Campo de Stueckelberg no formalismo de Utiyama

Campo de matéria

$$S = \int d^4x\, \mathcal{L}[\Phi^A, \partial_\mu \Phi^A] \longrightarrow \delta S = \int d^4x\, \delta\mathcal{L}[\Phi^A, \partial_\mu \Phi^A] = 0$$

$$\delta\mathcal{L}[\Phi^A, \partial_\mu \Phi^A] = \frac{\partial \mathcal{L}}{\partial(\Phi^A)}(\delta\Phi^A) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^A)}(\delta\partial_\mu \Phi^A) = 0$$

Transformações infinitesimais

Transformações nas coordenadas

Transformações nos campos

Teorema de Noether

Conservação do tensor energia-momento, tensor momento angular.

Conservação da carga eletromagnética.

Transformação de Gauge global

Transformação infinitesimal no Campo de matéria:

$$\delta\Phi^{A} = \epsilon^{a} I^{A}_{(a)B}\Phi^{B}$$

$$\left[I_{(a)}, I_{(b)}\right]^{A}_{C} = I^{A}_{(a)B} I^{B}_{(b)C} - I^{A}_{(b)B} I^{B}_{(a)C} = f^{c}_{ab} I^{A}_{(c)C}$$

$$\frac{\partial\mathcal{L}}{\partial(\Phi^{\mathrm{A}})} I^{A}_{(a)B}\Phi^{B} + \frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\Phi^{\mathrm{A}})} I^{A}_{(a)B}\partial_{\mu}\Phi^{B} = 0$$

Transformação de Gauge local

$$\delta\Phi^{A} = \epsilon^{a}(x)\, I^{A}_{(a)B}\Phi^{B}$$

$$\frac{\partial\mathcal{L}}{\partial(\Phi^{\mathrm{A}})}\left(\epsilon^{a} I^{A}_{(a)B}\Phi^{B}\right) + \frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\Phi^{\mathrm{A}})}\partial_{\mu}\left(\epsilon^{a} I^{A}_{(a)B}\Phi^{B}\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial(\Phi^A)}\left(\epsilon^a(x) I^A_{(a)B}\Phi^B\right) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^A)}\partial_\mu \epsilon^a(x) I^A_{(a)B}\Phi^B + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^A)}\epsilon^a(x) I^A_{(a)B}\partial_\mu \Phi^B = 0$$

$$\epsilon^a(x)\left[\frac{\partial \mathcal{L}}{\partial(\Phi^A)} I^A_{(a)B}\Phi^B + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\partial_\mu \Phi^B\right] + \partial_\mu \epsilon^a(x)\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\Phi^B\right] = 0$$

Introduzirmos o campo:

$$\delta A'^J = \epsilon^a(x) U^J_{(a)K} A'^K + \frac{1}{g} C^{J\mu}_a \partial_\mu \epsilon^a(x)$$

$$\mathcal{L}[\Phi^A, \partial_\mu \Phi^A] \longrightarrow \mathcal{L}'[\Phi^A, \partial_\mu \Phi^A, A'^J]$$

$$\epsilon^{a}(x)\left[\frac{\partial \mathcal{L}'}{\partial \Phi^{\mathrm{A}}} I^{A}_{(a)B}\Phi^{B} + \frac{\partial \mathcal{L}'}{\partial A'^{J}} U^{J}_{(a)K}A'^{K} + \frac{\partial \mathcal{L}'}{\partial\left(\partial_{\mu}\Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\partial_{\mu}\Phi^{B}\right] +$$

$$\partial_{\mu}\epsilon^{a}(x)\left[\frac{\partial \mathcal{L}'}{\partial\left(\partial_{\mu}\Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\frac{\partial \mathcal{L}'}{\partial A'^{J}} C^{J\mu}_{a}\right] = 0$$

$$\frac{\partial \mathcal{L}'}{\partial \Phi^{\mathrm{A}}} I^{A}_{(a)B}\Phi^{B} + \frac{\partial \mathcal{L}'}{\partial A'^{J}} U^{J}_{(a)K}A'^{K} + \frac{\partial \mathcal{L}'}{\partial\left(\partial_{\mu}\Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\partial_{\mu}\Phi^{B} = 0$$

