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$T \neq 0$

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& Ashoke Sen Lectures @ HRI

(YouTube)

$$Z(\beta) = \mathrm{Tr}\left(e^{-\beta H}\right) \qquad \beta = \frac{1}{T}$$

$$Z = \int dq \langle q | e^{-\beta H} | q \rangle =$$

$$= \int dq \langle q | e^{-\epsilon H}\, I\, e^{-\epsilon H} \cdots e^{-\epsilon H} | q \rangle$$

$$\int dq_i \frac{dp_i}{2\pi} | q_i \rangle \langle q_i | p_i \rangle \langle p_i |$$

$$Z = C(T) \int^{*} \mathcal{D}q \; e^{-S_E}$$

$$S_E = \int_0^{\beta} \frac{m}{2}\dot{q}^2 + V(q)$$

Freq. de Matsubara

$$\omega_n = \frac{2\pi n}{\beta}$$

$$\underline{q(0) = q(\beta)}$$

$$q(\tau) = \sum_{n \in \mathbb{Z}} \tilde{q}_n \, e^{i\omega_n \tau}$$

$$\tilde{q}_n = a_n + i\, b_n \qquad a_n = a_{-n}$$

$$b_n = -b_{-n}$$

$$S_E = \int_0^\beta \left\{ \frac{m}{2}|\dot{q}|^2 + \frac{m}{2}\omega^2 |q|^2 \right\} =$$

$$= \int_0^\beta d\tau \sum_{n,n'} \frac{m}{2}\left(\omega_n \omega_{n'} + \omega^2\right) \tilde{q}_n \tilde{q}_{n'}^*$$

$$\times\, e^{i(\omega_n - \omega_{n'})t}$$

$$\frac{1}{\beta}\,\delta_{nn'}$$

$$\sum_n |\tilde{q}_n|^2 (\omega_n^2 + \omega^2) \cdot \frac{m}{2\beta} =$$

$$\sum_n (a_n^2 + b_n^2) \cdot \frac{m}{2\beta} =$$

$$\sum_{n=1}^{\infty} \frac{m}{\beta} (a_n^2 + b_n^2)(\omega_n^2 + \omega^2) + \frac{m\omega^2}{2\beta} a_0^2$$

$$= S_E$$

$$\mathcal{D}q = B(T)\left(\prod_{i=1}^{\infty} da_n\, db_n\right) da_0$$

$$Z = \int c(T)\, B(T)\left(\prod_{i=1}^{\infty} da_n\, db_n\right) da_0 \;\times$$

$$\times \exp\left(-\sum_{n=1}^{\infty} \frac{m}{\beta}\left(a_n^2+b_n^2\right)\left(\omega_n^2+\omega^2\right) - \frac{m\omega^2}{2\beta}\, a_0^2\right)$$

$$= \underline{C(T|\beta(T))} \sqrt{\frac{2\pi\beta}{m\omega^2}} \prod_{n=1}^{\infty} \frac{\pi\beta}{m(\omega^2+\omega_n^2)}$$

$$\frac{Z}{Z_0} = \ldots$$

$$\frac{1}{\beta}\int_0^\beta q(\tau)\,d\tau = \frac{a_0}{\beta} = \langle q \rangle$$

$$a_0 = \beta \langle q \rangle \in [0, \beta L]$$

$$Z_0 = \lim_{\omega} Z = c \cdot B \cdot \beta L \cdot \prod_{n=1}^{\infty} \frac{\pi \beta}{m \omega_n^2}$$

$$\frac{Z}{Z_0} = \frac{1}{\beta L} \sqrt{\frac{2\pi\beta}{m\omega^2}} \prod_{n=1}^{\infty} \frac{\omega_n^2}{\omega^2 + \omega_n^2}$$

$$Z_0 = \mathrm{Tr}\left(e^{-\beta \frac{p^2}{2m}}\right) =$$

$$= \int \langle q | p \rangle \langle p | e^{-\beta \frac{p^2}{2m}} | q \rangle \, dq \, \frac{dp}{2\pi}$$

