GRAVITY AND THE QUANTUM: SOME CONCEPTUAL CONSIDERATIONS

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Concepción, Chile, 06 November 2006

GRAVITATION AND UNIVERSALITY

At the classical level, gravitation shows a quite peculiar property

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Particles with different masses and different compositions feel it in such a way that all of them acquire the same acceleration

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Universality of Free Fall

$$\Updownarrow$$

Weak Equivalence Principle

GENERAL RELATIVITY

The weak equivalence principle allows the definition of the strong equivalence principle

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General Relativity, Einstein's theory for the gravitational field,
is fundamentally based on the Strong Equivalence Principle

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The gravitational interaction is geometrized $\rightarrow$ geometry replaces the concept of force

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The presence of a gravitational field is supposed to produce a *curvature* in spacetime

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A particle in a gravitational field simply follows a geodesics of the curved spacetime

Important

In the eventual lack of universality, the equivalence principles will be no longer valid,
and general relativity breaks down

EQUIVALENCE VERSUS UNCERTAINTY PRINCIPLES

General relativity and quantum mechanics are not consistent with each other

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The seeds of discord are the very principles on which these theories take their roots

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General relativity: is based on the equivalence principle $\Rightarrow$ **local**

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Quantum mechanics: is based on the uncertainty principle $\Rightarrow$ **nonlocal**

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Question: is there a peaceful way of reconciling the
equivalence and the uncertainty principles?

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Answer: seems to be **no** as these two principles are fundamentally different,
and like darkness and lightness, they cannot hold simultaneously

Crucial Question

Is it possible to discard one of these principles?

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Difficult, because general relativity and quantum mechanics are two of the main pillars of modern physics, and discarding one of their underlying principles would mean to discard one of these pillars

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However, a more careful analysis of this question strongly suggests that the equivalence principle
is the weaker part of the building, and may eventually be discarded

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In the Preface of his classic textbook, Synge confess that

... I have never been able to understand this Principle ... Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according to as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer's world line.

He then compares the equivalence principle with the office of a midwife, and completes ...

... I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced

Further Problems Related to the Equivalence Principles

Even if it is true at the classical level, it is not possible to construct a quantum version of the strong equivalence principle

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On the other hand, the inconsistency of quantum mechanics with the weak equivalence principle is a matter of experiment

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Although it has passed all experimental tests at the classical level, there are compelling evidences that the weak equivalence principle might not be true at the quantum level

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Example: Colella-Overhauser-Werner Experiment

CAN WE DISPENSE WITH THE EQUIVALENCE PRINCIPLE?

Good reason: there are compelling indications that gravitation is not universal at the quantum level

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The equivalence principle is no more applicable at this level

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However, without this principle, the geometrical description of general relativity breaks down

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A new question then arises: are we able to manage without the equivalence principle, and consequently without general relativity?

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The answer to this question is YES

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Let us then see how this is possible

A GAUGE FORMULATION FOR GRAVITATION

Like the other fundamental interactions, gravitation can be described by a gauge theory

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Teleparallel Gravity

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Gauge theory for the translation group

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In general relativity, **curvature** represents the gravitational field.
In teleparallel gravity, instead of curvature, **torsion** is assumed to represent the gravitational field

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In spite of this difference, the two theories are found to yield
equivalent classical descriptions of gravitation

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Gravitation can be described *alternatively* in terms of curvature, as in general relativity,
or in terms of torsion, in which case we have teleparallel gravity

CONCEPTUAL DIFFERENCES

In general relativity, curvature is used to *geometrize* the gravitational interaction

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In teleparallel gravity, torsion accounts for gravitation not by geometrizing the interaction,

but by acting as a *force*

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As a consequence, there are no geodesics in teleparallel gravity, but only force equations

quite similar to the Lorentz force equation of electrodynamics

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Similarly to Maxwell's theory, which is also a gauge theory,

teleparallel gravity is found not to require the equivalence principle to describe the gravitational interaction

