1 Seiberg-Witten theory- an introduction. Notes by Horatiu Nastase

1.1 Classical moduli space of $\mathcal{N}=2$ SYM

Since the potential of $\mathcal{N}=2$ SU(2) SYM is

$$V=\frac{1}{2}([\phi^+,\phi])^2 \tag{1}$$

its moduli space is given by

$$[\phi^+,\phi]=0 \tag{2}$$

By a gauge transformation, one can always put the solution into the form

$$<\phi>=\frac{a\sigma_3}{2} \tag{3}$$

where a is a complex number, so that $\phi^a=a\delta^{a3}$. A gauge invariant parametrization of the moduli space is given by

$$u=tr<\phi^2>=\frac{a^2}{2} \tag{4}$$

which is thus a *modulus*, i.e. parametrizes gauge-inequivalent vacua. Note that therefore this is different than the usual Higgs mechanism for the mexican hat potential $\lambda(|\phi|^2-v^2)^2$, where the different vacua are gauge equivalent! Now on top of the usual gauge equivalent vacua we have also gauge inequivalent ones, parametrized by u.

At nonzero a, $SU(2)\simeq SO(3)$ is broken to U(1), thus we have 't Hooft monopoles, embedded into the supersymmetric theory. Instead of the 't Hooft mexican hat potential, we have the potential (1), but for fluctuations around a vacuum of fixed modulus u one gets a similar result. The BPS bound is true now not only in the $\lambda\rightarrow 0$ limit, but always, as it comes from supersymmetry (is part of the algebra).

Instead of u, we will use a to describe the moduli space, since we will see it is more convenient for the physics. But u is still the parameter of the moduli space. In terms of a, there is still a gauge equivalence $a\leftrightarrow -a$, but moreover there is still a Z_2 R-symmetry exchanging $u\leftrightarrow -u$, and at $a=0(u=0)$ SU(2) is unbroken, so the classical moduli space is the upper half plane punctured at 0, i.e.

$$H^+-\{0\} \tag{5}$$

One can compute the Witten index $Tr(-)^F$ and see that is nonzero, thus susy will be unbroken. That means that the low energy theory is a general $\mathcal{N}=2$ supersymmetric U(1) (unbroken gauge group) gauge theory. Thus its low energy Wilsonian effective action (i.e. effective action that one gets by integrating down in energy, until the IR scale μ) is the general abelian action

$$I = \frac{1}{16\pi} Im \int d^4x d^4\theta \mathcal{F}(\Psi) \tag{6}$$

Since the U(1) is the unbroken U(1) of SU(2), and one can easily find that the complex scalar ϕ in Ψ has no potential, it can be identified with the modulus a, i.e. $<\phi>=a$.

Then the scalar kinetic term is (as one can easily check by acting with $d^4\theta = D^2\tilde{D}^2$)

$$Im[\mathcal{F}''(\phi)|\partial_\mu \phi|^2] \tag{7}$$

thus we have a *nonlinear sigma model* with metric

$$g_{\phi\phi^+} = Im\mathcal{F}''(\phi) \tag{8}$$

which means that the "Zamolodchikov metric on moduli space" for $<\phi>=a$ is

$$ds^2 = Im\mathcal{F}''(a) da\, d\bar{a} \equiv Im\tau(a) da\, d\bar{a}; \quad \tau(a) \equiv \mathcal{F}''(a) \tag{9}$$

Unitarity of the scalar field theory (positivity of the kinetic term) implies

$$Im\tau(a) > 0 \tag{10}$$

but if $\mathcal{F}$ is a holomorphic function, $Im\tau$ is harmonic, thus it has no minimum on the complex plane, thus we can't have $Im\tau > 0$ everywhere. So $Im\tau(a) > 0$ can only be true in the quantum theory if $\mathcal{F}$ is defined *only locally* (on patches).

Note that the low energy effective action that we took is the most general one. One could a priori take also a non-chiral term

$$\int d^4\theta d^4\bar{\theta} \mathcal{H}(\Psi, \Psi^+) \tag{11}$$

but this is higher order in derivatives, as we can check by dimensional analysis: both θ's and $\tilde{\theta}$'s give a $D^2\bar{D}^2 = \Box$, thus the first component of the above action will be of the type $\Box^2 H(\phi, \phi^+)$. Thus this is a high energy correction term.

