MATHEMATICS Exam, High School entrance (8th grade), nationwide, january 2001

Every subject is required. You have 10 points for attending the exam: maximum grade 100. Work time is 2 hours. No calculators!

Part I (45 points)

1. The result of the calculation:

a) $(-2) \times (-5) =$... (3 pnts)

b) $[0.1 \times (-2) \times (-5) - 2]^{2001} =$..... (2 pnts)

2. The projections of the opposite sides of a right angle triangle on the hypothenuse are 2 cm and 8 cm.

a) The length of the perpendicular height corresponding to the hypothenuse is ... cm. (3 pnts)

b) The area of the given triangle is ... cm^2. (2 pnts)

3. a) After introduction of the factor under the square root, the number $2\sqrt{3}$ is equal to ... (3 pnts)

b) Between $3\sqrt{2}$ and 4, the greatest is .... (2 pnts)

4. An equilateral triangle is inscribed in a circle of radius 4 cm.

a) The side of the triangle has the length of ... cm. (3 pnts)

b) The perimeter of the given triangle is ... cm. (2 pnts)

5. 40% of x is equal to y.

a) The value of the ratio x/y is equal to .... (3 pnts)

b) The value of the ratio $\frac{2x+3y}{y}$ is equal to .... (2 pnts)

6. A cube ABCDA'B'C'D' has the side equal to 6 cm.

a) The length of the diagonal of a face is equal to ... cm. (3 pnts)

b) The value of the angle between the diagonals AD' and A'B is equal to ... degrees. (2 pnts)

7. a) Giving x as a common factor, the expression $E(x) = x^2 + 2x$ is equal to $E(x) =$..... (3 pnts)

b) Giving $x + 2$ as a common factor, the expression $E(x) = (x + 2)^3 + x^2 + 2x$ has the form $E(x) = (x + 2)(ax^2 + bx + c)$. The real numbers a, b, c are equal to .... (2 pnts)

8. Let one have a regular hexagon ABCDEF of side 2 cm. On its plane we draw a perpendicular to the plane, MA, of length 2 cm.

a) Draw a diagram corresponding to the text. (3 pnts)

b) The distance from M to the side CD is .... cm. (2 pnts)

9. Let the set A be A=$\{-2; 0; 3.1(6); \sqrt{12}\}$.

a) Written as an irreducible fraction, the umber 3.1(6) is equal to ... (3 pnts)

b) The set A $\setminus \mathbf{Q}$ is equal to ....

Part II (45 points)

10. The arithmetic mean of two natural numbers a and b is 35. Raising the number a by 11 and lowering the number b by 9, the obtained numbers are equal.

a) Find the sum $a + b$ (2 pnts).

b) Find the two numbers a and b. (6 pnts)

11. A natural number n divided by 12 gives a remainder of 5. The same number n divided by 10 gives a remainder of 3. Find the remainder of the division of n by 60. (5 pnts).

12. Consider the expressions

$$E_1(x) = \frac{x^3 - x^2}{x^2 - 2x + 1}, x \in \mathbf{R} \setminus \{1\} \quad \text{and} \quad E_2(x) = \frac{x-1}{x+1} + \frac{x+2}{x-1} + \frac{x+3}{x^2-1}, x \in \mathbf{R} \setminus \{\pm 1\} \quad (1)$$

a) Simplify the expression $E_1(x)$ by $x - 1$. (3 pnts)
b) Calculate $E_1(\sqrt{2} + 1)$, making rational the denominator of the result. (2 pnts).
c) Show that

$$E_2(x) = \frac{2x}{x-1} \quad (5 \ \text{pnts}) \tag{2}$$

d) Show that $\forall n \in \mathbf{N} \setminus \{1\}$, the value of the expression $E(n) = E_1(n) - E_2(n)/2$ represents a natural number. (2 pnts).

13. On the plane of a rectangle ABCD, on the same of it, we draw perpendiculars to the plane, AA', BB', CC', DD', such that AA'>DD'. The following legths are given: AD'= 6cm, AB=8cm, AA'=10cm, A'D'= $3\sqrt{5}$ cm and CC'= 1cm.
a) Draw a graph corresponding to the text (2pnts)
b) Calculate the length of AC'. (5 pnts)
c) Calculate the legth of DD'. (7 pnts)
d) Find out the length of BB' such that the points A,B,C,D are in the same plane. (6 pnts)