$$\frac{\partial \mathcal{L}'}{\partial\left(\partial_{\mu}\Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\frac{\partial \mathcal{L}'}{\partial A'^{J}} C^{J\mu}_{a} = 0$$

$$\frac{\partial \mathcal{L}'}{\partial A'^{1}} C^{1\mu}_{a} + \frac{\partial \mathcal{L}'}{\partial A'^{2}} C^{2\mu}_{a} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^{N}} C^{N\mu}_{a} = -g\frac{\partial \mathcal{L}'}{\partial\left(\partial_{\mu}\Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\Phi^{B}$$

Em coordenadas espaço-tempo:

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{10} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{20} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N0} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_0 \Phi^A)} I^A_{(a)B} \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{11} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{21} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N1} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_1 \Phi^A)} I^A_{(a)B} \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{12} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{22} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N2} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_2 \Phi^A)} I^A_{(a)B} \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{13} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{23} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N3} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_3 \Phi^A)} I^A_{(a)B} \Phi^B$$

$$\begin{bmatrix} C_1^{10} & C_1^{20} & & C_1^{N0} \\ C_1^{11} & C_1^{21} & \cdots & C_1^{N1} \\ C_1^{12} & C_1^{12} & & C_1^{N2} \\ \vdots & & \ddots & \vdots \\ C_n^{13} & C_n^{23} & \cdots & C_n^{N3} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{L}'}{\partial A'^1} \\ \frac{\partial \mathcal{L}'}{\partial A'^2} \\ \vdots \\ \frac{\partial \mathcal{L}'}{\partial A'^N} \end{bmatrix} = -g \begin{bmatrix} \frac{\partial \mathcal{L}'}{\partial(\partial_0 \Phi^A)} I^A_{(1)B} \Phi^B \\ \frac{\partial \mathcal{L}'}{\partial(\partial_1 \Phi^A)} I^A_{(1)B} \Phi^B \\ \vdots \\ \frac{\partial \mathcal{L}'}{\partial(\partial_3 \Phi^A)} I^A_{(n)B} \Phi^B \end{bmatrix}$$

$$C_{4nxN}\,[\vdots]_N = -g[\vdots]_{4n}$$

$$4n = N$$

$$C_a^{J\mu}(C^-)_{\mu K}^a = \delta_K^J$$

$$(C^-)_{\nu J}^a C_b^{J\mu} = \delta_b^a \delta_\nu^\mu$$

$$\delta\left((C^-)_{\nu J}^b A'^J\right) = \epsilon^a(x)(C^-)_{\nu J}^b U_{(a)K}^J A'^K + \frac{1}{g}(C^-)_{\nu J}^b C_a^{J\mu}\partial_\mu\,\epsilon^a(x) = \delta A_\nu^b$$

$$A'^K = C_c^{K\mu}A_\mu^c$$

$$\delta A_\nu^b = \epsilon^a(x)(C^-)_{\nu J}^b U_{(a)K}^J C_c^{K\mu}A_\mu^c + \frac{1}{g}(C^-)_{\nu J}^b C_a^{J\mu}\partial_\mu\,\epsilon^a(x)$$

$$\delta A_\mu^a = \epsilon^b(x)S_{bc\mu}^{a\nu}A_\nu^c + \frac{1}{g}\partial_\mu\,\epsilon^a(x)$$

$$\mathcal{L}[\Phi^A, \partial_\mu \Phi^A] \longrightarrow \mathcal{L}'[\Phi^A, \partial_\mu \Phi^A, A'^J] \longrightarrow \mathcal{L}''[\Phi^A, \partial_\mu \Phi^A, A_\mu^a]$$

$$\epsilon^a(x)\left[\frac{\partial \mathcal{L}''}{\partial \Phi^A} I^A_{(a)B}\Phi^B + \frac{\partial \mathcal{L}''}{\partial A_\mu^b} S^{b\nu}_{ac\mu} A_\nu^c + \frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\partial_\mu \Phi^B\right] +$$