Seiberg has calculated the perturbative form of $\mathcal{F}$ and found that it is 1-loop exact, namely one gets

$$\mathcal{F}_{pert}(\Psi;\Lambda)=\frac{\tau_{cl}\Psi^2}{2}(1+\frac{e^2}{4\pi^2}ln\frac{\Psi^2}{\Lambda^2}) \tag{12}$$

which should be plugged in the Wilsonian effective action $S_W[\phi;\mu]$, from which we get a holomorphic dependence of the coupling $e_{eff}(\mu)$. Thus one renormalizes by exchanging μ by the VEV of the Higgs, i.e. first defining the coupling e in (12) at the Wilson scale, $e(\mu)$, which is different from the usual effective action.

The translation between the two pictures is given by the definitions

$$\begin{aligned}&\tau(a)\equiv\mathcal{F}''(a;\mu)\\ &Re\tau(\mu)=\frac{\theta(\mu)}{2\pi};\quad Im\tau(\mu)=\frac{4\pi}{e^2(\mu)}\end{aligned} \tag{13}$$

Then one has

$$\frac{4\pi}{e^2(a)}\equiv Im\tau(a)=Im\mathcal{F}''(a;\mu)=\frac{4\pi}{e^2(\mu)}+\frac{1}{\pi}ln\frac{a^2}{\mu^2}+\frac{3}{\pi} \tag{14}$$

and one defines the renormalization group invariant scale Λ by putting $e^2(\mu=\Lambda)=\infty$, i.e.

$$\frac{4\pi}{e^2(a)}\equiv\frac{1}{\pi}ln\frac{a^2}{\Lambda^2}+\frac{3}{\pi}\Rightarrow\Lambda^2=\mu^2exp\{-\frac{4\pi^2}{e^2(\mu)}\} \tag{15}$$

Thus $e_{eff}(\mu)$ was turned into $e_{eff}(a)$, which at $a\to\infty$, i.e. in the UV, where we have small coupling, thus perturbation theory is valid, takes the form

$$\tau(a)\simeq\frac{i}{\pi}(log\frac{a^2}{\Lambda^2}+3) \tag{16}$$

Thus the classical moduli space is the the upper half plane punctured at the origin, and the perturbative quantum corrections at infinity are given by (16).

1.2 Quantum moduli space, duality and monodromies

We know that on the $\mathcal{N}=2$ abelian SYM moduli space there is a duality transformation. Defining the dual field

$$\Phi_D=\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi}\Rightarrow a_D=\mathcal{F}'(a) \tag{17}$$

after the duality transformation from Φ to Φ_D one finds

$$\mathcal{F}_D''(\Phi_D) = -\frac{d\Phi}{d\Phi_D} = -\frac{1}{\mathcal{F}''(\Phi)} \Rightarrow \tau_D(a_D) = -\frac{1}{\tau(a)} \tag{18}$$

which is the S element of Sl(2,Z) acting on Φ, Φ_D, i.e.

$$S: \begin{pmatrix} \Phi_D \\ \Phi \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \Phi_D \\ \Phi \end{pmatrix} \tag{19}$$

But one can also shift the action by

$$S \rightarrow S + \frac{b}{16\pi} Im \int d^4x d^2\theta W^\alpha W_\alpha = S - \frac{b}{16\pi}\int d^4x F_{\mu\nu} {*F}^{\mu\nu} = S - 2\pi b n \tag{20}$$

where n is the instanton number. This shift is irrelevant if b is an integer. It corresponds to $\theta \rightarrow \theta + 2\pi b$ (a shift in the theta parameter), or equivalently, in adding a term $b\Psi^2/2$ to $\mathcal{F}$, thus it also shifts $\Phi_D \rightarrow \Phi_D + b\Phi$, which means

$$T: \begin{pmatrix} \Phi_D \\ \Phi \end{pmatrix} \rightarrow \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Phi_D \\ \Phi \end{pmatrix} \tag{21}$$

Together, S and T (for b=1) generate the duality group Sl(2,Z) acting on Φ, Φ_D, or on the moduli space, described by a, a_D.

