1 Susy in 4d- Notes by Horatiu Nastase A very basic introduction (survival guide)

1.1 Algebras

2-component notation for 4d spinors

$$\psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{1}$$

C matrix

$$C_{AB} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}; \epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2}$$

Gamma matrix

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \quad (\sigma^\mu)_{\alpha\dot{\alpha}} = (1, \vec{\sigma})_{\alpha\dot{\alpha}}; \quad (\bar{\sigma}^\mu)^{\alpha\dot{\alpha}} = (1, -\vec{\sigma})^{\alpha\dot{\alpha}} \tag{3}$$

N-extended susy algebra (I,J=1,N)

$$\begin{aligned} \{Q_\alpha^I, \bar{Q}_{\dot{\alpha}J}\} &= 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \delta^I_J \\ \{Q_\alpha^I, Q_\beta^J\} &= \epsilon_{\alpha\beta} Z^{IJ} \\ \{\bar{Q}_{I\dot{\alpha}}, \bar{Q}_{J\dot{\beta}}\} &= \epsilon_{\dot{\alpha}\dot{\beta}} Z^*_{IJ} \end{aligned} \tag{4}$$

Here Z^{IJ} is complex and antisymmetric and is a central charge, i.e. commmutes with all generators.

For $\mathcal{N} = 2$, $Z^{IJ} = 2\epsilon^{IJ} Z$. Then the redefinitions

$$a_\alpha = \frac{1}{\sqrt{2}}[Q^1_\alpha + \epsilon_{\alpha\beta}(Q^2_\beta)^+]; \quad b_\alpha = \frac{1}{\sqrt{2}}[Q^1_\alpha - \epsilon_{\alpha\beta}(Q^2_\beta)^+] \tag{5}$$

imply for a massive representation in the rest frame (no momentum)

$$\{a_\alpha, a^+_\beta\} = 2(M + |Z|)\delta_{\alpha\beta}; \quad \{b_\alpha, b^+_\beta\} = 2(M - |Z|)\delta_{\alpha\beta} \tag{6}$$

and the rest of the (anti)commutators are zero. Which means that we have the BPS bound: $M \geq |Z|$. Indeed, otherwise we would have negative norm states (ghosts) by acting with the creation operators on the vacuum. Note that the BPS bound can be proven under more general conditions, when we are in higher dimensions and we have all sorts of central p-form charges, but the principle is the same.

Also note that representing a (susy or not) algebra in terms of oscillators in order to classify representations is a very useful procedure in representation theory (the high energy theorists's version), that has been heavily used in more complicated theories.

We see that when the BPS bound is saturated, one half of the oscillators dissappear from the spectrum, so BPS representations are shorter (half size). In fact, they are the same size as a massless representation (a fact that can be obtained from dimensional reduction from a higher dimensional susy algebra).

1.2 $\mathcal{N} = 1$ susy

In the $\mathcal{N} = 1$ case, we can represent the algebra by derivatives as

$$
\begin{aligned}
Q_\alpha &= \partial_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\
\bar{Q}_{\dot{\alpha}} &= -\partial_{\dot{\alpha}} + i\sigma^\mu_{\dot{\alpha}\alpha}\theta^\alpha\partial_\mu; \quad P_\mu = i\partial_\mu
\end{aligned}
\tag{7}
$$

We can also define susy invariant derivatives, that are obtained just by switching the relative sign in the two susy charges, i.e.

$$
\begin{aligned}
D_\alpha &= \partial_\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\
\bar{D}_{\dot{\alpha}} &= -\partial_{\dot{\alpha}} - i\sigma^\mu_{\dot{\alpha}\alpha}\theta^\alpha\partial_\mu
\end{aligned}
\tag{8}
$$

We can check that indeed these derivatives commute with the supercharges.

