Campo escalar interagente $(\lambda\phi^4)$

$$\mathcal{L} = \mathcal{L}_0 - \frac{\lambda}{4!}\phi^4 \qquad \mathcal{L}_0 = \frac{1}{2}\partial_\nu\phi\,\partial^\nu\phi - \frac{1}{2}m^2\phi^2$$

Para o campo livre, fizemos a transição de 21.2 para 22.1 :

$$\boxed{Z_0[J] = \frac{\int D\phi\, \exp\left\{-i\int d^4x\left[\frac{1}{2}\phi\left(\Box + m^2 - i\varepsilon\right)\phi - \phi J\right]\right\}}{\int D\phi\, \exp\left\{-i\int d^4x\left[\frac{1}{2}\phi\left(\Box + m^2 - i\varepsilon\right)\phi\right]\right\}}} \Longrightarrow \boxed{Z_0[J] = \exp\left\{-\frac{i}{2}\int d^4x\, d^4y\; J(x)\,\Delta_F(x-y)\,J(y)\right\}}$$

A expressão da direita é mais conveniente para fazermos a derivação funcional em J. Gostaríamos de conseguir algo equivalente para a lagrangeana com interação.

$$Z[J] = \frac{\int D\phi\, \exp\left\{i\int d^4x\left[\mathcal{L} + J\phi\right]\right\}}{\int D\phi\, \exp\left\{i\int d^4x\,\mathcal{L}\right\}} \Longrightarrow\ ??$$

(essencialmente precisamos separar ϕ de J)

$Z_0[J] \equiv$ func. gerador p/ campos livres

$Z[J] \equiv$ func. gerador p/ campos com interação

Vamos encontrar e resolver (em função de Z_0) uma equação diferencial para Z:

$$\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} = \frac{\int D\phi\,\phi(x)\, \exp\left\{i\int d^4x\left[\mathcal{L} + J\phi\right]\right\}}{\int D\phi\, \exp\left\{i\int d^4x\,\mathcal{L}\right\}} \qquad \text{(eq. 28.1)}$$

$$\boxed{\frac{\delta}{\delta J(x)}\, i\int d^4x'\, J(x')\,\phi(x') = i\,\phi(x)}$$

$$\boxed{\hat{Z}[\phi] \equiv \frac{e^{i\int d^4x\,\mathcal{L}(\phi)}}{\int D\phi'\, e^{i\int d^4x\,\mathcal{L}(\phi')}}}$$

$$\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} = \int D\phi\,\phi(x)\,\hat{Z}[\phi]\,\exp\left[i\int d^4x\, J\phi\right] \qquad \text{(eq. 28.2)}$$

$$Z[J] = \int D\phi\;\; \hat{Z}[\phi]\; \exp\left[i\int d^4x'\; J(x')\,\phi(x')\right]$$

(isto é basicamente a transformada de Fourier para funcionais)

$$i\frac{\delta}{\delta\phi(x)}\hat{Z}[\phi] = i\frac{\delta}{\delta\phi(x)} \mathrm{EXP}\Big[-i\int d^4x\, \underbrace{\left(\tfrac{1}{2}\phi(\Box+m^2)\phi\right.}_{} - \mathcal{L}_{INT}\Big)\Big]\left[\int D\phi'\, e^{i\int d^4x\,\mathcal{L}(\phi')}\right]^{-1} =$$

(reescrevemos a lagrangeana como no começo da pag 17)

$$= \hat{Z}[\phi]\,\frac{\delta}{\delta\phi(x)}\left[\int d^4x\left(\tfrac{1}{2}\phi(\Box+m^2)\phi - \mathcal{L}_{INT}\right)\right]$$

$$\frac{\delta}{\delta\phi(y)}\int \tfrac{1}{2}\phi(x)(\Box+m^2)\phi(x)\,d^4x =$$

$$= \frac{\delta}{\delta\phi(y)}\int \tfrac{1}{2}m^2\phi^2(x)\,d^4x + \frac{\delta}{\delta\phi(y)}\int\tfrac{1}{2}\phi(x)\,\partial_\mu\partial^\mu\phi(x)\,d^4x =$$