As in the SO(3) Georgi-Glashow model, besides 't Hooft monopoles and electrons, one has also dyons, and we saw there that there is a Sl(2,Z) duality invariance of the spectrum, if we act on the space of dyon charges, together with the action on a, a_D. Then

$$\frac{M}{\sqrt{2}} \geq |Z| = |an_e + a_D n_m| = | \begin{pmatrix} n_e & n_m \end{pmatrix} \begin{pmatrix} a \\ a_D \end{pmatrix} | \tag{22}$$

and classically $a_D = a\tau_{cl}$. The fact that quantum mechanically, one has still

$$Z = an_e + a_D n_m \tag{23}$$

is justified as follows. The coupling of dual photons to hypermultiplets is susy, thus it can only be of type

$$\sqrt{2} n_e \Phi Q \tilde{Q} \tag{24}$$

but the electrons will live in hypermultiplets, thus they $(Q\tilde{Q})$ have mass $\sqrt{2}n_e a$. By S duality (which was proven to be true for the low energy abelian theory), the next term is $a_D n_m$. We notice that $\tau_{cl} = a_D/a$, but $\tau_{qu} = \partial a_D/\partial a$.

The metric on the quantum moduli space is

$$ds^2 = Im\mathcal{F}''(a)dad\bar{a} = Im\frac{da_D}{da}dad\bar{a} = Imda_Dd\bar{a} \tag{25}$$

which is also Sl(2,Z) invariant. Near $u \to \infty$, we get

$$a_D = \frac{\partial \mathcal{F}_{1loop}}{\partial a} = \frac{2ia}{\pi}ln\frac{a}{\Lambda} + \frac{ia}{\pi} \tag{26}$$

and classically, $u = a^2/2$. That means that if we rotate u by 2π at infinity, we get the *monodromy*

$$\begin{aligned} a_D &\to -a_D + 2a \\ a &\to -a \end{aligned} \tag{27}$$

thus a, a_D is acted on by the monodromy matrix

$$M_\infty = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} = PT^{-1} \tag{28}$$

where $P = -1$. Complex theory implies that there are p singularities in the complex u plane, and a, a_D have monodromies M_i, such that

$$M_\infty = M_1 ... M_p \tag{29}$$

where

$$[M_i, M_\infty] = 0 \tag{30}$$

(abelian monodromy). Classically, $u = a^2$ is a good global coordinate on the moduli space. We need at least two punctures, because of $u \leftrightarrow -u$ symmetry (which is a remnant of a classical $U(1)_R$ symmetry). But one could still have maybe only u=0 as a single puncture. However, then $M_\infty = M_0$, and then one could take $u = a^2$ quantum mechanically also, valid everywhere, as a global coordinate on moduli space. By the unitarity argument given before, that cannot happen. So the simplest possibility is

$$M_\infty = M_{u_0} M_{-u_0} \tag{31}$$

But what happens at a singularity? A singularity on the moduli space is a place where the nonlinear sigma model (kinetic term for moduli) (7) is singular. But that can only happen if we have integrated out degrees of freedom in the Wilsonian effective action, that are massive almost everywhere but become massless at the singularities. Then we have done something illegal, and the singularity is the result.

What states could become massless? Let us assume it is a state (n_e, n_m) that becomes massless at the singularity u_i with monodromy matrix M_i. Then

$$(n_m \;\; n_e)M_i = (n_m \;\; n_e) \tag{32}$$

or $(n_m, \;\; n_e)$ is a left-eigenvector of M_i. The reason for that is that we want

$$Z = \begin{pmatrix} n_e & n_m \end{pmatrix} \begin{pmatrix} a \\ a_D \end{pmatrix} \tag{33}$$

to be invariant under the monodromy

$$\begin{pmatrix} a \\ a_D \end{pmatrix} \rightarrow M \begin{pmatrix} a \\ a_D \end{pmatrix} \tag{34}$$

For a state $(n_m, \;\; n_e)$ to be a left-eigenvector of M, we find that M is given by

$$M = M(n_m; n_e) = \begin{pmatrix} 1 + 2n_m n_e & 2n_e^2 \\ -2n_m^2 & 1 - 2n_m n_e \end{pmatrix} \tag{35}$$

It is a number theory result that then for any $p > 2$, given the monodromy at infinity that we found in (28), there is no solution for $M_\infty = M_{u_1} ... M_{u_p}$. Thus the only possibility is the simplest one we took.