Being susy invariant (commute with susy), we can impose susy preserving constraints by acting with them. In particular, on a general superfield we can impose the chirality constraint

$$
\bar{D}_{\dot{\alpha}}\Phi = 0 \tag{9}
$$

obtaining a *chiral superfield.* Similarly, a superfield satisfying $D_\alpha\Phi = 0$ is called *anti-chiral.*

A chiral superfield can be expanded as

$$
\Phi = \Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{10}
$$

where ϕ, ψ, F are arbitrary functions and F is an auxiliary field called F term. Thus the chiral (or Wess-Zumino) off-shell multiplet is made of a complex scalar, a Majorana spinor and an auxiliary complex scalar. These functions depend on the combination $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$, since $\bar{D}_{\dot{\alpha}} y^\mu = 0$. Then $\phi(x) = \Phi|_{\theta=0}$ (since $y^\mu|_{\theta=0} = x^\mu$ and $D_\alpha y^\mu|_{\theta=0} = 0$), and also $\psi(x) = D_\alpha\Phi|_{\theta=0}/\sqrt{2}$, $F(x) = D^2\Phi|_{\theta=0}$. We can also expand to find out the components in terms of x and θ. One Taylor expands the fields ϕ, ψ, F and then keeps in Φ only terms that have at most one of each independent spinor. Specifically, one gets

$$\Phi = \phi(x)+\sqrt{2}\Psi(x)+\theta^2 F(x)+i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x)-\frac{i}{\sqrt{2}}\theta^2(\partial_\mu\psi(x)\sigma^\mu\bar{\theta})-\frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi(x) \tag{11}$$

The derivatives are also important since they satisfy

$$\begin{aligned}\int d^4x\int d^2\theta &= \int d^4x D^2|_{\theta=0} \equiv \int d^4x D^\alpha D_\alpha|_{\theta=0}\\ \int d^4x\int d^2\bar{\theta} &= \int d^4x \bar{D}^2|_{\theta=0} \equiv \int d^4x \bar{D}^\alpha \bar{D}_\alpha|_{\theta=0}\end{aligned} \tag{12}$$

(true only under space integration!) where (by an abuse of notation) the $\mathcal{N}=1$ fermionic integration is

$$d^4\theta = d^2\theta d^2\bar{\theta} \tag{13}$$

and we can find component actions from superfield actions by converting the integrations into D's and $\bar{D}$'s and acting with them on various terms in the action.

The most general action for the chiral superfield is

$$\mathcal{L} = \int d^4\theta K(\Phi,\bar{\Phi}) + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\bar{W}(\bar{\phi}) \tag{14}$$

where the first term is called Kahler potential and the second is called superpotential (with its conjugate). If we have several superfields transforming in the representation of a group, we write the same formula, with $\Phi = \Phi^a T^a$ and an overall Tr to the action (T^a= generator in the particular representation).

To find the component action, as we said, we can convert integrals to covariant derivatives and act on the Lagrangean. But this is easier for the superpotential, for which we only need to use $\Phi(y,\theta)$, not $\Phi(x,\theta)$. For the Kahler potential, it is possible, but more difficult, since we need to remember that

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^\mu\partial_\mu \tag{15}$$

so we have to be careful of the order in which we take the derivatives. It is probably simpler for the Kahler potential to just do the fermionic integrations. However, when one does super-Feynman rules for supergraphs, one uses only D^2 and $\bar{D}^2$, as it turns out to be simpler. We should also remember that

$$\bar{D}^2 D^2 \Phi = 16 \Box \Phi \tag{16}$$

on a chiral superfield.

The only renormalizable terms in the chiral superfield action are: For the Kahler potential, the kinetic term

$$K = \Phi^+ \Phi \tag{17}$$

For the superpotential, the terms

$$W = \lambda \Phi + m \Phi^2 + g \Phi^3 \tag{18}$$

where m gives masses, g is a Yukawa coupling and λ gives a constant F-term, being just λF in components.