(usamos a propriedade do ex5, pag 10)

$$= m^2\phi(y) + \int\tfrac{1}{2}\delta(x-y)\,\partial_\mu\partial^\mu\phi(x)\,d^4x + \int\tfrac{1}{2}\phi(x)\frac{\delta}{\delta\phi(y)}\left[\partial_\mu\partial^\mu\phi(x)\right]d^4x =$$

$$= m^2\phi(y) + \tfrac{1}{2}\Box\phi(y) + \tfrac{1}{2}\underbrace{\int\phi(x)\,\partial_\mu\partial^\mu\left[\frac{\delta\phi(x)}{\delta\phi(y)}\right]d^4x}_{} =$$

podemos usar integração por partes (duas vezes) para transferir as derivadas para $\phi(x)$

$$= (\Box+m^2)\phi(y)$$

$$\frac{\delta}{\delta\phi(y)}\left[-\int d^4x\,\mathcal{L}_{INT}(\phi(x))\right] = -i\int d^4x\,\delta(x-y)\left.\frac{\partial\mathcal{L}_{INT}(\phi)}{\partial\phi}\right|_{\phi=\phi(x)} =$$

$$= -i\left.\frac{\partial\mathcal{L}_{INT}(\phi)}{\partial\phi}\right|_{\phi=\phi(y)} = \mathcal{L}'_{INT}(\phi(y))$$

$$i\frac{\delta}{\delta\phi(x)}\hat{Z}[\phi] = \hat{Z}[\phi]\left[(\Box+m^2)\phi(x) - \mathcal{L}'_{INT}(\phi(x))\right]$$

Multiplicando os dois lados desta equação por: $\mathrm{EXP}\left[i\int J(x)\phi(x)\,d^4x\right]$

e integrando em ϕ:

$$\overbrace{\int \mathcal{D}\phi \; i \frac{\delta \hat{Z}[\phi]}{\delta \phi(x)} \, \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right]}^{LHS} =$$

$$= \underbrace{\int \mathcal{D}\phi \; \hat{Z}[\phi] \left[(\Box + m^2)\phi(x) - \mathcal{L}'_{INT}(\phi(x)) \right] \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right]}_{RHS}$$

RHS =

$$= (\Box + m^2) \underbrace{\int \mathcal{D}\phi \; \hat{Z}[\phi]\, \phi(x)\, \text{EXP}[\;]}_{\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)}} - \int \mathcal{D}\phi \; \underbrace{\hat{Z}[\phi]}_{\text{NÃO TEM } J} \underbrace{\mathcal{L}'_{INT}(\phi(x))\, \text{EXP}[\;]}_{*}$$

(conforme eq. 28.2)

$$\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} = \int \mathcal{D}\phi\, \phi(x)\, \hat{Z}[\phi]\, \text{EXP}\left[i\int d^4x\, J\phi \right]$$

Aqui aparece o passo fundamental

$$* \quad \mathcal{L}'_{INT}(\phi(x))\, \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right] = \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J} \right) \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right]$$

$$\text{Ex:} \quad \frac{\lambda}{4!}\phi^4(x) \longrightarrow \frac{\lambda}{4!}\left(\frac{1}{i}\right)^4 \frac{\delta^4}{\delta J(x)^4}$$

$$\frac{\lambda}{4!}\left(\frac{1}{i}\right)^4 \frac{\delta^4}{\delta J(x)^4} \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right] = \frac{\lambda}{4!}\, i\, \phi(x) \frac{\delta^3}{\delta J(x)^3} \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right] =$$

$$= -\frac{\lambda}{4!}\phi^2(x)\frac{\delta^2}{\delta J(x)^2} \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right] = -\frac{i\lambda}{4!}\phi^3(x)\frac{\delta}{\delta J(x)} \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right] =$$

$$= \frac{\lambda}{4!}\phi^4(x)\, \text{EXP}\left[i\int J(x)\phi(x)\, d^4x \right]$$