Classically, at u=0, the gauge bosons $(0, 1)$ become massless, but that is not good at u_0!

Quantum mechanically, we thus want

$$M_\infty = M(n_m; n_e) M(n'_m; n'_e) \tag{36}$$

and we find that we need $n_m = n'_m = 1$, and the simplest solution is $(n_m \;\; n_e) = (1, 0)$ (monopole) and $(n'_m \;\; n'_e) = (1, -1)$ (dyon), and correspondingly

$$M_1 = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}; \quad M_{-1} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix} \tag{37}$$

Another check of the correctness of this result is the following. If a monopole becomes massles, it will have a U(1) effective theory, with the beta function

$$\mu \frac{d}{d\mu} g_D = \frac{g_D^3}{8\pi} \tag{38}$$

and the scale μ is proportional to a_D which is the only scale there and will go to zero, since it is the VEV of the dual (monopole) scalar field. Then $a_D \simeq c_0(u-u_0)$ is a good coordinate on the moduli space near the singularity (we have seen that at $n_e = 0$, $Z = n_m a_D$ and for BPS states $M = |Z|$). From the beta function one finds

$$\tau_D|_{\theta_D=0} = \frac{4\pi i}{g_D^2(a_D)} \rightarrow a = -\frac{i}{\pi} a_D(a) ln \frac{a_D(a)}{\Lambda} + \frac{i}{\pi} \tag{39}$$

out of which one can calculate the monodromy M_{u_0} and find the one from before.

Then M_{u_0} and M_{-u_0} generate together the group

$$\Gamma(2) = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sl(2,Z) | b = 0 \ \ mod \ \ 2\} \tag{40}$$

so the quantum moduli space is the upper half plane (due to the $u \leftrightarrow -u$ R-symmetry) divided by the $\Gamma(2)$ monodromy group,

$$\mathcal{M}_{qu} = H^+/\Gamma(2) \tag{41}$$

and we have reduced the problem of finding the solution to the low energy effective action (solving the model by finding $\mathcal{F}(\Psi)$) to the problem of finding the metric on moduli space, i.e. finding $a(u), a_D(u)$, such that there are the given monodromies at the singularities u_0, $-u_0$ and ∞, a purely mathematical problem.

Before we do that, let us comment on the physics. We have found singularities on the moduli space, due to states becoming massless. One should resolve these singularities by not integrating out the massless states. In our formalism, that will involve adding a term $\int d^8\theta \mathcal{H}(\Psi, \Psi^+)$ to the low energy effective action, that we saw is higher derivative. Everywhere except at the singularities, it should be neglected, but there it will be important.

Instead of that, what one must do near the singularity is go to a dual description, that will be smooth. Near the singularity where monopoles become massless, one should do an S duality and go to the dual description in terms of a_D (which is smooth) and $\mathcal{F}_D$ (remember, a_D is the VEV of the monopole superfield scalar ϕ_D).

So near singularities, one should go to a dual description in terms of the fields becoming massless at that point. That description will be smooth.

1.3 Solution of the theory by Riemann surface method

This is the method adopted by Seiberg and Witten. One takes an auxiliary Riemann surface≡ elliptic (complex) curve, which will have a moduli space of parameters $\mathcal{M}_{qu}$ and a period matrix $\tau(u)$, and is given by

$$y^2 = (x-1)(x+1)(x-u) \tag{42}$$

which is to be understood as a (x,y) curve, i.e. y(x). Then the x space is doubly covered and has branch points at ±1, u and ∞. So there is a branch cut between +1 and -1 and one between u and ∞. This space is nothing but a (complex) torus, with the independent torus cycles: a cycle surrounds +1 and -1 and the cut between them; b cycle surrounds +1 and u.

The first homology group= group of closed cycles on the surface,

$$V_u \equiv H^1(E_u, C) \tag{43}$$

where the surface E_u is a function of the parameter u, is generated by $\nu = n\alpha + m\beta$, where α and β are the a and b cycles of the torus.

