The Unruh Effect

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Outline

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Pieces Which Look Flat

- A **manifold** is a set M, together with a collection of subsets $\{U_\alpha\}$, such that:
    - Each U_α has a bijection ϕ_α with an $\underbrace{\text{open subset of } \mathbf{R}^n}_{\text{a bunch of open balls}}$ (coordinate systems)
        - What we call x^μ is short for $(\phi_\alpha)^\mu$
    - Each point of M is in at least one U_α (every point is described by a coordinate system)
    - There are C^∞ transition functions between coordinate systems (coordinate transformations)
- Summary: a manifold is a set made of subsets which look like $\mathbf{R}^n$

Coordinate Systems and Transformations

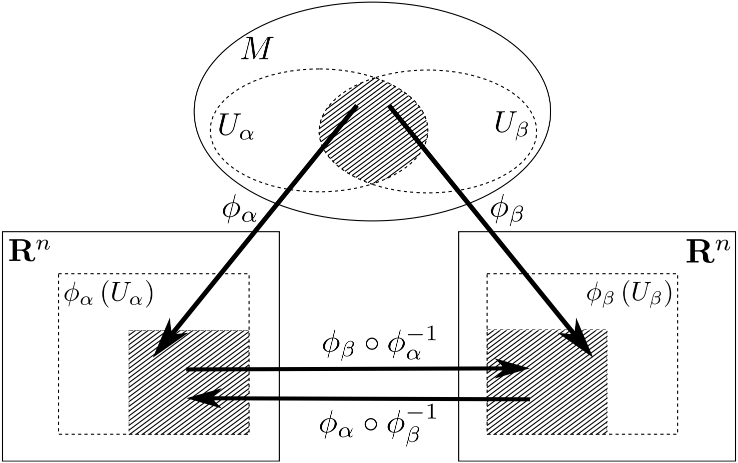

Figure: Coordinate systems and transformations.

Outline

Directional Derivative Operators

- A **vector** at a point p is a linear function $V : \mathcal{F} \to \mathbf{R}$ obeying the Leibniz rule ($\mathcal{F}$ is the set of all C^∞ $\underbrace{\text{scalar fields}}_{f:M\to\mathbf{R}}$)
- Intuition: every curve $\gamma : \mathbf{R} \to M$ parametrized by λ and which goes through p defines a vector

$$V(f) \equiv \frac{d}{d\lambda}(f \circ \gamma)\Big|_p \in \mathbf{R}$$

  - Leibniz rule:

$$V(fg) = \frac{df}{d\lambda}\Big|_p g(p) + f(p)\frac{dg}{d\lambda}\Big|_p$$

The Tangent Space

- The set of all vectors at a point p forms a vector space called the **tangent space** T_p
- Given a coordinate system x^μ, partial derivatives ∂_μ form a basis for T_p:

$$V = \frac{d}{d\lambda} = \underbrace{\frac{dx^\mu}{d\lambda}}_{\equiv V^\mu}\partial_\mu$$

- We denote these vectors generically by their components V^μ in some coordinate system

Vector Transformation Law

- From the chain rule of derivatives we find the **vector transformation law**:

$$V' = V \Rightarrow V'^{\mu}\partial'_{\mu} = V^{\mu}\partial_{\mu} = V^{\mu}\frac{\partial x'^{\nu}}{\partial x^{\mu}}\partial'_{\nu}$$

$$\Rightarrow \boxed{V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}}$$

- Compare with the Lorentz transformation

$$V'^{\mu} = \Lambda^{\mu}{}_{\nu} V^{\nu}$$

Coordinate Basis

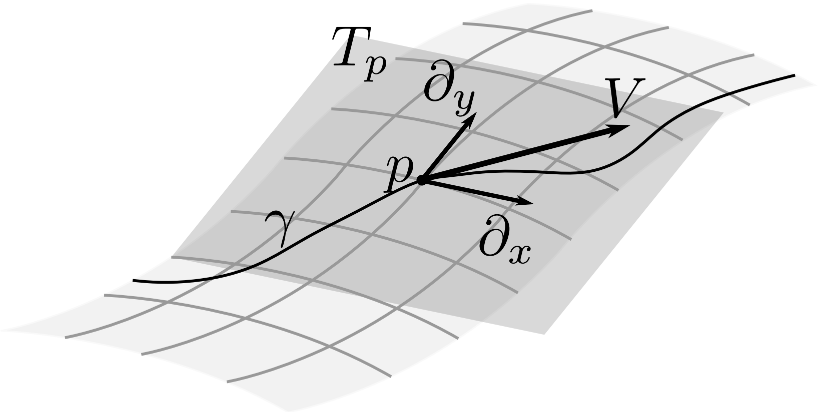

Figure: Coordinate basis.