... voltando à conta:

$$RHS = (\Box + m^2)\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J} \right) \underbrace{\int \mathcal{D}\phi\; \hat{Z}[\phi]\, \text{EXP}[\;]}_{Z[J]} =$$

$$= (\Box + m^2)\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) Z[J]$$

$$LHS = \int D\phi \; i \frac{\delta}{\delta \phi(x)} \hat{Z}[\phi] \, EXP\left[i \int J(x)\phi(x)\, d^4x \right] =$$

$\to$ INTEGRAL POR PARTES

$$= i \, \hat{Z}[\phi] \, EXP\left[i \int J(x)\phi(x)\, d^4x \right] \Big| - i \int \underbrace{D\phi \; iJ(x)\hat{Z}[\phi] \, EXP\left[i \int J(x)\phi(x)\, d^4x \right]}_{\to Z[J]}$$

O

Este é um termo de superfície no espaço dos campos (não no espaço de Minkowsky) pensando que estamos sempre fazendo a conta com $\mathcal{H} \to (1 + i\epsilon)\mathcal{H}$ e este termo desaparecerá na "borda" $(\phi \to \infty)$

$$= J(x) Z[J]$$

Finalmente:

$$(\Box + m^2)\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) Z[J] = J(x) Z[J]$$

(eq. 31.1)

$\downarrow$ $\mathcal{L}_{INT} = 0$

$$(\Box + m^2)\frac{1}{i}\frac{\delta Z_0[J]}{\delta J(x)} = J(x) Z_0[J]$$

(eq. 31.2)

Queremos resolver 31.1. Provaremos que a solução é:

$$\boxed{Z[J] = N \, EXP\left\{ i \int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) dx \right\} Z_0[J]}$$

(eq. 31.3)

$$J(x)Z[J] = N\, J(x)\, \mathrm{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right] Z_0[J] =$$

$$\exp\left[-i\int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right] J(x) \exp\left[+i\int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right]$$

$$= J(x) - \mathcal{L}'_{int}\left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)$$

(provado na pag 199 do Ryder)

$$= N\, \mathrm{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right]\left[J(x) - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)\right] Z_0[J] =$$

eq. 31.2 $\quad (\Box+m^2)\frac{1}{i}\frac{\delta Z_0[J]}{\delta J(x)} = J(x)Z_0[J]$

$$= N\, \mathrm{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right]\left[(\Box+m^2)\frac{1}{i}\frac{\delta}{\delta J(x)} - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)\right] Z_0[J] =$$

$$Z[J] = N\, \mathrm{EXP}\left\{i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)dx\right\} Z_0[J]$$

$$= (\Box+m^2)\frac{1}{i}\frac{\delta Z[J]}{\delta J(x)} - \mathcal{L}'_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right) Z[J]$$

Em suma, para teorias interagentes, usaremos o funcional gerador (normalizado segundo o mesmo critério usado para o caso livre):

$$\boxed{Z[J] = \frac{\mathrm{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right]\mathrm{EXP}\left[-\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)\,dx\,dy\right]}{\left\{\mathrm{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(y)}\right)dy\right]\mathrm{EXP}\left[-\frac{i}{2}\int J(x)\Delta_F(x-y)J(y)\,dx\,dy\right]\right\}\Big|_{J=0}}}$$

(eq. 32.1)

Expansão perturbativa para $\lambda\phi^4$:

$$\mathcal{L}_{INT} = -\frac{\lambda}{4!}\phi^4$$

Só conseguimos tratar a exponencial de L_{int} que aparece na eq 32.1 por meio de uma série (que podemos truncar se λ é pequeno). Essa é a expansão perturbativa no formalismo funcional:

$$\text{EXP}\left[i\int \mathcal{L}_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)dz\right] =$$

$$= \text{Exp}\left[-\frac{i\lambda}{4!}\int\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4 dz\right] \simeq 1 - \frac{i\lambda}{4!}\int\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4 dz + \mathcal{O}(\lambda^2)$$

$$\frac{1}{i}\frac{\delta}{\delta J(z)}\text{EXP}\Big[-\frac{i}{2}\underbrace{\int J(x)\Delta_F(x-y)J(y)dxdy}_{I}\Big] = -\int \Delta_F(z-x)J(x)dx\, e^{-\frac{i}{2}I}$$