A homology basis for V_u, $\gamma_i(\alpha,\beta)$, can be mapped to a basis for 1-forms on the space, λ_i, by

$$\oint_{\gamma_j} \lambda_i = \delta_i^j \tag{44}$$

One such basis in the case at hand is

$$\lambda_1 = \frac{dx}{y}; \quad \lambda_2 = \frac{xdx}{y} \tag{45}$$

where λ_1 is the unique holomorphic differential on the space. Then one can define

$$b_i = \oint_{\gamma_i'} \lambda_1 \tag{46}$$

called "period integrals" and

$$b_1/b_2 = \tau_u \tag{47}$$

called "period matrix", which has the property that $Im\tau_u > 0$. One can define

$$\begin{aligned} &\lambda = a_1(u)\lambda_1 + a_2(u)\lambda_2 \\ &a_D(u) \equiv \oint_{\gamma_1} \lambda \\ &a(u) \equiv \oint_{\gamma_2} \lambda \end{aligned} \tag{48}$$

and then if one chooses

$$\frac{d\lambda}{du} = f(u)\lambda_1 = f(u)\frac{dx}{y} \tag{49}$$

one gets

$$\frac{da_D}{da} = \tau_u \tag{50}$$

That means that if one fixes f such that the monodromies around the singularities (on the moduli space, not on the Riemann surface) obtained when u becomes +1, -1 or ∞ for $a(u), a_D(u)$ coincide with what we had before, we will get the correct $a(u), a_D(u)$, defined everywhere, and $\tau_u = \tau$ has $Im\tau > 0$, giving a correct solution of the theory. One finds

$$\begin{aligned} a(u) - \frac{\sqrt{2}}{\pi}\int_{-1}^{1}\frac{dx\sqrt{x-u}}{x^2-1} \\ a_D(u) = \frac{\sqrt{2}}{\pi}\int_{1}^{u}\frac{dx\sqrt{x-u}}{x^2-1} \end{aligned} \tag{51}$$

At the singularities, there is a vanishing cycle on the Riemann surface (torus), given by

$$\nu = n_m\beta + n_e\alpha \tag{52}$$

thus giving

$$\oint_\nu \lambda = 0 = n_m a_D + n_e a \tag{53}$$

Also, the intersection number of two general torus cycles (elements in the homology group) will be

$$\#(\nu_i, \nu_j) = n_m^i n_e^j - n_m^i n_e^i \in Z \tag{54}$$

giving the Dirac quantization condition (or DZS, rather).

1.4 Solution by differential equations (Picard-Fuchs)

Consider the Schrodinger-like proplem on the complex plane

$$[-\frac{d^2}{dz} + V(z)]\Psi(z) = 0 \tag{55}$$

If the potential V(z) has poles $z_1, ...z_p$ (and ∞), and is meromorphic at (i.e. has no monodromies around) them, it means that any set of two independent solutions to the above must satisfy also

$$\begin{pmatrix} \psi_a \\ \psi_2 \end{pmatrix} (x + e^{2\pi i}(z - z_i)) = M_i \begin{pmatrix} \psi_a \\ \psi_2 \end{pmatrix} (z) \tag{56}$$

i.e. there should be monodromy matrices acting on them.

For us, we want the functions $a(u), a_D(u)$, with certain monodromies at +1, -1, ∞. So we should invent a Schrodinger equation that gives them. Given the meromorphicity and singularity conditions, it can only be of the type

$$V(z) = \frac{\alpha}{(z+1)^2} + \frac{\beta}{(z-1)^2} + \frac{\gamma}{(z+1)(z-1)} \tag{57}$$

If we rename α, β, γ as

$$\alpha = \frac{1-\lambda_1^2}{4}; \quad \beta = \frac{1-\lambda_2^2}{4}; \quad \gamma = \frac{1-\lambda_1^2-\lambda_2^2-\lambda_3^2}{4} \tag{58}$$

The transformation

$$\psi(z) = (z+1)^{1/2(1-\lambda_1)}(z-1)^{1/2(1-\lambda_2)} f(\frac{z+1}{2}) \tag{59}$$

takes the equation to the hypergemoetrical equation

$$x(1-x)f''(x) + [c - (a+b+1)x]f'(x) - abf(x) = 0 \tag{60}$$

with

$$a = \frac{1-\lambda_1-\lambda_2+\lambda_3}{2}; \quad b = \frac{1-\lambda_1-\lambda_2-\lambda_3}{2}; \quad c = 1-\lambda_1 \tag{61}$$