Gauge superfields are introduced as follows. A general (arbitrary function of superspace) *real* scalar superfield V (thus $V = V^+$) will contain the gauge field A_μ. But we impose the super-gauge symmetry

$$V \to V + i\Lambda - i\Lambda^+ \tag{19}$$

defined by the chiral superfield Λ. V has many components, but we can fix a gauge by using the super-gauge symmetry, such that V becomes very simple:

$$V = -\theta \sigma^\mu \bar{\theta} A_\mu + i\theta^2 (\bar{\theta} \bar{\lambda}) - i\bar{\theta}^2 (\theta \lambda) + \frac{1}{2} \theta^2 \bar{\theta}^2 D \tag{20}$$

So the susy multiplet is made up of a gauge field, a Majorana fermion and a complex auxiliary scalar, called a D term.

This gauge is called the Wess-Zumino (WZ) gauge and breaks manifest (superspace) susy (but we still have component susy!), but keeps as residual symmetry the usual bosonic gauge symmetry $A_\mu \to A_\mu + \partial_\mu \Lambda(x)$. The superfield strength is

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V \tag{21}$$

and is a chiral superfield, is Majorana: $(W_\alpha)^+ = W_{\dot{\alpha}}$ and satisfies the reality constraint

$$D^\alpha W_\alpha = D^{\dot{\alpha}} W_{\dot{\alpha}} \leftrightarrow Im(D^\alpha W_\alpha) = 0 \tag{22}$$

which is the susy generalization of the Bianchi identity.

In the nonabelian case (YM), we have the gauge transformation

$$e^{-2eV} \to e^{i\Lambda^+} e^{-2eV} e^{-i\Lambda} \tag{23}$$

and the super-field strength is

$$W_\alpha = \frac{1}{8e}\bar{D}^2(e^{2eV} D_\alpha e^{-2eV}) \tag{24}$$

and every superfield is contracted with the generators T^a.

The SYM kinetic action is

$$\begin{aligned} I_{SYM} &= -\frac{1}{4}\int d^4x d^2\theta tr W_\alpha W^\alpha + h.c. \\ &= \int d^4x tr[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{4}F_{\mu\nu} * F^{\mu\nu} - i\bar{\lambda}\sigma^\mu \nabla_\mu \lambda + \frac{D^2}{2}] + h.c. \end{aligned} \tag{25}$$

where we have written the an imaginary term, because we can write an action with the complex coupling

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \tag{26}$$

that reads

$$\begin{aligned} I_{SYM} &= \frac{1}{16\pi} Im[\tau \int d^4x d^2\theta tr W_\alpha W^\alpha] \\ &= \frac{1}{g^2}\int d^4x\ tr[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\bar{\lambda}\sigma^\mu\nabla_\mu\lambda + \frac{D^2}{2}] + \frac{\theta}{32\pi^2}\int d^4x F_{\mu\nu} * F^{\mu\nu} \end{aligned} \tag{27}$$

and the kinetic action for the matter multiplet (chiral scalar field), coupled to the YM (nonabelian) multiplet, is

$$\begin{aligned} I_{matter} &= \frac{1}{4g^2}\int d^4x d^2\theta d^2\bar{\theta} tr(\Phi^+ e^{-2gV}\Phi) \\ &= \frac{1}{2g^2}\int d^4x\ tr[|\nabla_\mu\phi|^2 - i\bar{\psi}\sigma^\mu\nabla_\mu\psi + F^+F - \phi^+[D,\phi] \\ &-ig\sqrt{2}\phi^+\{\lambda,\psi\} ig\sqrt{2}\bar{\psi}[\bar{\lambda},\phi]] \end{aligned} \tag{28}$$

To this, as we saw, we can add a renormalizable cubic superpotential, but there is no renormalizable term we can add for the YM multiplet.

