In Search of the Spacetime Torsion

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Gravitation and Universality

At the classical level, gravitation shows a quite peculiar property

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Particles with different masses and different compositions feel it in such a way that all of them acquire the same acceleration

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Universality of Free Fall

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Weak Equivalence Principle

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Since the inertial forces are also universal, this allows the introduction of the so called

Strong Equivalence Principle

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Establishes the **local** equivalence between inertia and gravitation

GENERAL RELATIVITY

General Relativity, Einstein's theory for the gravitational field,
is fundamentally based on the Strong Equivalence Principle

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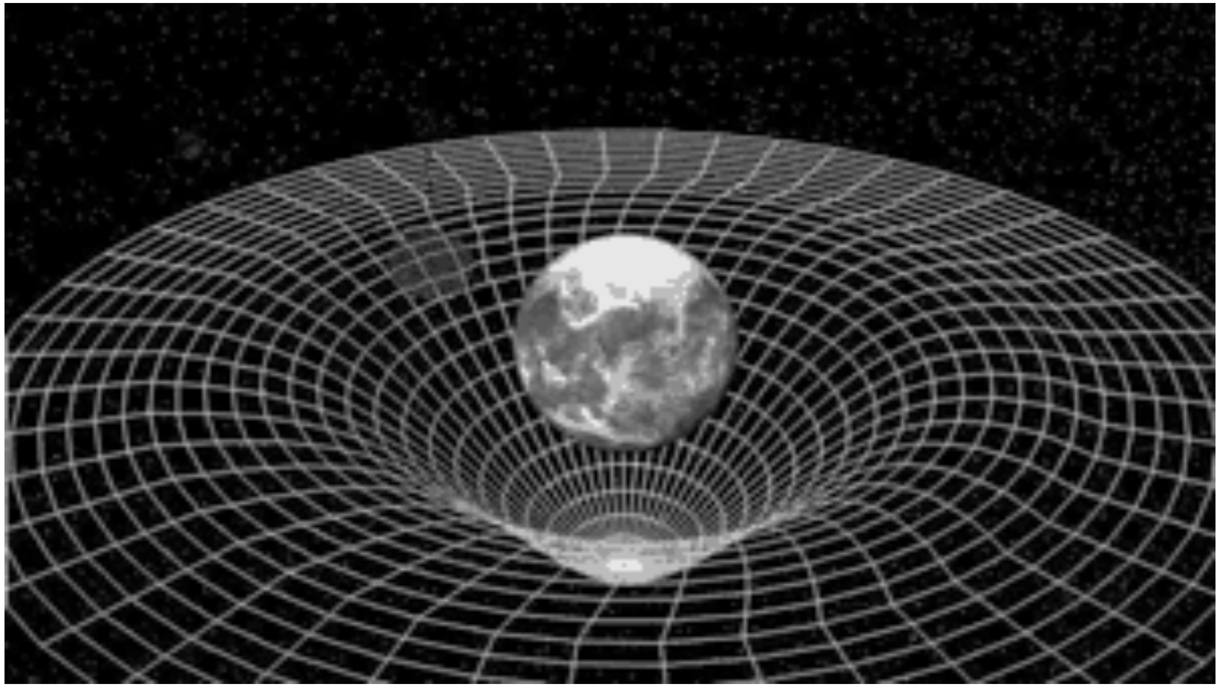
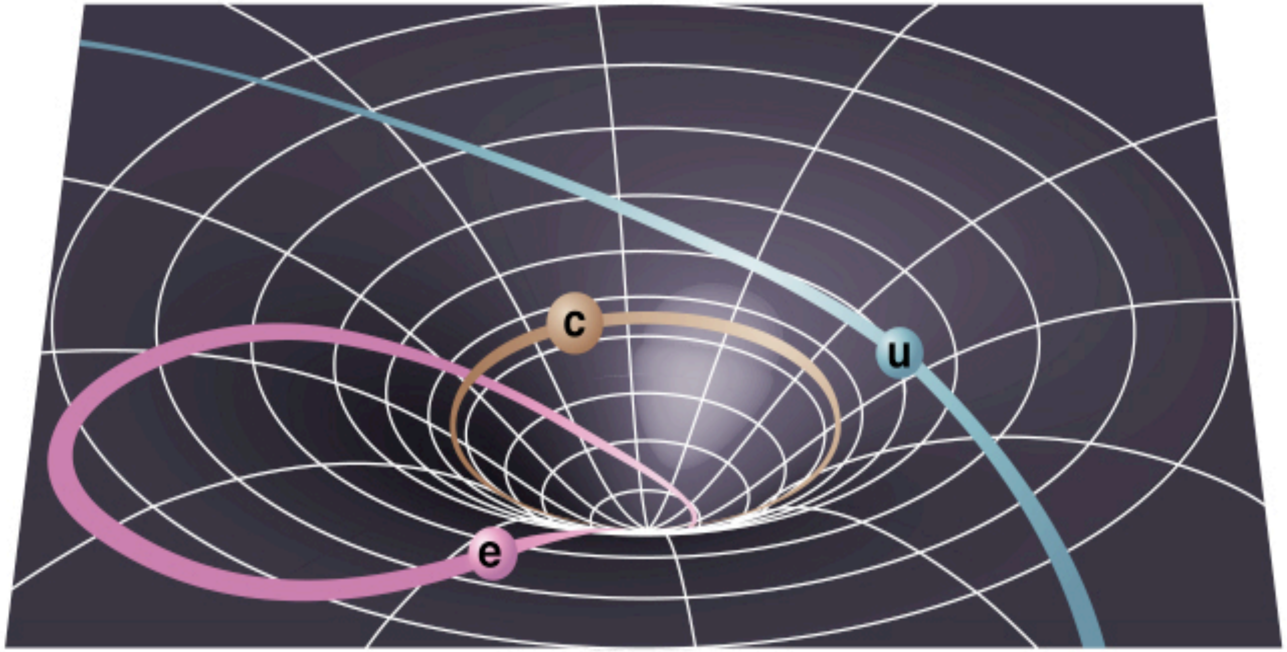
To comply with universality, the presence of a gravitational field
is supposed to produce a *curvature* in spacetime