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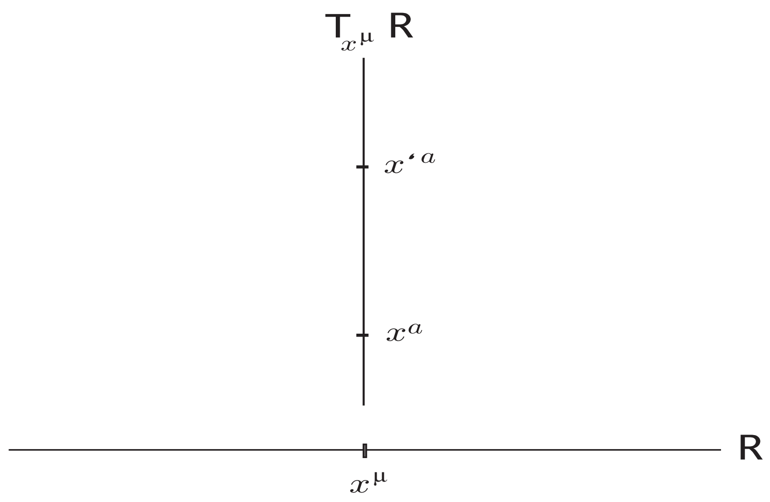
Newtonian mechanics does not require universality either

FUNDAMENTALS OF TELEPARALLEL GRAVITY

According to the gauge structure of teleparallel gravity, to each point of spacetime there is attached a Minkowski tangent space, on which the translation (gauge) group acts



- The Greek alphabet $\mu, \nu, \rho, \ldots$ denote spacetime indices
- The Latin alphabet $a, b, c, \ldots$ denote algebraic indices related to the tangent Minkowski spaces

$$\Downarrow$$

Gauge transformations are defined as local translations of the Minkowski tangent space coordinates:

$$x^a \rightarrow x'^a = x^a + \epsilon^a(x^\mu)$$

As a gauge theory, the fundamental field of teleparallel gravity is the translational gauge potential ${B^a}_\mu$, a 1-form assuming values in the Lie algebra of the translation group

$$B_\mu = {B^a}_\mu \, P_a$$

with $P_a = \partial/\partial x^a \to$ generators of infinitesimal translations

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Under a gauge transformation $\delta x^a = \epsilon^a(x) \equiv \epsilon^a$, the gauge potential transforms according to

$$B'^a{}_\mu = {B^a}_\mu - \partial_\mu \epsilon^a$$

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It appears naturally as the nontrivial part of the tetrad field ${h^a}_\mu$

$$h^a{}_\mu = \partial_\mu x^a + {B^a}_\mu$$

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If the tangent space indices are raised and lowered with the Minkowski metric η_{ab}, therefore, the spacetime indices will raised and lowered with the spacetime metric

$$g_{\mu\nu} = \eta_{ab} \, {h^a}_\mu \, {h^b}_\nu$$

The fundamental connection of teleparallel gravity is the

Weitzenböck Connection

$$\overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} = h_a{}^{\rho} \partial_\nu h^a{}_\mu$$

$$\Downarrow$$

It is a connection presenting torsion, but no curvature:

$$\overset{\bullet}{T}{}^{\rho}{}_{\mu\nu} \equiv \overset{\bullet}{\Gamma}{}^{\rho}{}_{\nu\mu} - \overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} \neq 0$$

$$\overset{\bullet}{R}{}^{\lambda\rho}{}_{\mu\nu} = 0$$

$$\Downarrow$$

Torsion coincides with the translational field strength $\overset{\bullet}{T}{}^{a}{}_{\mu\nu}$:

$$\overset{\bullet}{T}{}^{a}{}_{\mu\nu} \equiv \partial_\mu B^a{}_\nu - \partial_\nu B^a{}_\mu = h^a{}_\rho \, \overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}$$