$$\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^2 e^{-\frac{i}{2}I} = \Big(i\Delta_F(\underbrace{z-z}_{?!}) + \int\Delta_F(z-x)J(x)dx\int\Delta_F(z-x_2)J(x_2)dx_2\Big)e^{-\frac{i}{2}I}$$

$$= \left[i\Delta_F(0) + \left(\int\Delta_F(z-x)J(x)dx\right)^2\right]e^{-\frac{i}{2}I}$$

$$\boxed{\frac{1}{i}\frac{\delta}{\delta J(z)}\left(\int\Delta_F(z-x)J(x)dx\right)^n e^{-\frac{i}{2}I} = \left\{-ni\Delta_F(0)\left(\int\Delta_F(z-x)J(x)dx\right)^{n-1} - \left(\int\Delta_F(z-x)J(x)dx\right)^{n+1}\right\}e^{-\frac{i}{2}I}}$$

$$\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^3 e^{-\frac{i}{2}I} = \Big[-i\Delta_F(0)\int\Delta_F(z-x)J(x) +$$

$$-2i\Delta_F(0)\left(\int \Delta_F(z-x)\,J(x)\,dx\right) - \left(\Delta_F(z-x)\,J(x)\right)^3\Big]\, e^{-\frac{i}{2}I} =$$

$$= \left\{-3i\Delta_F(0)\int \Delta_F(z-x)\,J(x)\,dx - \left(\Delta_F(z-x)\,J(x)\right)^3\right\} e^{-\frac{i}{2}I}$$

$$\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4 e^{-\frac{i}{2}I} = \Big\{-3i\Delta_F(0)\Big[i\Delta_F(0) - \left(\Delta_F(z-x)\,J(x)\,dx\right)^2\Big] +$$

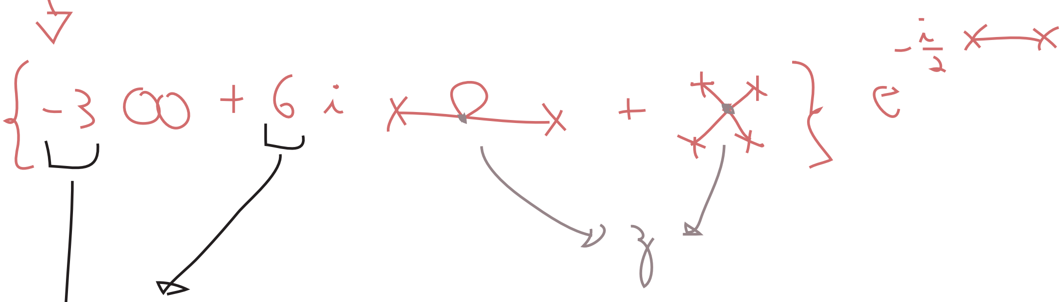
$$-\Big[-3i\Delta_F(0)\left(\int \Delta_F(z-x)\,J(x)\,dx\right)^2 - \left(\int \Delta_F(z-x)\,J(x)\,dx\right)^4\Big]\Big\} =$$

$$= \left\{-3\Delta_F^2(0) + 6i\Delta_F(0)\left(\int \Delta_F(z-x)\,J(x)\,dx\right)^2 + \left(\int \Delta_F(z-x)\,J(x)\,dx\right)^4\right\} e^{-\frac{i}{2}I}$$