Geometric Description of the Interaction

A particle in a gravitational field simply follows the geodesics of the curved spacetime



In general Relativity, geometry replaces the concept of force

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The responsibility of describing the gravitational interaction is transferred to spacetime

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There is no the concept of "gravitational force"

Equation of Motion

The fundamental connection of **general relativity** is the **Christoffel Connection**

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\, g^{\rho\lambda}(\partial_{\mu} g_{\lambda\nu} + \partial_{\nu} g_{\lambda\mu} - \partial_{\lambda} g_{\mu\nu})$$

$$\Downarrow$$

It is a connection with vanishing torsion, but non-vanishing curvature:

$$\overset{\circ}{T}{}^{\rho}{}_{\nu\mu} \equiv \overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\circ}{\Gamma}{}^{\rho}{}_{\nu\mu} = 0$$

$$\overset{\circ}{R}{}^{\rho}{}_{\lambda\nu\mu} \equiv \partial_{\nu}\overset{\circ}{\Gamma}{}^{\rho}{}_{\lambda\mu} - \partial_{\mu}\overset{\circ}{\Gamma}{}^{\rho}{}_{\lambda\nu} + \overset{\circ}{\Gamma}{}^{\rho}{}_{\eta\nu}\overset{\circ}{\Gamma}{}^{\eta}{}_{\lambda\mu} - \overset{\circ}{\Gamma}{}^{\rho}{}_{\eta\mu}\overset{\circ}{\Gamma}{}^{\eta}{}_{\lambda\nu} \neq 0$$

In general relativity, the equation of motion of a test particle is given by the

Geodesic Equation

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$$\boxed{\frac{du_{\mu}}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_{\theta}\, u^{\nu} = 0}$$

WHAT ABOUT TORSION?

Why should matter (energy and momentum) produce **only curvature** in spacetime?

Was Einstein wrong when he made this assumption?

Does torsion play any role in gravitation?

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There are TWO different answers to these questions

First Possibility

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EINSTEIN-CARTAN TYPE THEORIES

(including gauge theories for the Poincaré group and other more general groups)

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The underlying spacetimes of these models present both curvature and torsion

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Energy and momentum $\longrightarrow$ Source of Curvature

Spin of matter $\longrightarrow$ Source of Torsion

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According to these theories, **curvature and torsion** represent independent degrees of freedom of the gravitational field

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At the macroscopic level, where spins vanish, they coincide with general relativity

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At the microscopic level, where spins are relevant they show different results from general relativity

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These theories presuppose new physics (or phenomena) associated to torsion

Second Possibility

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TELEPARALLEL GRAVITY

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Gauge Theory for the Translation Group

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- In general relativity, **curvature** represents the gravitational field
- In teleparallel gravity, **torsion** represents the gravitational field

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In spite of this difference, the two theories are found to yield

Equivalent Descriptions of Gravitation

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Torsion appears as an alternative to curvature

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Gravitation can be described *alternatively* in terms of curvature, as in general relativity, or in terms of torsion, in which case we have teleparallel gravity

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Teleparallel gravity does not presupposes new physics associated with torsion

A Glimpse on Teleparallel Gravity

The fundamental connection of **teleparallel gravity** is the **Weitzenböck Connection**

$$\overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} = h_a{}^{\rho}\,\partial_\nu h^a{}_\mu$$

$$\Downarrow$$

It is a connection with non-vanishing torsion, but vanishing curvature:

$$\overset{\bullet}{T}{}^{\rho}{}_{\nu\mu} \equiv \overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\bullet}{\Gamma}{}^{\rho}{}_{\nu\mu} \neq 0$$

$$\overset{\bullet}{R}{}^{\lambda\rho}{}_{\nu\mu} \equiv \partial_\nu \overset{\bullet}{\Gamma}{}^{\rho}{}_{\lambda\mu} - \partial_\mu \overset{\bullet}{\Gamma}{}^{\rho}{}_{\lambda\nu} + \overset{\bullet}{\Gamma}{}^{\rho}{}_{\eta\nu} \overset{\bullet}{\Gamma}{}^{\eta}{}_{\lambda\mu} - \overset{\bullet}{\Gamma}{}^{\rho}{}_{\eta\mu} \overset{\bullet}{\Gamma}{}^{\eta}{}_{\lambda\nu} = 0$$