The scalar potential of the general $\mathcal{N}=1$ system is then given by

$$g^2 V = \sum_i |F_i|^2 - \frac{g^2}{2} D^a D^a \tag{29}$$

where the F terms (auxiliary) F_i are replaced by their equation of motion and similarly for the D terms D^a. In the general $\mathcal{N}=1$ system, $I_{SYM} + I_{matter} +$ superpotential, we have

$$F_i = \frac{\partial W}{\partial \Phi_i}; \quad D^a = \Phi^+ T^a \Phi \equiv \Phi^{+^i} (T^a)_{ij} \Phi^j \tag{30}$$

For an *abelian* theory, one can still add a term to the susy action, called Fayet-Iliopoulos term (FI term)

$$\mathcal{L}_{FI} = \int d^2\theta d^2\bar{\theta} \xi^a V^a \tag{31}$$

which just shifts the D term:

$$D^a = \xi^a + \Phi^{+^i} (T^a)_{ij} \Phi^j \tag{32}$$

1.3 $\mathcal{N}=2$ and $\mathcal{N}=4$ susy

An $\mathcal{N}=2$ YM superfield Ψ is simpler when expressed in terms of two $\mathcal{N}=1$ superspaces (as opposed to an $\mathcal{N}=2$ superspace), so by an expansion in a $\tilde{\theta}$ spinor, with coefficients that are $\mathcal{N}=1$ superfields (in θ). Specifically, one has

$$\begin{aligned}
\Psi &= \Phi(\tilde{y}, \theta) + \sqrt{2} \tilde{\theta}^\alpha W_\alpha(\tilde{y}, \theta) + \tilde{\theta}^2 G(\tilde{y}, \theta) \\
G(\tilde{y}, \theta) &= -\frac{1}{2} \int d^2\bar{\theta} [\Phi(\tilde{y}, \theta)]^+ e^{-2eV(\tilde{y}, \theta)} \\
\tilde{y}^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} + i\tilde{\theta}\sigma^\mu\bar{\tilde{\theta}} = y^\mu + i\tilde{\theta}\sigma^\mu\bar{\tilde{\theta}}
\end{aligned} \tag{33}$$

and the most general action for Ψ is the an arbitrary function

$$\frac{1}{16\pi} Im \int d^4x d^2\theta d^2\tilde{\theta} \mathcal{F}(\Psi) \tag{34}$$

(where the 16π and the Im as opposed to Re are of course conventional). With $\Psi = \Psi^a T^a \to \Phi = \Phi^a T^a, W_\alpha = W_\alpha^a T^a$, we can do the $d^2\tilde{\theta}$ integration as

$$Im \int d^4x [\int d^2\theta \mathcal{F}_{ab}(\Phi) W^{a\alpha} W_\alpha^b + \int d^2\theta d^2\bar{\theta} (\Phi^+ e^{-2gV})^a \mathcal{F}_a(\Phi)] \tag{35}$$

where $\mathcal{F}_a = \partial\mathcal{F}/\partial\Phi^a$, etc.

Classically, the unique renormalizable $\mathcal{N}=2$ action is given by

$$\mathcal{F}(\Psi) = \mathcal{F}_{class}(\Psi) = \frac{1}{2}\tau tr\Psi^2 \tag{36}$$

and it coincides with $I_{SYM}+I_{matter}$. There is no renormalizable $\mathcal{N}=2$ invariant self-interaction allowed (for $\mathcal{N}=1$, we had a nontrivial superpotential Φ^3).

Still, the model is nontrivial, as there is a nontrivial D term (even if the F terms are trivial). Since Φ is in the adjoint representation, $(T^a)_{bc} = f^a{}_{bc}$ and we can subsitute it in the general formula. Thus the F and D term action is

$$I_{aux} = \frac{1}{g^2}\int d^4x tr[\frac{D^2}{2} - g\phi^+[D,\phi] + F^+F] \tag{37}$$

giving the scalar potential

$$V = \frac{1}{2}([\phi^+,\phi])^2 \tag{38}$$

which has a nontrivial minimum if

$$[\phi^+,\phi] = 0; \quad \phi \neq 0 \tag{39}$$

One the most general $\mathcal{N}=2$ SYM lagrangean would also involve a term

$$\int d^4x d^4\theta d^4\bar{\theta} H(\Psi,\Psi^+) \tag{40}$$

but this will be a higher derivative lagrangean, as we can check by dimensional analysis: For each set of $\mathcal{N}=1$ θ's, we have seen that $D^2\bar{D}^2 = \Box$, thus for the first component of the previous action we would get something like $\Box^2 H(\phi,\phi^*)$.