DIAGRAMATICAMENTE: $\Delta_F(x-y) \Rightarrow$ x —— y

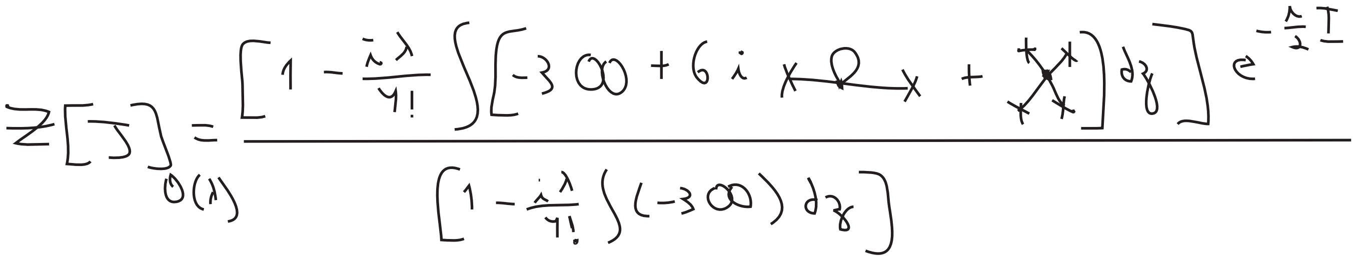
$\Delta_F(0) \Rightarrow$

$J(x) \Rightarrow \times$



fatores de simetria

É fácil obter Z em ordem λ, já que o denominador é obtido fazendo J = 0:



$$\left[1-\frac{i\lambda}{4!}\int(-3\,\infty)\,dz\right]\left[1-\frac{i\lambda}{4!}\int\left(6i\;\times\!\!-\!\!\bigcirc\!\!-\!\!\times+\times\!\!\!\!\!\!\times\right)dz\right]=$$

$$=1-\frac{i\lambda}{4!}\int(-3\,\infty)\,dz-\frac{i\lambda}{4!}\int\left(6i\;\times\!\!-\!\!\bigcirc\!\!-\!\!\times+\times\!\!\!\!\!\!\times\right)dz+\mathcal{O}(\lambda^2)$$

$$Z[J]_{\mathcal{O}(\lambda)}=\frac{\cancel{\left[1-\frac{i\lambda}{4!}\int(-3\,\infty)\,dz\right]}\left[1-\frac{i\lambda}{4!}\int\left(6i\;\times\!\!-\!\!\bigcirc\!\!-\!\!\times+\times\!\!\!\!\!\!\times\right)dz\right]}{\cancel{\left[1-\frac{i\lambda}{4!}\int(-3\,\infty)\,dz\right]}}\;e^{-\frac{i}{2}I}$$

$$\boxed{Z[J]_{\mathcal{O}(\lambda)}=\left[1-\frac{i\lambda}{4!}\int\left(6i\;\times\!\!-\!\!\bigcirc\!\!-\!\!\times+\times\!\!\!\!\!\!\times\right)dz\right]e^{-\frac{i}{2}\int J\Delta_F J}}$$

(eq. 35.1)

A partir desta, podemos obter os correlatores (em ordem λ)

Correlator de 2 pontos:

$$\langle 0|T\{\phi(x_1)\,\phi(x_2)\}|0\rangle=-\left.\frac{\delta^2 Z[J]}{\delta J(x_2)\,\delta J(x_1)}\right|_{J=0}$$

O termo com 4 Js $\left(\times\!\!\!\!\!\!\times\right)$ não vai contribuir quando J = 0, o outro termo nos dá:

$$Z[J]_{\mathcal{O}(\lambda)}=-\frac{i\lambda}{4!}\int 6i\,\Delta_F(0)\left(\int\Delta_F(z-x)\,J(x)\,dx\right)^2dz\;e^{-\frac{i}{2}I}$$

$$\frac{\delta Z[J]}{i\,\delta J(x_1)}=\frac{\lambda}{4i}\Delta_F(0)\int\left\{2\,\Delta_F(z-x_1)\int\Delta_F(z-x)\,J(x)\,dx+\dots\right\}dz\;e^{-\frac{i}{2}I}=$$

nesse termo a potência de J aumentou

$$\boxed{\frac{\delta}{\delta J(x)}e^{-\frac{i}{2}I}=-i\int\Delta_F(x-x_1)\,J(x_1)\,dx_1\;e^{-\frac{i}{2}I}}$$

$$\frac{1}{i}\frac{\delta}{\delta J(x_2)}\frac{1}{i}\frac{\delta Z[J]}{\delta J(x_1)} = -\frac{\lambda}{2}\Delta_F(0)\int \Delta_F(z-x_1)\Big\{\Delta_F(z-x_2) + \underbrace{\cdots}_{\mathcal{O}(J^2)}\Big\}\, dz\, e^{-\frac{i}{2}I}$$