Fundamental Relation

The Weitzenböck and the Christoffel connections are related by

$$\boxed{\overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} + \overset{\bullet}{K}{}^{\rho}{}_{\mu\nu}}$$

$$\Downarrow$$

$$\overset{\bullet}{K}{}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\left(\overset{\bullet}{T}{}_{\mu}{}^{\rho}{}_{\nu} + \overset{\bullet}{T}{}_{\nu}{}^{\rho}{}_{\mu} - \overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}\right) \quad \rightarrow \quad \text{Contortion Tensor}$$

Geodesics Versus Force Equation

Let us consider the geodesic equation of general relativity

$$\boxed{\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^\theta{}_{\mu\nu}\, u_\theta\, u^\nu = 0}$$

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Using the relation between the Weitzenböck and the Christoffel connections

$$\overset{\circ}{\Gamma}{}^\rho{}_{\mu\nu} = \overset{\bullet}{\Gamma}{}^\rho{}_{\mu\nu} - \overset{\bullet}{K}{}^\rho{}_{\mu\nu}$$

the geodesic equation can be written in the alternative form

$$\boxed{\frac{du_\mu}{ds} - \overset{\bullet}{\Gamma}{}^\theta{}_{\mu\nu}\, u_\theta\, u^\nu = \overset{\bullet}{T}{}^\theta{}_{\mu\nu}\, u_\theta\, u^\nu}$$

This is a **force equation**, with torsion playing the role of **gravitational force**

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The geodesic equation of general relativity and the force equation of teleparallel gravity yields the same physical trajectory

Although equivalent ...

... there are conceptual differences between GR and TG

In general relativity, curvature is used to **geometrize** the gravitational interaction

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In teleparallel gravity, torsion accounts for gravitation, not by geometrizing the interaction, but by acting as a **force**

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As a consequence, there are no geodesics in teleparallel gravity, but only force equations

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This is similar to Maxwell's theory, in which the interaction of a charged particle with the electromagnetic field is described by the Lorentz force

Why Gravitation Presents Two Alternative Descriptions?

Like any other interaction of nature, gravitation has a description in terms of a gauge theory

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Teleparallel gravity is a gauge theory for the translation group

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On the other hand, **universality of gravitation** allows an alternative description in terms of a geometrization of spacetime

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General Relativity is a geometric theory for gravitation

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Universality of the gravitational interaction is then the responsible for the existence of the alternative geometric description

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Only a universal interaction can present a geometric description

DISCUSSION

According to **generalizations of general relativity** (like, for example, Einstein-Cartan and gauge theories for the Poincaré group), torsion is supposed to represent additional degrees of freedom of gravity, and consequently there might be new physics associated to its presence

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If this is true, Einstein made a mistake when he did not include torsion in general relativity

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However, from the point of view of **teleparallel gravity**, torsion does not represent additional degrees of freedom, but simply an alternative to curvature in the description of gravitation

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In this case, no new physics is associated with torsion, which means essentially that general relativity is a complete theory

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If this is true, then Einstein was right when he did not include torsion in general relativity

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Which of these interpretations is the correct one?

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The answer to this question can only be given by experiment

Experimental Status

There are no experimental data on the coupling of angular momentum, either orbital or spinorial, with gravitation

• Concerning **new gravitational effects** produced by the coupling of torsion with **rotation**, there has been recently a proposal to look for these effects using the data of Gravity Probe B

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Yi Mao, Max Tegmark, Alan Guth and Serkan Cabi
Constraining Torsion with Gravity Probe B (*Preprint* gr-qc/0608121)

• Concerning **new gravitational effects** produced by the coupling of torsion with **spin**, they could eventually be probed near a neutron star — like a binary pulsar, for example — where a macroscopic spin might be present

Up to now, however, no evidences for these phenomena have ever been reported

Final Message ...

There is a widespread belief according to which torsion is related to the spin of matter.

Although this could be true, it is not the only possibility

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Teleparallel gravity: curvature and torsion are simply alternative ways
of describing the very same gravitational field

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From this point of view: torsion has already been detected!

It is the responsible for all gravitational effects, including the physics of the solar system,
which can be reinterpreted in terms of a force equation, with torsion playing the role of force

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According to teleparallel gravity, there are no new effects associated with torsion.
The search for these effects, therefore, is a nonsense

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**We can say that the existing experimental data
favour the teleparallel point of view**

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**Of course, this is not a definitive result.
The ultimate answer can only be given by further experiments**

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