The other $\mathcal{N}=2$ multiplet is the hypermultiplet, which is composed of two $\mathcal{N}=1$ chiral multiplets, Q and $\tilde{Q}$. The most general $\mathcal{N}=2$ preserving coupling of the N_f hypermultiplets characterized by an index i and in the fundamental N_c of the gauge group, to the SYM multiplet is, in $\mathcal{N}=1$ language

$$\int d^2\theta d^2\bar{\theta}(Q_i^+ e^{-2V} Q_i + \tilde{Q}_i e^{2V}\tilde{Q}_i^+) + \int d^2\theta(\sqrt{2}\tilde{Q}_i\Phi Q_i + m_i\tilde{Q}_i Q_i) + h.c. \tag{41}$$

N=4 SYM is the theory with

$$\mathcal{F}(\Psi) = \mathcal{F}_{class}(\Psi) = \frac{1}{2}\tau tr\Psi^2 \tag{42}$$

coupled to one massless adjoint hypermultiplet similarly to the coupling above: The index i is now also in the adjoint of the gauge group, so we need to put a trace outside the integral . The superpotential is now

$$W = \epsilon_{ijk}Tr\Phi^i[\Phi^j, \Phi^k]; \quad i = 1, 2, 3 \tag{43}$$

It is the unique lagrangean allowed by $\mathcal{N} = 4$ susy, and the theory is superconformal.

1.4 Susy breaking (Witten index)

Since always we have in the algebra

$$\{Q, Q\} \propto H \tag{44}$$

and the vacuum should be susy by itself, thus

$$Q|0>= 0 \Rightarrow ||Q|0> || = 0 \Rightarrow E_0 = 0 \tag{45}$$

so a necessary condition for susy to be unbroken is that the vacuum energy is zero (just from the susy algebra).

At any nonzero energy level, we need the same number of bosons and fermions (hence the counting of bosonic and fermionic states to determine the field content of a susy theory), because we will have in general

$$Q|b>= \sqrt{E}|f>; \quad Q|f>= \sqrt{E}|b> \tag{46}$$

thus we can only have a susy invariant state if it is a linear superposition of bosonic and fermionic states $|b> - |f>$. And the number of purely bosonic and purely fermionic states has to be the same, $n_B^{E\neq 0} = n_F^{E\neq 0}$. However, for the ground state, bosonic and fermionic states are invariant by themselves:

$$Q|b>= 0; \quad Q|f>= 0 \tag{47}$$

So when we say $|b>$ and $|f>$ we really mean bosonic and fermionic zero modes (creation operators) acting on a particular vacuum, so $b^+_{vac}|0>$, $f^+_{vac}|0>$.

Thus the number of fermionic and bosonic zero modes (vacuum states) need not be the same. The quantity

$$n_B^{E=0} - n_F^{E=0} \tag{48}$$

is also invariant under changes in the parameters of the theory. Indeed, if say a boson acquires a mass, then to balance the massive state equality, a fermion will acquire a mass also.

But

$$Tr(-1)^F \equiv < i|(-1)^F|i >= n_B^{E=0} - n_F^{E=0} \tag{49}$$

(since at nonzero level, the bosons match the fermions), so

$$Tr(-1)^F = Tre^{2\pi i J_z} \tag{50}$$

is called the Witten index and is independent of the parameters of the theory, and characterizes susy breaking. From what we said, we deduce that

- If the Witten index $Tr(-1)^F$ is nonzero, susy is unbroken
- If the Witten index is zero, but $n_B^{E=0} = n_F^{E=0} \neq 0$, susy is still unbroken, because that means there is an operator C such that $Tr((-1)^F C) \neq 0$, and we just need to redefine the fermion number.
- If the Witten index is zero and $n_B^{E=0} = n_F^{E=0} = 0$, then susy is broken, because that means that the true vacuum has higher energy.