$J=0$

$$\downarrow$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle_{\mathcal{O}(\lambda)} = -\frac{\lambda}{2}\Delta_F(0)\int dz\; \Delta_F(z-x_1)\Delta_F(z-x_2)$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle = i\,\Delta_F(x_1-x_2) - \frac{\lambda}{2}\Delta_F(0)\int dz\; \Delta_F(z-x_1)\Delta_F(z-x_2) + \mathcal{O}(\lambda^2)$$

$= i$ $-\frac{\lambda}{2}$  $+\,\mathcal{O}(\lambda^2)$

$$\Delta_F(x-y) = \frac{1}{(2\pi)^4}\int \frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}\, d^4k$$

$$-\frac{1}{2}\lambda\,\Delta_F(0)\int \Delta_F(x_1-z)\Delta_F(x_2-z)\, dz =$$

$$= -\frac{\lambda}{2}\frac{\Delta_F(0)}{(2\pi)^8}\int \frac{e^{-ip(x_1-z)-iq(x_2-z)}}{(p^2-m^2+i\varepsilon)(q^2-m^2+i\varepsilon)}\, d^4p\, d^4q\, d^4z =$$

$\therefore\ \boxed{\int dz \to \delta(p+q)}$

$$= -\frac{\lambda}{2}\frac{\Delta_F(0)}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}}{(p^2-m^2+i\varepsilon)^2}\, d^4p$$

$$\langle 0|T[\phi(x_1),\phi(x_2)]|0\rangle = \frac{1}{(2\pi)^4}\int \frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}\left[i - \frac{\lambda}{2}\frac{\Delta_F(0)}{k^2-m^2+i\varepsilon}\right] d^4k$$

$$= \frac{i}{(2\pi)^4}\int \frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}\left[1 + \frac{i\lambda}{2}\frac{\Delta_F(0)}{k^2-m^2+i\varepsilon}\right] d^4k$$

Se ficamos em ordem λ podemos escrever:

$$\left[1-\frac{\frac{1}{2}i\lambda\Delta_F(0)}{k^2-m^2+i\varepsilon}\right]^{-1}\underset{\lambda\to 0}{\sim} 1+\frac{\frac{1}{2}i\lambda\Delta_F(0)}{k^2-m^2+i\varepsilon}+\mathcal{O}(\lambda^2)$$

$$\langle 0|T[\phi(x_1),\phi(x_2)]|0\rangle = \frac{i}{(2\pi)^4}\int \frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}\cdot\frac{1}{\left[1-\frac{\frac{1}{2}i\lambda\Delta_F(0)}{k^2-m^2+i\varepsilon}\right]}\,d^4k =$$

$$= \frac{i}{(2\pi)^4}\int\frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon-\frac{1}{2}i\lambda\Delta_F(0)}$$

$$m_r^2 = m^2+\frac{1}{2}i\lambda\Delta_F(0)$$

De fato isso também valeria para ordens superiores, já que teríamos uma série geométrica:

$$\frac{i}{k^2-m^2+i\varepsilon}+\frac{i}{k^2-m^2+i\varepsilon}(-im^2)\frac{i}{k^2-m^2+i\varepsilon}+\frac{i}{k^2-m^2+i\varepsilon}\left[(-im^2)\frac{i}{k^2-m^2+i\varepsilon}\right]^2+\dots$$

$$=\frac{i}{k^2-m^2+i\varepsilon}\cdot\frac{1}{1-\frac{m^2}{k^2-m^2+i\varepsilon}}=\frac{i}{k^2-m^2-m^2+i\varepsilon}$$

$$\langle 0|T[\phi(x_1),\phi(x_2)]|0\rangle = \frac{i}{(2\pi)^4}\int\frac{e^{-ik(x-y)}}{k^2-m_r^2+i\varepsilon}$$

Isto é exatamente o que espereríamos como o propagador de uma partícula livre de massa m_r. O efeito das correções introduzidas pelos loops têm o efeito de modificar a massa da partícula.