Another Kind of Vector

- A **dual vector** at a point p is a linear function $\omega : T_p \to \mathbf{R}$
- The set of all dual vectors at a point p forms a vector space called the **cotangent space** T_p^*
  - The set $\left(T_p^*\right)^*$ can be identified with T_p (so a vector is a linear function $V : T_p^* \to \mathbf{R}$)

Example

Given a coordinate system x^μ, the functions defined by $\mathrm{d}x^\mu\left(\partial_\nu\right) = \delta^\mu_\nu$ form a **coordinate basis** for the cotangent space.

It Transforms as a Covariant Vector

- Expansion in coordinate basis: $\omega = \omega_\mu \mathrm{d}x^\mu$
- Component notation: ω_μ
- **Dual vector transformation law**:

$$\omega'_\mu = \left(\Lambda^{-1}\right)^\nu{}_\mu \omega_\nu \longrightarrow \boxed{\omega'_\mu = \frac{\partial x^\nu}{\partial x'^\mu}\omega_\nu}$$

Outline

A Multilinear Map

- A (k, l) **tensor** at a point p is a multinear function
  $T : \underbrace{T_p^* \times \ldots \times T_p^*}_{k} \times \underbrace{T_p \times \ldots \times T_p}_{l} \to \mathbf{R}$
- The set of all tensors at p forms a vector space

Examples

A vector is a $(1, 0)$ tensor and a dual vector is a $(0, 1)$ tensor.

A Product Between Tensors

- The **outer product** of a (k, l) tensor T with an (m, n) tensor S is a $(k+m, l+n)$ tensor $T \otimes S$ defined by

$$\boxed{\begin{aligned} & T \otimes S\left(\omega^1, \ldots, \omega^k, \omega^{k+1}, \ldots, \omega^{k+m}; v_1, \ldots, v_l, v_{l+1}, \ldots, v_{l+n}\right) \\ = & \, T\left(\omega^1, \ldots, \omega^k, v_1, \ldots, v_l\right) S\left(\omega^{k+1}, \ldots, \omega^{k+m}, v_{l+1}, \ldots, v_{l+n}\right) \end{aligned}}$$

(things we are used to writing as $T^{\mu_1 \cdots \mu_k}{}_{\nu_1 \ldots \nu_l} S^{\mu_{k+1} \cdots \mu_{k+m}}{}_{\nu_{l+1} \ldots \nu_{l+n}}$)

Example

The outer product $\partial_{\mu_1} \otimes \ldots \otimes \partial_{\mu_k} \otimes \mathrm{d}x^{\nu_1} \otimes \ldots \otimes \mathrm{d}x^{\nu_l}$ forms a **coordinate basis** for the vector space of tensors at a point.

It Transforms as a Tensor

- Expansion in coordinate basis:
  $T = T^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l} \partial_{\mu_1} \otimes \dots \otimes \partial_{\mu_k} \otimes \mathrm{d}x^{\nu_1} \otimes \dots \otimes \mathrm{d}x^{\nu_l}$
- Component notation: $T^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l}$
- **Tensor transformation law**:

$$T'^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l} = \Lambda^{\mu_1}{}_{\sigma_1} \dots \Lambda^{\mu_k}{}_{\sigma_k} \left(\Lambda^{-1}\right)^{\rho_1}{}_{\nu_1} \dots \left(\Lambda^{-1}\right)^{\rho_l}{}_{\nu_l} T^{\sigma_1 \dots \sigma_k}{}_{\rho_1 \dots \rho_l}$$

$$\longrightarrow \boxed{T'^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l} = \frac{\partial x'^{\mu_1}}{\partial x^{\sigma_1}} \dots \frac{\partial x'^{\mu_k}}{\partial x^{\sigma_k}} \frac{\partial x^{\rho_1}}{\partial x'^{\nu_1}} \dots \frac{\partial x^{\rho_l}}{\partial x'^{\nu_l}} T^{\sigma_1 \dots \sigma_k}{}_{\rho_1 \dots \rho_l}}$$