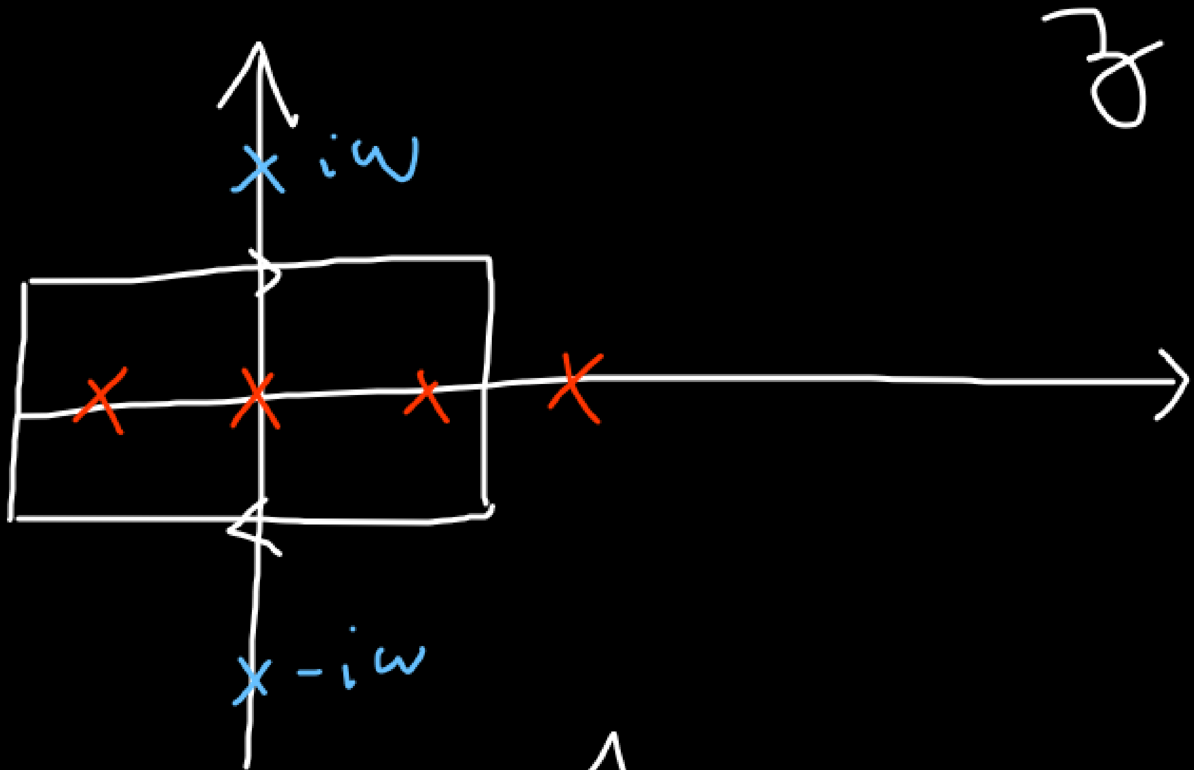
$$= \int e^{-\beta \frac{p^2}{2m}} \, dq \, \frac{dp}{2\pi} = \frac{L}{2\pi} \sqrt{\frac{2\pi m}{\beta}}$$

$$Z = \frac{\cancel{L}}{\cancel{2}\pi} \sqrt{\frac{\cancel{2\pi} m}{\cancel{\beta}}} \frac{1}{\beta \cancel{L}} \sqrt{\frac{\cancel{2\pi} i \cancel{\beta}}{\cancel{m} \omega^2}} \prod_{n=1}^{\infty} \frac{\omega_n^2}{\omega^2 + \omega_n^2}$$

$$= \frac{1}{\beta \omega} \prod_{n=1}^{\infty} \frac{\omega_n^2}{\omega^2 + \omega_n^2} \quad , \quad \omega_n = \frac{2\pi n}{\beta}$$

$$\omega^{-1} \frac{\partial}{\partial \omega} \log Z = - \sum_{n \in \mathbb{Z}} \frac{1}{\omega^2 + \omega_n^2} =$$

$$= - \frac{1}{2\pi i} \oint_C dz \, \frac{1}{\omega^2 + z^2} \, \frac{i\beta}{e^{i\beta z} - 1}$$



$$Z = \frac{1}{2\sinh\frac{\beta\omega}{2}}$$

$$Z = \operatorname{tr} e^{-\beta H} = \sum_{n=0}^{\infty} e^{-\beta\left(\frac{1}{2}+n\right)\omega} = \frac{e^{-\frac{\beta\omega}{2}}}{1-e^{-\beta\omega}}$$

$$= \left(2\sinh\frac{\beta\omega}{2}\right)^{-1}$$

$$S_E = \int_0^\beta \underset{\downarrow \atop -it}{d\tau} \int dV \left\{ \partial_\mu \phi \, \partial_\mu \phi + \frac{m}{2} \phi \right\} =$$

$$= \exp\left(- \frac{T}{2V} \sum_{\vec{k}, \omega_n} \left(\omega_n^2 + \vec{k}^2 + m^2 \right) \left| \tilde{\phi}(\omega_n, k) \right|^2 \right)$$