A massa m_r é chamada de massa física ou massa renormalizada, voltaremos a ela em breve, basta no momento notar que a contribuição:

$$\Delta_F(0)=\Delta_F(x-x)=\frac{1}{(2\pi)^4}\int\frac{1}{k^2-m^2+i\varepsilon}\,d^4k$$

é claramente divergente para $k\to\pm\infty$

Correlator de 4 pontos:

$$\langle 0 | T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\} | 0 \rangle = \left. \frac{\delta^4 Z[J]}{\delta J(x_1)\, \delta J(x_2)\, \delta J(x_3)\, \delta J(x_4)} \right|_{J=0}$$

$$\lambda^0 \Rightarrow -\left[\Delta_F(x_1-x_2)\,\Delta_F(x_3-x_4) + \Delta_F(x_1-x_3)\,\Delta_F(x_2-x_4) + \Delta_F(x_1-x_4)\,\Delta_F(x_2-x_3)\right]$$

(veja eq. 27.1)

$$-\frac{i\lambda}{4!}\int \Big(6i \underbrace{}_{A}$$  $$+ \underbrace{}_{B}$$  $$\Big)\, dz \; e^{-\frac{i}{2} I}$$

$$\lambda^1 \Rightarrow \quad A \Rightarrow \frac{\delta^4}{\delta J \ldots \delta J} \left\{ \frac{\lambda}{4} \Delta_F(0) \int dz \left[\underbrace{\int dx\, \Delta_F(z-x)\, J(x)}_{I_J(z)} \right]^2 e^{-\frac{i}{2} I} \right. =$$

$$= \frac{\delta^3}{i\delta J_1\, i\delta J_2\, i\delta J_3} \left\{ \frac{\lambda}{4} \Delta_F(0) \int dz \left[\frac{1}{i}\, 2\, \Delta_F(z-x_4)\, I_J(z) - I_J^2(z)\, I_J(x_4) \right] e^{-\frac{i}{2} I} \right\} =$$

$$= \frac{\delta^2}{i\delta J_1\, i\delta J_2} \left\{ \frac{\lambda}{4} \Delta_F(0) \int dz \left[-2\, \Delta_F(z-x_4)\, \Delta_F(z-x_3) + 2i\, \Delta_F(z-x_4)\, I_J(z)\, I_J(x_3) - \frac{1}{i}\, 2\, \Delta_F(z-x_3)\, I_J(z)\, I_J(x_4) - \frac{1}{i} I_J^2(z)\, \Delta_F(x_4-x_3) + \ldots \right] e^{-\frac{i}{2} I} \right\} =$$

termos com potência maior que 2 em J (como só restam duas derivadas estes termos vão dar 0 quando J = 0)

$$= \frac{\delta}{i\,\delta J_1}\left\{ \frac{\lambda}{4}\Delta_F(0)\int dz \left[2\,\Delta_F(z-x_4)\,\Delta_F(z-x_3)\, I_J(x_2) + \right.\right.$$

$$+2\Delta_F(z-x_4)\Big(\Delta_F(z-x_2)\, I_J(x_3)+\Delta_F(x_3-x_2)\, I_J(z)\Big)+$$

$$+2\,\Delta_F(z-x_3)\,\Delta_F(z-x_2)\, I_J(x_4)+2\,\Delta_F(z-x_3)\, I_J(z)\,\Delta_F(x_4-x_2)+$$

$$\left.\left.+2\,\Delta_F(z-x_2)\, I_J(z)\,\Delta_F(x_4-x_3)+\underbrace{\ldots}_{\text{termos com potência maior que 1 em J}}\right] e^{-\frac{i}{2}I}\right\}=$$

$$= \frac{\lambda}{4}\Delta_F(0)\int dz\left[\frac{2}{i}\Delta_F(z-x_4)\,\Delta_F(z-x_3)\,\Delta_F(x_2-x_1)+\right.$$

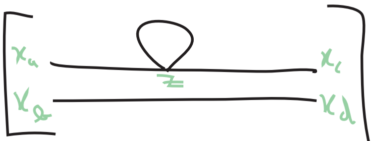
$$+\frac{2}{i}\,\Delta_F(z-x_4)\Big(\Delta_F(z-x_2)\,\Delta_F(x_3-x_1)+\Delta_F(x_3-x_2)\,\Delta_F(z-x_1)\Big)+$$