Outline

A Special Kind of Tensor

- A **metric** is a $(0,2)$, symmetric, nondegenerate $\underbrace{\text{tensor field}}_{\text{a tensor } \forall p \in M}$
  - Rank $(0,2)$: takes two vectors and gives a real number (inner product in tangent spaces)
  - **Symmetric**: $g_{\mu\nu} = g_{\nu\mu}$
  - **Nondegenerate**: determinant doesn't vanish (inverse metric)
- For every point $p \in M$, one can always find a set of **locally inertial coordinates** such that $g_{\mu\nu}$ is in **canonical form** ($+1$'s and -1's)
  - Convention:
    - **Riemannian**: all plus (positive-definite)
    - **Lorentzian**: one single minus $\longrightarrow$ spacetime!

A Norm

- We classify a vector V^μ in the following way

$$\text{if } g_{\mu\nu}V^\mu V^\nu \text{ is } \begin{cases} < 0, & V^\mu \text{ is } \textbf{timelike} \\ = 0, & V^\mu \text{ is } \textbf{null} \\ > 0, & V^\mu \text{ is } \textbf{spacelike} \end{cases}$$

- In locally inertial coordinates (t, x, y, z) at a point p, a timelike vector is said to be
  - **Future-directed** if it has a component in the direction of ∂_t
  - **Past-directed** if it has a component in the direction of $-\partial_t$

Example

Trivial examples are future-directed ∂_t and past-directed $-\partial_t$.

Outline

Partial Derivatives Do Not Transform As We Want

- Partial derivatives do not transform properly
  - Scalars:

$$\partial'_\mu \phi' = \frac{\partial x^\nu}{\partial x'^\mu} \partial_\nu \phi \quad ✔$$

  - Vectors:

$$\begin{aligned} \partial'_\mu V'^\nu &= \frac{\partial x^\sigma}{\partial x'^\mu} \partial_\sigma \left(\frac{\partial x'^\nu}{\partial x^\rho} V^\rho \right) \\ &= \frac{\partial x^\sigma}{\partial x'^\mu} \frac{\partial x'^\nu}{\partial x^\rho} \partial_\sigma V^\rho + \frac{\partial x^\sigma}{\partial x'^\mu} \frac{\partial^2 x'^\nu}{\partial x^\sigma \partial x^\rho} V^\rho \quad ✘ \end{aligned}$$

Generalization

- The **covariant derivative** ∇_μ generalizes the partial derivative ∂_μ and is defined from the Christoffel symbols

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right)$$

  - Scalars: $\nabla_\mu \phi = \partial_\mu \phi$
  - Contravariant vectors: $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^{\nu}_{\mu\lambda} V^\lambda$
  - Covariant vectors: $\nabla_\mu \omega_\nu = \partial_\mu \omega_\nu - \Gamma^{\lambda}_{\mu\nu} \omega_\lambda$
  - Tensors:

$$\begin{aligned}\nabla_\sigma T^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l} =& \partial_\sigma T^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \nu_l} \\ & + \Gamma^{\mu_1}_{\sigma\lambda} T^{\lambda \dots \mu_k}{}_{\nu_1 \dots \nu_l} + \dots + \Gamma^{\mu_k}_{\sigma\lambda} T^{\mu_1 \dots \lambda}{}_{\nu_1 \dots \nu_l} \\ & - \Gamma^{\lambda}_{\sigma\nu_1} T^{\mu_1 \dots \mu_k}{}_{\lambda \dots \nu_l} - \dots - \Gamma^{\lambda}_{\sigma\nu_l} T^{\mu_1 \dots \mu_k}{}_{\nu_1 \dots \lambda}\end{aligned}$$

Outline

The Definition

- The **Riemann tensor field** is defined from the Christoffel symbols as

$$\boxed{R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}}$$

- **Flat spacetime** $\Leftrightarrow R^{\rho}{}_{\sigma\mu\nu} = 0$ everywhere

Outline

Killing Vectors

- A **Killing vector field** K^μ is one which satisfies **Killing's equation**
$$\nabla_{(\mu}K_{\nu)} = 0$$
- Given K^μ, there's a coordinate system such that $K = \partial_{\sigma^*} \Leftrightarrow \partial_{\sigma^*} g_{\mu\nu} = 0$, for some coordinate x^{σ^*}
  - Symmetry under $x^{\sigma^*} \to x^{\sigma^*} + a^{\sigma^*}$