$$\partial_\mu \epsilon^a(x)\left[\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\Phi^B + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A_\mu^a}\right] = 0$$

$$\frac{\partial \mathcal{L}''}{\partial \Phi^A} I^A_{(a)B}\Phi^B + \frac{\partial \mathcal{L}''}{\partial A_\mu^b} S^{b\nu}_{ac\mu} A_\nu^c + \frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\partial_\mu \Phi^B = 0$$

$$\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^A)} I^A_{(a)B}\Phi^B + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A_\mu^a} = 0$$

$$\frac{\partial \mathcal{L}''}{\partial\left(\partial_\mu \Phi^{\mathrm{A}}\right)} I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A^{a}_{\mu}} = 0$$

$$\nabla_\mu \Phi^{\mathrm{A}} = \partial_\mu \Phi^{\mathrm{A}} - g A^{a}_{\mu} I^{A}_{(a)B}\Phi^{B}$$

$$\frac{\partial \mathcal{L}''}{\partial\left(\partial_\mu \Phi^{\mathrm{A}}\right)} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{\mathrm{B}})}\frac{\partial\left(\nabla_\nu \Phi^{\mathrm{B}}\right)}{\partial\left(\partial_\mu \Phi^{\mathrm{A}}\right)} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{\mathrm{B}})}\delta^{\mu}_{\nu}\delta^{B}_{A} = \frac{\partial \mathcal{L}''}{\partial\left(\nabla_\mu \Phi^{A}\right)}$$

$$\frac{\partial \mathcal{L}''}{\partial A^{a}_{\mu}} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{A})}\frac{\partial(\nabla_\nu \Phi^{A})}{\partial A^{a}_{\mu}} = -g\frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{A})}\delta^{b}_{a}\delta^{\mu}_{\nu} I^{A}_{(b)B}\Phi^{B} = -g\frac{\partial \mathcal{L}''}{\partial\left(\nabla_\mu \Phi^{A}\right)} I^{A}_{(a)B}\Phi^{B}$$

$$\frac{\partial \mathcal{L}''}{\partial\left(\nabla_\mu \Phi^{A}\right)}\left[I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\left(-g I^{A}_{(a)B}\Phi^{B}\right)\right] = 0$$

$$\mathcal{L}[\Phi^A, \partial_\mu \Phi^A] \longrightarrow \mathcal{L}''[\Phi^A, \partial_\mu \Phi^A, A^a_\mu] \longrightarrow \mathcal{L}_{int}[\Phi^A, \nabla_\mu \Phi^A]$$

$$\delta(\nabla_\mu \Phi^A) = \partial_\mu (\delta\Phi^A) - g(\delta A^a_\mu) I^A_{(a)B} \Phi^B - g A^a_\mu I^A_{(a)B} (\delta\Phi^B)$$

$$\delta(\nabla_\mu \Phi^A) = \epsilon^a(x)\, I^A_{(a)B} \partial_\mu \Phi^B - g\epsilon^b(x) S^{a\nu}_{bc\mu} A^c_\nu I^A_{(a)B} \Phi^B - g\epsilon^a(x) A^b_\mu I^A_{(b)B} I^B_{(a)C} \Phi^C$$

$$I^A_{(a)B} I^B_{(b)C} - I^A_{(b)B} I^B_{(a)C} = f^c_{ab} I^A_{(c)C}$$

$$\delta(\nabla_\mu \Phi^A) = \epsilon^a(x)\, I^A_{(a)B} [\partial_\mu \Phi^B - g A^b_\mu I^B_{(b)C} \Phi^C] - g\epsilon^b(x) [S^{a\nu}_{bc\mu} - \delta^\nu_\mu f^a_{bc}] A^c_\nu I^A_{(a)B} \Phi^B$$

$$\delta(\nabla_\mu \Phi^A) = \epsilon^a(x)\, I^A_{(a)B} \nabla_\mu \Phi^B - g\epsilon^b(x) [S^{a\nu}_{bc\mu} - \delta^\nu_\mu f^a_{bc}] A^c_\nu I^A_{(a)B} \Phi^B$$