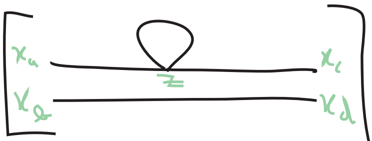
$$+\frac{2}{i}\,\Delta_F(z-x_3)\,\Delta_F(z-x_2)\,\Delta_F(x_4-x_1)+\frac{2}{i}\,\Delta_F(z-x_3)\,\Delta_F(z-x_1)\,\Delta_F(x_4-x_2)+$$

$$\left.+\frac{2}{i}\,\Delta_F(z-x_2)\,\Delta_F(z-x_1)\,\Delta_F(x_4-x_3)+\underbrace{\ldots}_{\text{termos com J}}\right]e^{-\frac{i}{2}I}=$$

$\boxed{J=0}$

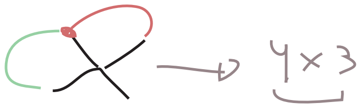
$$= -\frac{i\lambda}{4}\cdot 2\,\Delta_F(0)\int dz\left[\underbrace{\sum_{\text{PERMS}}\Delta_F(z-x_a)\,\Delta_F(z-x_b)\,\Delta_F(x_c-x_d)}_{\text{São seis termos com todas as combinações de a,b,c,d=1,2,3,4}}\right]=$$

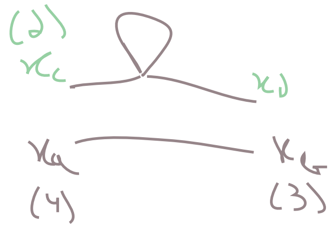
$$= -\frac{i\lambda}{4}\;12\left[\vphantom{\frac{a}{b}}\right.$$


$$\left.\vphantom{\frac{a}{b}}\right] = -\frac{i\lambda}{4!}\underbrace{(12\times 6)}_{\text{Fator de Simetria}}\left[\vphantom{\frac{a}{b}}\right.$$


$$\left.\vphantom{\frac{a}{b}}\right] =$$

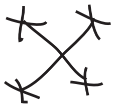
Fator de Simetria





$$\frac{4\times 3}{2}\cdot\frac{2}{2} = 6$$

$$\Delta_F(x_1-x_2) = \Delta_F(x_2-x_1)$$

Temos ainda a contribuição do termo  (batizado de "B" na pag 38):

$$-\frac{i\lambda}{4!}\int\Big(\underbrace{6i\,\times\!\!-\!\!\bigcirc\!\!-\!\!\times}_{A} + \underbrace{\times\!\!\!\!\times}_{B}\Big)\,dz\; e^{-\frac{i}{2}I}$$

$$B \Rightarrow \frac{\delta^4}{\delta J\ldots\delta J}\left\{-\frac{i\lambda}{4!}\int dz\; I_J^{4}(z)\, e^{-\frac{i}{2}I}\right\} =$$

Não vou fazer esta conta, mas vale a pena fazer uma delas na vida, então vou colocar isso em uma lista

$$= -\frac{i\lambda}{4!}\,24\int \Delta_F(z-x_1)\,\Delta_F(z-x_2)\,\Delta_F(z-x_3)\,\Delta_F(z-x_4)\,dz =$$

$$= -\frac{i\lambda}{4!}\,\underbrace{24}_{\text{Fator de Simetria}}\Big[\times\Big]$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\}|0\rangle =$$

$$= -3\Big[\,=\,\Big] - \frac{i\lambda}{4!}\Big[72\big(\,\underline{\overline{\;\;\bigcirc\;\;}}\,\big) + 24\big(\times\big)\Big] + O(\lambda^2)$$

(eq. 40.1)

Com isso em mãos já conseguimos ver as regras de Feynman da teoria (no espaço das coordenadas):

$\Rightarrow \Delta_F(x-y)$


$\Rightarrow -i\lambda$

Fator de simetria $\Rightarrow S/4!$