Outline

Generalizing a Theory

- Flat spacetime:

$$S = \int d^n x \mathcal{L}\left(\phi_i, \partial_\mu \phi_i\right)$$

- Curved spacetime:
  - $\eta_{\mu\nu} \longrightarrow g_{\mu\nu}$
  - Require coordinate-invariance

$$\partial_\mu \longrightarrow \nabla_\mu \qquad d^n x' \longrightarrow d^n x \sqrt{-g}$$

  - Assert that the theory remains true:

$$S = \int d^n x \underbrace{\sqrt{-g}\hat{\mathcal{L}}\left(\phi_i, \nabla_\mu \phi_i\right)}_{\equiv \mathcal{L}}$$

$$\frac{\partial \hat{\mathcal{L}}}{\partial \phi_i} - \nabla_\mu \left[\frac{\partial \hat{\mathcal{L}}}{\partial\left(\nabla_\mu \phi_i\right)}\right] = 0$$

The Scalar Field in Flat Spacetime

- Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2$$

- EOM:

$$\left(\Box - m^2\right)\phi = 0$$

  where $\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$

- Positive-frequency modes:

$$\partial_t f_k = -i\omega_k f_k$$

  in some globally inertial coordinate system

- Field mode expansion:

$$\phi \propto \int dk\left(a_k f_k + a_k^\dagger f_k^*\right)$$

The Scalar Field in Curved Spacetime

- Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$$

- EOM:

$$\left(\Box - m^2\right)\phi = 0$$

where $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$

- Positive-frequency modes:

$$\partial_{\sigma^*} f_k = -i\omega_k f_k$$

where ∂_{σ^*} is a future-directed Killing vector in some coordinate system

- Field mode expansion:

$$\phi \propto \int dk \left(a_k f_k + a_k^\dagger f_k^*\right)$$

The Problem

- Is there any such future-directed Killing vector in my spacetime at all?
- If there are more than one, which one I should I use to define positive-frequency modes?
  - Each one is a partial derivative in a particular coordinate system, is there any coordinate system which is preferred?

Two Sets of Modes

- Suppose two such Killing vectors:
    - ∂_{σ^*} in some coordinate system x^μ
    - ∂_{ρ^*} in some coordinate system y^μ
- Positive-frequency modes:

$$\partial_{\sigma^*} f_i = -i\omega_i f_i \qquad \partial_{\rho^*} g_i = -i\omega_i g_i$$

(i is just notation, can be discrete or continuous)

- Field mode expansion:

$$\phi \propto \sum_i \left(a_i f_i + a_i^\dagger f_i^* \right) \qquad \phi \propto \sum_i \left(b_i g_i + b_i^\dagger g_i^* \right)$$

- Vacuum states:

$$\boxed{a_i |0_f\rangle = 0} \qquad \boxed{b_i |0_g\rangle = 0}$$

Bogolubov Transformations

- Both sets must be a basis for the same function space
  - The transformations between these two sets are called the **Bogolubov transformations**:

$$g_i = \sum_j \left(\alpha_{ij} f_j + \beta_{ij} f_j^*\right)$$

$$f_i = \sum_j \left(\alpha_{ji}^* g_j - \beta_{ji} g_j^*\right)$$

  - The expansion coefficients (now promoted to creation and annihilation operators) must transform accordingly in order to describe the same field:

$$a_i = \sum_j \left(\alpha_{ji} b_j + \beta_{ji}^* b_j^\dagger\right)$$

$$b_i = \sum_j \left(\alpha_{ij}^* a_j - \beta_{ij}^* a_j^\dagger\right)$$

Vacuum Is Relative

- Number operator of i-th g-mode:

$$n_{gi} = b_i^\dagger b_i = \sum_{jk} \left(\alpha_{ij} a_j^\dagger - \beta_{ij} a_j\right) \left(\alpha_{ik}^* a_k - \beta_{ik}^* a_k^\dagger\right)$$

- Calculating f-VEV of n_{gi}:

$$\langle 0_f | n_{gi} | 0_f \rangle = \sum_{jk} \beta_{ij} \beta_{ik}^* \langle 0_f | a_j a_k^\dagger | 0_f \rangle \propto \sum_{jk} \beta_{ij} \beta_{ik}^* \delta_{jk} = \sum_j |\beta_{ij}|^2$$