Both $Tr(-1)^F$ and any other $Tr((-1)^F C)$ needed can be in general calculated as topological invariants, and we can thus determine if a theory will (dynamically or spontaneously) break susy.

In conclusion, we have the following cases for the shape of the susy gauge theory scalar (Higgs) potential:

- True vacuum at $\phi = 0$ has E=0 (and maybe false vacuum at $\phi \neq 0$ and $E \neq 0$): susy unbroken, gauge symmetry unbroken.
- True vacuum at $\phi = 0$ has $E_1 > 0$ (and maybe false vacuum at $\phi \neq 0$ and $E_2 > E_1$): susy broken, gauge symmetry unbroken.
- False vacuum at $\phi = 0$ has $E_1 > 0$ but true vacuum at $\phi \neq 0$ has $E_2 = 0$: susy unbroken, gauge symmetry broken.

- False vacuum at $\phi = 0$ has $E_1 > 0$ and true vacuum at $\phi \neq 0$ has $0 < E_2 < E_1$: susy broken, gauge symmetry broken.

Witten did an analysis of the Witten index of susy gauge theories and has found that the only possibility for susy breaking is (maybe!- we need to treat case by case) is of matter in a complex representation of the gauge group and of coupling to gravity.

In particular, rigid $\mathcal{N} = 2$ gauge theories (that have matter in real representations) cannot have susy broken. That is the reason why we need $\mathcal{N} = 1$ for phenomenology (MSSM)- we can only break rigid susy in $\mathcal{N} = 1$ theories with complex representations.

1.5 Susy breaking- types

Susy breaking can a priori be:

-tree level (spontaneous)

-perturbative (spontaneous)

-nonperturbative: spontaneous or dynamical. Dynamical means that there is a dynamical mechanism that makes the variables at low energy different from what they are at high energy. An example of that is the QCD phenomenon of formation of VEV for $< \bar{q}q >$ at low energies, or any fermion bilinear condensate for that matter.

Tree level:

The condition for susy to be unbroken is the minimization of the scalar potential to give zero energy. For the general tree-level (renormalizable) $\mathcal{N} = 1$ theory, that just means that the F terms and the D terms have to vanish (all of them!)

$$\begin{aligned} F_i &= \frac{\partial W}{\partial \Phi_i} = 0 \\ D^a &= \xi^a + \Phi^{+i} (T^a)_{ij} \Phi^j = 0 \end{aligned} \tag{51}$$

A simple analysis shows that there are as many equations as unknowns, so for a generic superpotential it is possible to find solutions to these equations, and susy is unbroken.

If there are no U(1) factors, it is always possible. If we have U(1)'s, but the FI factors ξ^a vanish, then the vanishing of the F terms implies there is another solution that vanishes both the F and the D terms. *The same* *is true even in non-renormalizable theories (thus for quantum corrections).* Thus generically, only the FI terms allow spontaneous breaking of susy, or otherwise if there is no overall minimum of the superpotential.

There are cases when there is no overall minimum of the superpotential. The most common is a generalization of a class of models due to O'Raifeartaigh. COnsider two sets of scalars, X_n and Y_i, and

$$W = \sum_i Y_i f(X_n) \tag{52}$$

Then the F term vanishing gives

$$f_i(x_n) = 0; \quad \sum_i y_i \frac{\partial f_i(x)}{\partial x_n} = 0 \tag{53}$$

and we can always solve the second set by $y_i = 0$, but if there are more Y's than X's, there is no generic solution to the first (more conditions than variables).