LAGRANGIAN AND FIELD EQUATIONS

$$\overset{\bullet}{\mathcal{L}} = \frac{h}{8k^2}\left[\overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}\overset{\bullet}{T}{}_{\rho}{}^{\mu\nu} + 2\,\overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}\overset{\bullet}{T}{}^{\nu\mu}{}_{\rho} - 4\,\overset{\bullet}{T}{}_{\rho\mu}{}^{\rho}\overset{\bullet}{T}{}^{\nu\mu}{}_{\nu}\right]$$

$$\Downarrow$$

$$\Longrightarrow \quad k^2 = 8\pi G/c^4 \quad \text{and} \quad h \equiv \sqrt{-g} \quad \text{with } g = \det(g_{\mu\nu})$$

$$\Downarrow$$

The teleparallel Lagrangian can be rerwritten in the form

$$\overset{\bullet}{\mathcal{L}} = \frac{h}{4k^2}\,\overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}\,\overset{\bullet}{S}{}_{\rho}{}^{\mu\nu}$$

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$\overset{\bullet}{S}{}^{\rho\mu\nu} \Rightarrow$ superpotential: an appropriate combination of the torsion tensor

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The corresponding field equation is

$$\partial_\sigma(h\overset{\bullet}{S}{}_{\lambda}{}^{\rho\sigma}) - k^2\,(h\overset{\bullet}{t}{}_{\lambda}{}^{\rho}) = 0$$

$$\Downarrow$$

$h\,\overset{\bullet}{t}{}_{\lambda}{}^{\rho} \Rightarrow$ is the energy-momentum *pseudotensor* of the gravitational field

EQUIVALENCE WITH GENERAL RELATIVITY

The Einstein-Hilbert Lagrangian of general relativity is

$$\overset{\circ}{\mathcal{L}} = -\frac{\sqrt{-g}}{2k^2}\ \overset{\circ}{R}$$

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It is related to the teleparallel Lagrangian through

$$\overset{\circ}{\mathcal{L}} = \overset{\bullet}{\mathcal{L}} + \partial_\mu \left(2\, h\, k^{-2}\, \overset{\bullet}{T}{}^{\nu\mu}{}_{\nu}\right)$$

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Up to a divergence, therefore, the teleparallel Lagrangian is equivalent to the Einstein–Hilbert Lagrangian of general relativity

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Due to the equivalence between the corresponding Lagrangians, the field equations coincide:

$$\partial_\sigma(h \overset{\bullet}{S}{}_\lambda{}^{\rho\sigma}) - k^2\,(h \overset{\bullet}{t}{}_\lambda{}^{\rho}) \equiv h\left(\overset{\circ}{R}{}_\lambda{}^{\rho} - \tfrac{1}{2}\,\delta_\lambda{}^{\rho}\ \overset{\circ}{R}\right)$$

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In spite of the conceptual differences, teleparallel gravity and general relativity yield equivalent descriptions of the gravitational interaction

GEODESICS VERSUS FORCE EQUATION

Let us consider the motion of a spinless particle of mass m in a gravitational field $B^a{}_\mu$
Analogously to the electromagnetic case, the action integral is

$$\mathcal{S} = \int_a^b \left[-m\,c\,d\sigma - m\,c\,B^a{}_\mu\,u_a\,dx^\mu \right]$$

- $d\sigma = (\eta_{ab} dx^a dx^b)^{1/2}$ is the Minkowski invariant interval
- $u^a = h^a{}_\mu\,u^\mu$ is the anholonomic particle four-velocity
- $u^\mu = dx^\mu/ds$ is the particle four-velocity
- $ds = (g_{\mu\nu} dx^\mu dx^\nu)^{1/2}$ is the spacetime invariant interval

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The first term of the action represents the action of a free particle, and the second the coupling of the particle with gravitation

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Notice that the separation of the action in these two terms is possible only in a gauge theory, like teleparallel gravity, being not possible in general relativity