- No reason to believe it's zero:

$$\boxed{\langle n_{gi} \rangle_{f\text{-vacuum}} \neq 0}$$

Outline

1. Manifolds
   - The Definition
   - Vectors
   - Tensors
   - The Metric
2. Curvature
   - Covariant Derivatives
   - The Riemann Curvature Tensor
   - Killing Vectors
3. Quantum Field Theory in Curved Spacetime
   - The Scalar Field in Curved Spacetime
   - The Unruh Effect

Massless Scalar in 2D

- Massless scalar field:

$$\Box\phi = 0$$

- Two-dimensional Minkowski space in globally inertial coordinates:

$$ds^2 = -\mathrm{d}t^2 + \mathrm{d}x^2$$

An Accelerated Observer

- Trajectory of an observer with acceleration α:

$$t(\tau) = \frac{1}{\alpha}\sinh(\alpha\tau)$$
$$x(\tau) = \frac{1}{\alpha}\cosh(\alpha\tau)$$

- Indeed, we can show that

$$a^{\mu} a_{\mu} = \alpha^2$$

where $a^{\mu} \equiv d^2 x^{\mu} / d\tau^2$ is 4-acceleration

Rindler Coordinates

- From globally inertial coordinates (t, x) to **Rindler coordinates** (η, ξ):

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta) \qquad x = \frac{1}{a} e^{a\xi} \cosh(a\eta) \qquad x > |t|$$

  where a is some constant parameter

- In these coordinates, the accelerated trajectory becomes

$$\eta(\tau) = \frac{\alpha}{a}\tau \qquad \text{(varies)}$$

$$\xi(\tau) = \frac{1}{a}\ln\left(\frac{a}{\alpha}\right) \qquad \text{(constant)}$$

- Something similar can be shown for $x < -|t|$:

$$t = -\frac{1}{a} e^{a\xi} \sinh(a\eta) \qquad x = -\frac{1}{a} e^{a\xi} \cosh(a\eta) \qquad x < -|t|$$

A Timelike Killing Vector

- In these coordinates, our metric looks like

$$ds^2 = e^{2a\xi}\left(-\mathrm{d}\eta^2 + \mathrm{d}\xi^2\right)$$

- We immediately see $\partial_\eta g_{\mu\nu} = 0 \Rightarrow \partial_\eta$ is a Killing vector
- Performing a coordinate transformation on this vector,

$$\begin{aligned}\partial_\eta &= \frac{\partial t}{\partial \eta}\partial_t + \frac{\partial x}{\partial \eta}\partial_x \\ &= a\left(x\partial_t + t\partial_x\right)\end{aligned}$$

- For $x > |t|$, ∂_η is future-directed, for $x < -|t|$, past-directed

Rindler Coordinates

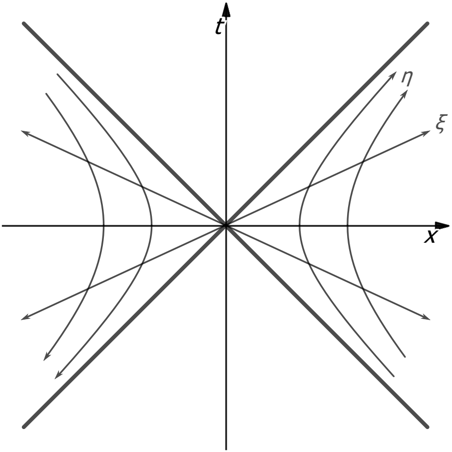

Figure: Minkowski spacetime in Rindler coordinates.

A Set of Modes

- In these coordinates, our massless KG looks like

$$e^{-2a\xi}\left(-\partial_\eta^2+\partial_\xi^2\right)\phi=0$$

- The mode $g_k=(4\pi\omega)^{-1/2}\,e^{-i\omega\eta+ik\xi}$ solves this equation and satisfies $\partial_\eta g_k=-i\omega g_k$
- But we need $-\partial_\eta g_k=-i\omega g_k$ for $x<-|t|$, since there ∂_η is past-directed, so we impose

$$g_k^{(1)}=\begin{cases}\frac{1}{\sqrt{4\pi\omega}}e^{-i\omega\eta+ik\xi} & \text{for } x>|t|\\ 0 & \text{for } x<-|t|\end{cases}$$

$$g_k^{(2)}=\begin{cases}0 & \text{for } x>|t|\\ \frac{1}{\sqrt{4\pi\omega}}e^{i\omega\eta+ik\xi} & \text{for } x<-|t|\end{cases}$$