For the Fayet-Iliopoulos mechanism, consider a U(1) model with two chiral superfields of opposite charges $\pm e$, $\Phi_\pm$ and superpotential (most general U(1)-invariant renormalizable) $W = m\Phi_+\Phi_-$. Then

$$V(\phi_+, \phi_-) = m^2|\phi_+|^2 + m^2|\phi_-|^2 + (\xi + e^2|\phi_+|^2 - e^2|\phi_-|^2)^2 \tag{54}$$

and clearly there is no zero energy minimum if $\xi \neq 0$.

Tree level susy breaking implies the sum rule (is proven exactly)

$$\sum_{spin\ 0} \text{mass}^2 - 2\sum_{spin\ 1/2} \text{mass}^2 + 3\sum_{spin\ 1} \text{mass}^2 = 0 \tag{55}$$

holding separately for each set of given conserved quantum numbers.

An analysis of these sum rules for MSSM shows that it contradicts experiment. It would imply the existence of light particles that are excluded experimentally. So tree level susy breaking is ruled out in the Standard Model.

Perturbative level

As was proven by Seiberg, because of the holomorphy of the superpotential, one can prove that there are only 1-loop corrections to it (thus the F terms).

From it one can prove that if there are no FI terms and if if there is an overall minimum to the superpotential (no F terms), then susy is not broken *at any finite order in perturbation theory.*

Susy breaking terms in the Wilsonian (low energy) effective interaction can induce tadpoles that are quadratically divergent only if there are scalars that are neutral with respect to all exact symmetries. In theories without such scalars, all superrenormalizable interactions are soft, i.e. there are no quadratic or higher divergencies. In particular, the Minimally Supersymmetric Standard Model (MSSM) is such a model, and that is the main phenomenological reason for wanting it. The absence of quadratic divergencies implies that one can maintain a separation of scales between the electroweak and GUT scales. The other reason is the coupling unification, which is not perfect in the Standard Model and becomes better in MSSM.

So as we saw, tree level susy breaking is excluded experimentally in MSSM, and that implies that *it is not broken perturbatively either, at any finite order in perturbation theory.* So we are left with nonperturbative breaking as the only solution for MSSM.

Nonperturbative level

That is actually phenomenologically good, since it allows for a separation of scales between the unification scale M_X and the susy breaking scale M_S. From coupling running we get $M_S = M_X exp(-8\pi^2 b/\mathcal{G}^2(M_X))$, where b is of order 1 and $\mathcal{G}(\mu)$ is an asymptotically free gauge coupling that becomes strong at M_S. This is analogous to chiral symmetry breaking in QCD.

So we assume a strong force that becomes strong and breaks susy, but since we don't see such a force, it must be in a *hidden sector*. Then the phenomenlogy is in how is the susy breaking communicated to MSSM.

The mechanism by which this can happen are

-the mediation is done by the $SU(3)\times SU(2)\times U(1)$ fields themselves, also known as **gauge mediation**. Then the mass splittings in quarks, leptons, gauge bosons and superpartners would be of order $g_i^2/16\pi^2$ (where g_i are the Standard Model couplings) with respect to the susy breaking scale. Thus if the masses will be in the 100 GeV- 10 TeV range, M_S would be around **100 TeV**.

-the mediation is done by gravity, or rather the auxiliary fields that are superpartners of gravity, also know as **gravity mediation**. We expect then that the mass splittings between observed particles and superparticles to be of order $\sqrt{G}M_S^2$ or GM_S^3, corresponding to $M_S = 10^{11}$ **GeV** and 10^{13} **GeV**, respectively.

Finally, susy breaking itself can happen as we said, either *spontaneously*, namely that there is a nonperturbative superpotential (because there is no perturbative one, as we saw) that gives a nonzero vacuum energy, or *dynamically*, in that the correct low energy fields are not supersymmetric.

Seiberg's analysis of $\mathcal{N} = 1$ models generated such a nonperturbative superpotential.