Variation of the action yields the equation of motion

$$\frac{du_\mu}{ds} - \overset{\bullet}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = \overset{\bullet}{T}{}^{\theta}{}_{\mu\rho}\, u_\theta\, u^\rho$$

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This is the force equation governing the motion of the particle in teleparallel gravity

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Torsion $\overset{\bullet}{T}{}^{\rho}{}_{\mu\rho}$ plays the role of gravitational force

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It can be rewritten in terms of the Christoffel connection $\overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}$,
in which case it becomes the geodesic equation of general relativity:

$$\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = 0$$

$\Downarrow$

The force equation of teleparallel gravity and the geodesic equation of general relativity
yields the same physical trajectories. In fact, it can be shown that

$$\mathcal{S} = \int_a^b \left[-m\, c\, d\sigma - m\, c\, B^a{}_\mu\, u_a\, dx^\mu\right] = -m\, c \int_a^b ds$$

DOING WITHOUT THE EQUIVALENCE PRINCIPLE

If the gravitational (m_g) and inertial (m_i) masses are different, gravitation is no more universal,

and consequently general relativity breaks down

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Teleparallel gravity, however, remains as a consistent theory. The particles's action, in this case, is

$$\mathcal{S} = \int_a^b \left(-m_i \, c \, d\sigma - m_g \, c \, B^a{}_\mu \, u_a \, dx^\mu \right)$$

$\Downarrow$

Variation of this action yields the equation of motion

$$\left(\partial_\mu x^a + \frac{m_g}{m_i} \; B^a{}_\mu \right) \frac{du_a}{ds} = \frac{m_g}{m_i} \; \overset{\bullet}{T}{}^a{}_{\mu\rho} \, u_a \, u^\rho$$

$\Downarrow$

If the particle has in addition an electric charge q

$$\left(\partial_\mu x^a + \frac{m_g}{m_i} \; B^a{}_\mu \right) \frac{du_a}{ds} = \frac{m_g}{m_i} \; \overset{\bullet}{T}{}^a{}_{\mu\rho} \, u_a \, u^\rho + \frac{q}{m_i} \; F_{\mu\rho} \, u^\rho$$

$\Downarrow$

Similarly to the electromagnetic Lorentz force, which depends on the relation q/m_i, the gravitational force depends explicitly on the relation m_g/m_i of the particle

CONCLUSIONS

One of the fundamental problems of gravitation is the inconsistency of Einstein's general relativity with quantum mechanics

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The conceptual reason behind such inconsistency is related to the very principles on which these theories are based:

- General relativity is based on the equivalence principle, which is essentially *local*
- Quantum mechanics is based on the uncertainty principle, which is a *nonlocal* principle

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On this fundamental difference lies one of the roots of the difficulty in reconciling these two theories

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The replacement of general relativity by the gauge approach of teleparallel gravity

- **Means that we can dispense with the equivalence principle**
- **May lead to a conceptual reconciliation of gravity with quantum mechanics**

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Colella-Overhauser-Werner (COW) phenomenon

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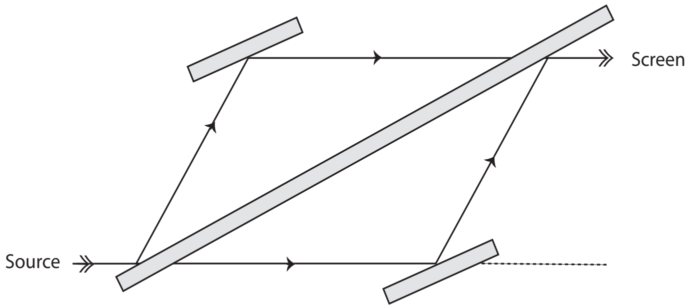
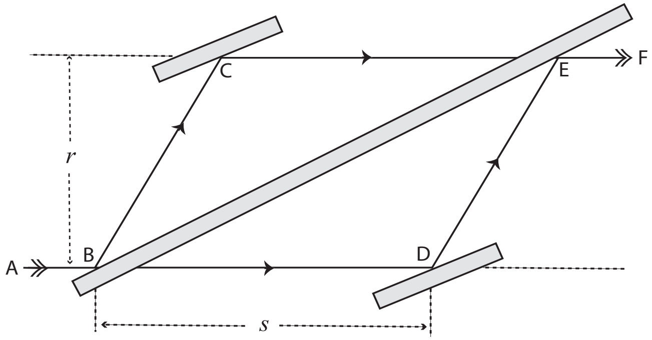
It consists in using a neutron interferometer to observe the quantum mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field