Mode Expansion

- This way, we expand

$$\phi = \int dk \left(b_k^{(1)} g_k^{(1)} + b_k^{(1)\dagger} g_k^{(1)*} + b_k^{(2)} g_k^{(2)} + b_k^{(2)\dagger} g_k^{(2)*} \right)$$

- The Rindler vacuum will be defined by

$$b_k^{(1,2)} |0_R\rangle = 0$$

- Our job:
    - Find Bogolubov coefficients relating Minkowski and Rindler
    - Calculate Minskowski VEV of the Rindler number operator
- Shortcut: find a set of modes which share the same vacuum as Minkowski

A Shortcut

- From the definition of the Rindler coordinates,

$$e^{-a(\eta-\xi)} = \begin{cases} a\left(-t+x\right) & x > |t| \\ a\left(t-x\right) & x < -|t| \end{cases}$$

$$e^{a(\eta+\xi)} = \begin{cases} a\left(t+x\right) & x > |t| \\ a\left(-t-x\right) & x < -|t| \end{cases}$$

Finding New Modes

- Assuming $k > 0$, we have, for $x > |t|$,

$$\sqrt{4\pi\omega} g_k^{(1)} = a^{i\omega/a} (-t + x)^{i\omega/a}$$

- For $x < -|t|$,

$$\sqrt{4\pi\omega} g_k^{(2)} = a^{-i\omega/a} (-t - x)^{-i\omega/a}$$

- Taking complex conjugate and reversing momentum of $g_k^{(2)}$,

$$\sqrt{4\pi\omega} g_{-k}^{(2)*} = a^{i\omega/a} e^{\pi\omega/a} (-t + x)^{i\omega/a}$$

- So the combination

$$\sqrt{4\pi\omega} \left[g_k^{(1)} + e^{-\pi\omega/a} g_{-k}^{(2)*} \right] = 2a^{i\omega/a} (-t + x)^{i\omega/a}$$

works for both $x > |t|$ and $x < -|t|$ (an identical result is obtained for $k < 0$)

A New Set of Modes

- A similar reasoning leads to

$$\sqrt{4\pi\omega}\left[g_k^{(2)} + e^{-\pi\omega/a} g_{-k}^{(1)*}\right] = 2a^{i\omega/a}\left(-t-x\right)^{i\omega/a}$$

- A new set of modes which share the same vacuum as Minkowski:

$$\boxed{\begin{aligned} h_k^{(1)} &= \frac{1}{\sqrt{2\sinh\left(\pi\omega/a\right)}}\left[e^{\pi\omega/2a} g_k^{(1)} + e^{-\pi\omega/2a} g_{-k}^{(2)*}\right] \\ h_k^{(2)} &= \frac{1}{\sqrt{2\sinh\left(\pi\omega/a\right)}}\left[e^{\pi\omega/2a} g_k^{(2)} + e^{-\pi\omega/2a} g_{-k}^{(1)*}\right] \end{aligned}}$$

The Bogolubov Transformation

- These define a Bogolubov transformation from $g_k^{(1,2)}$ to $h_k^{(1,2)}$
- We can use this to write the Bogolubov transformation between operators:

$$\boxed{\begin{aligned} b_k^{(1)} &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\left[e^{\pi\omega/2a} c_k^{(1)} + e^{-\pi\omega/2a} c_{-k}^{(2)\dagger}\right] \\ b_k^{(2)} &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\left[e^{\pi\omega/2a} c_k^{(2)} + e^{-\pi\omega/2a} c_{-k}^{(1)\dagger}\right] \end{aligned}}$$

where $c_k^{(1,2)}$ annihilates the Minkowski vacuum:

$$\boxed{c_k^{(1,2)}|0_M\rangle = 0}$$

Thermal Radiation

- In the same manner as before,

$$\boxed{\langle n_R \rangle_{\text{M-vacuum}} = \frac{1}{e^{2\pi\omega/a} - 1}\delta(0)}$$

- The delta has to do with our use of nonsquare-integrable modes
  - It is possible to obtain a finite result restricting the size of spacetime and thus using wave packet modes
- The factor we obtain reminds us of thermal radiation with temperature

$$\boxed{T = \frac{a}{2\pi}}$$

- Interpretation: an accelerated observer measures thermal radiation in the Minkowski vacuum

For Further Reading

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Thanks! :)