With solutions as hypergeometric functions. From the asymptotic behaviour at +1, -1 and ∞ of these solution, matched over the known asymptotics of $a(u), a_D(u)$, one gets $\lambda_1 = \lambda_2 = 1, \lambda_3 = 0$. Thus

$$\begin{aligned} V(z) &= -\frac{1}{4}\frac{1}{(z+1)(z-1)} \\ a_D(u) &= i\psi_2(u) = i\frac{u-1}{2}F(\frac{1}{2},\frac{1}{2},2;\frac{1-u}{2}) \\ a(u) &= -2i\psi_1(u) = \sqrt{2}(u+1)^{1/2}F(-\frac{1}{2},\frac{1}{2},1;\frac{2}{u+1}) \end{aligned} \tag{62}$$

The connection to the solution in the previous subsection is given by the Picard-Fuchs equation, satisfied by the period integrals on a surface:

$$\tilde{\mathcal{L}}_{PF}\psi(u) = 0; \quad \tilde{\mathcal{L}}_{PF}(u) = (1-u^2)\partial_u^2 - 2u\partial_u - \frac{1}{4} \tag{63}$$

We can check explicitly that $a(u), a_D(u)$ satisfies this equation, but it is a general property. Then the Picard-Fuchs equation can be turned into the Schrodinger form (55).

1.5 Monopole condensation and confinement

To study confinement, which was one of the reasons for looking at Seiberg-Witten theory anyway, one should go to the $\mathcal{N}=1$ theory, that is expected to confine. One adds the soft breaking term (no quadratic divergencies) to the superpotential for the low energy abelian theory

$$mTr\Phi^2 \tag{64}$$

that breaks $\mathcal{N}=2$ susy to $\mathcal{N}=1$ susy. This abelian $\mathcal{N}=1$ gauge theory is believed to have confinement of electric charge, due to the fact that the dual (magnetic) photon becomes massive, via the dual Higgs effect. The vacuum Z_4 chiral symmetry is spontaneously broken to Z_2, so we have also chiral symmetry breaking. This story is expected thus to be the analog of the dual superconductor and the 't Hooft picture for the SU(N) gauge theory confinement, but in a supersymmetric setting. The dual Higgs is the scalar in the monopole superfield, and thus we have a "condensation of monopoles" (the monopole scalar acquires a VEV). Let's see how this happens.

The superpotential for the low energy theory (dual) monopole fields is expected to be of the form

$$W(M,\tilde{M}) = mf_a(\Phi_D) + M\tilde{M}f_2(\Phi_D) \tag{65}$$

As $m \to 0$, we expect the first term to be just the bare term (64), i.e. if we lift the low energy relation

$$u = < tr\phi^2 > \tag{66}$$

to a superfield $U(\Phi_D)$ in the low energy susy field theory, we expect $f_1(\Phi_D) = U(\Phi_D)$. We also expect that at m=0,

$$M\tilde{M}f_2(\Phi_D) = \sqrt{2}\Phi_D M\tilde{M} \tag{67}$$

which is just the $\mathcal{N} = 2$ invariant coupling of the dual photon with the monopole fields. Thus in the dual description, the $\mathcal{N} = 1$ superpotential is expected to be

$$W = \sqrt{2}\Phi_D M\tilde{M} + mU(\Phi_D) \tag{68}$$

(at least at small mass m). Then the vacuum is defined by $dW = 0$, i.e. by the absence of D terms. For m=0, one gets $M = \tilde{M} = 0$ and a_D arbitrary, thus the $\mathcal{N} = 2$ moduli space. At nonzero m, one gets

$$< M >=< \tilde{M} >= \sqrt{-m\frac{du}{da_D}|_{a_D=0}} \tag{69}$$

which is exactly what we advocated: the dual Higgs gets a VEV, which is a magnetic order parameter. As a consequence, the dual (magnetic) photon becomes massive (mass gap) and the abelian electric charge is confined, just like in the usual dual superconductor.

But unlike the 't Hooft story, the confinement is only for the abelian effective theory on the moduli space (low energy).