The gravitational field is assumed to be Newtonian:

$$\phi \equiv g\, z$$

- g is the gravitational acceleration, supposed not to change significantly in the region of the experience
- z is the distance from some reference point



Since the segments BD and CE are at different distance from Earth, and consequently at different value of the gravitational potential ϕ, there will be a gravitationally induced quantum phase-shift between the two trajectories

$$\delta\varphi \equiv \varphi_{\mathsf{BCE}} - \varphi_{\mathsf{BDE}} = \frac{grs}{\hbar v}\, m$$

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The quantum phase difference induced by the gravitational field explicitly depends on the mass of the particle

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At the quantum level, therefore, due to this dependence, gravitation seems to be no more universal

Gravitational Aharonov-Bohm Effect

The usual (electromagnetic) Aharonov-Bohm effect consists in a shift, by a constant amount, of the electron interferometry wave pattern, in a region where there is no magnetic field, but there is a nontrivial electromagnetic potential

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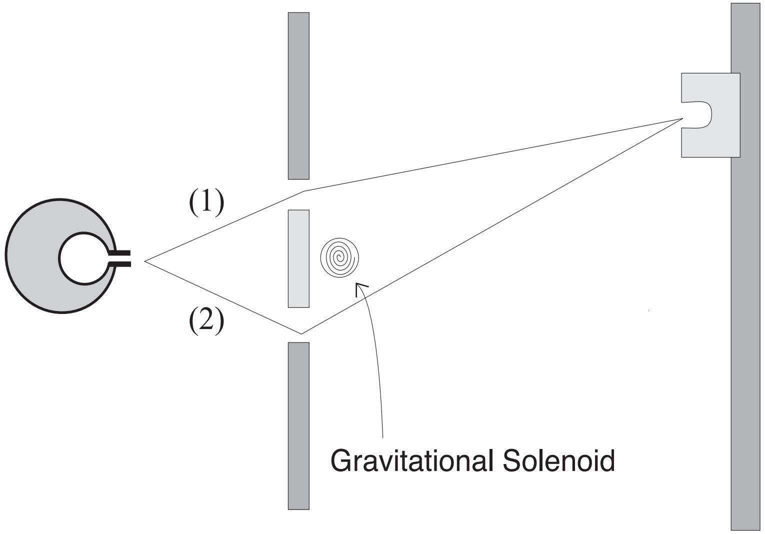
Analogously, the gravitational Aharonov-Bohm effect will consist in a similar shift of the same wave pattern, but produced by the presence of a gravitational potential, in a region where there is no gravitational field



The infinite "gravitational solenoid" produces a gravitomagnetic field flux Ω concentrated in its interior. In the ideal situation, the gravitational field outside the solenoid vanishes completely, but there is a nontrivial gravitational potential

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When the electrons move outside the solenoid, phase factors corresponding to paths lying on different sides of the solenoid will interfere, which produces an additional phase shift at the screen

$$\delta\varphi \equiv \varphi_{(2)} - \varphi_{(1)} = \frac{\mathcal{E}\,\Omega}{\hbar\,c}$$

- $\mathcal{E} = \gamma m c^2$ is the electron kinetic energy
- $\gamma \equiv [1 - (v^2/c^2)]^{-1/2}$ is the relativistic factor

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Also in the gravitational Aharonov-Bohm effect, the phase shift depends on the mass of the particle

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This is one more indication that, at the quantum level, gravitation seems to be no more universal.