) 14 c) -9 d) 9

20. If $p\sqrt{2} + q\sqrt{5} + r\sqrt{10} + s = 0,\ p, q, r, s \in \mathbf{Q}$, then the value of the expression $2p + 5p + 10r + s$ equals:
a) 0 b) 5 c) 7 d) 2

21. The set $A \setminus B$ is:
a) $\emptyset$
b) Infinite;
c) Finite, having at least two elements;
d) Made up of a single element

Consider the matrices

$$\mathbf{A} \in M_{3,4}(\mathbf{C}),\ A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \text{ and } I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

22. The rank of matrix A is:

a) 4 b) 1 c) 3 d) 2

23. The solution of the system of equations

$$\begin{aligned} x+y+z+t &= 1 \\ y+z+t &= 0 \\ z+t &= 0 \end{aligned} \tag{10}$$

where $(x, y, z, t) \in \mathbf{C}^4$, is:

a) (-1,1,-1,1) b) (1,1,-1,-1) c) $(1, 0, \lambda, -\lambda), \lambda \in \mathbf{C}$ d) (1,-1,1,-1)

24. The equation $AX = I_3, X \in M_{3,4}(\mathbf{C})$ has

a) No solutions;

b) An infinity of solutions;

c) A single solutions;

d) An infinite number of solutions strictly larger than 1.

25. The matrix I_3A has the sum of its elements equal to:

a) 9 b) 10 c) 0 d) 12

26. The set $\{Y \in M_{4,3}(\mathbf{C}) | det(YA) \neq 0\}$ is:

a) Empty;

b) Made up of a single element;

c) Made up of a finite number of elements, at least 2;

d) Infinite

27. How many solutions has the equations $\hat{3}x = \hat{0}$ in the ring $\mathbf{Z}_6$?

a) 1 b) 4 c) 2 d) 3

28. The sum $\hat{1} + \hat{2} + \hat{3} + \hat{4} + \hat{5}$, calculated in the ring $\mathbf{Z}_6$, is:

a) $\hat{3}$ b) $\hat{2}$ c) $\hat{0}$ d) $\hat{1}$

29. The product $\hat{1} \cdot \hat{2} \cdot \hat{3} \cdot \hat{4} \cdot \hat{5}$, calculated in the ring $\mathbf{Z}_6$, is:

a) $\hat{1}$ b) $\hat{2}$ c) $\hat{3}$ d) $\hat{0}$

30. What is the order of the element $\hat{2}$ in the group $(\mathbf{Z}_6, +)$?

a) 2 b) 4 c) 6 d) 3