It is actually not easy to find examples where the susy breaking is not of the dynamical type.

For example, take the case of adding quark superfields in the fundamental representation of $SU(N_C)$ and also in the fundamental of a flavor group $SU(N_f)$. The quarks are characterized by the chiral superfield Q in the fundamental and a chiral superfield $\tilde{Q}$ in the antifundamental, such that $D^a = Q^+ T^a Q - \tilde{Q} T^a \tilde{Q}^+$. For $N_f < N_c$ one can take $Q = \tilde{Q}^+$. Then, for no tree level superpotential, Seiberg found that for $N_C > N_f$ the Wilsonian effective action is for an effective gauge group $SU(N_C - N_f)$ and gauge invariant combinations of the quarks. The effective superpotential is found to be

$$W_{eff}(Q, \tilde{Q}) = (N_c - N_f) \left(\frac{\Lambda^{3N_C - N_f}}{det(\tilde{Q}Q)} \right)^{\frac{1}{N_C - N_f}} \tag{56}$$

and while this superpotential doesn't break susy and is actually written in terms of Q and $\tilde{Q}$ it is so in an non-analytic way, moreover the gauge theory is only effective, so it is a dynamical superpotential.

More to the point, if one takes a susy generation of Standard Model quarks and leptons without the right-handed leptons and without the $U(1)_Y$, so $SU(3) \times U(1)$ with Q, $\bar{u}$, $\bar{d}$, L, one can prove that the low energy is given by the gauge invariant quantities: $X_1 = Q\bar{u}L$, $X_2 = Q\bar{d}L$, $Y = Q^2 \bar{u}\bar{d}$, with effective superpotential (depending also a priori on the scales of the two gauge groups, Λ_2 and Λ_3 for SU(2) and SU(3) respectively)

$$W_{eff}(X_1, X_2, Y, \Lambda_2, \Lambda_3) = c \frac{\Lambda_3^7}{Y} \tag{57}$$

and one can check that there is no supersymmetric vacuum. In fact, there is no vacuum at all, so that is maybe not so good phenomenologically. In any case, this is a relevant example of *dynamical susy breaking*.

Gravitational mediation of susy breaking can be of first or second order in $k = \sqrt{8\pi G}$. If it is first order, it gives things like gaugino masses. Putting auxiliary scalars in the supergravity multiplet to their VEVs will generate susy terms. This will give susy-breaking terms in the Lagrangean of the type

$$\mathcal{L} = -2Re[\tilde{m}_g^* \frac{\Lambda \partial W}{\partial \Lambda}] \tag{58}$$

due to a dependence on a scale Λ of the (effective) superpotential, that can be either explicit or due to renormalization of couplings. The latter comes from an anomaly in the trace of the energy-momentum tensor, and then the previous susy-breaking term is called *anomaly mediated*. Note that here the 'complex gravitino mass' $\tilde{m}_g^*$ is of order k.

Gravitational mediation of order k^2 is also of two types:

-There is the *observable sector* and a *hidden sector* that becomes strong at an intermediate scale $m_W \ll \Lambda \ll m_{Pl}$ and produces a spontaneous breaking of susy in the hidden sector. The $k\Lambda^2$ is of order TeV and then $\Lambda \sim 10^{11}$ GeV.

-Besides the *observable* and *hidden sectors*, there are also *modular superfields* (superfields that include moduli, e.g. compactification moduli). The hidden sector is not spontaneously broken, but the gauge couplings of the hidden sector (that become strong at $\Lambda\ llm_{Pl}$ also) give a nonperturbative superpotential for the moduli superfields. So nonperturbative hidden sector effects such as gaugino condensation (VEVs for bilinears in gauginos) produce W. Then susy breaking in the modular sector is transmitted gravitationally to the observable sector. Then $k^2\Lambda^3 \sim$ TeV, implying $\Lambda \sim 10^{13} GeV$.