$$\frac{\partial \mathcal{L}_{int}}{\partial \Phi^A} \epsilon^a(x)\, I^A_{(a)B} \Phi^B + \frac{\partial \mathcal{L}_{int}}{\partial(\nabla_\mu \Phi^A)} \epsilon^a(x)\, I^A_{(a)B} \nabla_\mu \Phi^B -$$

$$S^{a\nu}_{bc\mu} = \delta^\nu_\mu f^a_{bc}$$

$$\frac{\partial \mathcal{L}_{int}}{\partial(\nabla_\mu \Phi^A)} g \epsilon^b(x) \left[S^{a\nu}_{bc\mu} - \delta^\nu_\mu f^a_{bc} \right] A^c_\nu I^A_{(a)B} \Phi^B = 0$$

$$\frac{\partial \mathcal{L}_{int}}{\partial \Phi^A} I^A_{(a)B} \Phi^B + \frac{\partial \mathcal{L}_{int}}{\partial(\nabla_\mu \Phi^A)} I^A_{(a)B} \nabla_\mu \Phi^B = 0$$

$$\delta A^a_\mu = \epsilon^b(x) f^a_{bc} A^c_\mu + \frac{1}{g} \partial_\mu \epsilon^a(x)$$

Campo de Gauge

$$\mathcal{L}_A \longrightarrow \mathcal{L}_A\left[A_\mu^a, \partial_\nu A_\mu^a\right]$$

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,\delta A_\mu^a + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)}\,\partial_\nu\left(\delta A_\mu^a\right) = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,\epsilon^b(x) f_{bc}^a A_\mu^c + \frac{1}{g}\partial_\mu\,\epsilon^a(x)\frac{\partial \mathcal{L}_A}{\partial A_\mu^a} + \epsilon^b(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a \partial_\nu A_\mu^c +$$
$$\partial_\nu\,\epsilon^b(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a A_\mu^c + \frac{1}{g}\partial_\nu\,\partial_\mu\,\epsilon^a(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} = 0$$

$$\epsilon^b(x)\left[\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,f_{bc}^a A_\mu^c + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a \partial_\nu A_\mu^c\right] + \partial_\nu\,\epsilon^b(x)\left[\frac{1}{g}\frac{\partial \mathcal{L}_A}{\partial A_\nu^b} + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a A_\mu^c\right] +$$
$$\partial_\nu\,\partial_\mu\,\epsilon^a(x)\frac{1}{2g}\left[\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\mu A_\nu^a\right)} + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)}\right] = 0$$

Equações Hierárquicas:

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a} f_{bc}^a A_\mu^c + \frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} f_{bc}^a \partial_\nu A_\mu^c = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} f_{bc}^a A_\mu^c + \frac{1}{g}\frac{\partial \mathcal{L}_A}{\partial A_\nu^b} = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial(\partial_\mu A_\nu^a)} + \frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} = 0$$

$$\mathcal{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{bc}^a A_\mu^b A_\nu^c$$

$$\delta \mathcal{F}_{\mu\nu}^a = \epsilon^b(x) f_{bc}^a \mathcal{F}_{\mu\nu}^c$$

$$\mathcal{L}_A \longrightarrow \mathcal{L}_A\left[A_\mu^a, \partial_\nu A_\mu^a\right] \longrightarrow \mathcal{L}'_A\left[\mathcal{F}_{\mu\nu}^a\right]$$

$$\frac{\partial \mathcal{L}'_A}{\partial \mathcal{F}_{\mu\nu}^a} f_{bc}^a \mathcal{F}_{\mu\nu}^c = 0$$

$$\frac{\partial \mathcal{L}'_A}{\partial A_\mu^a} = 0$$

Grupo de Transformações U(1)

$$\phi \longrightarrow e^{i\epsilon}\phi$$

$$\phi' = e^{i\epsilon}\phi \cong \phi + i\epsilon\phi$$

$$\phi^* \longrightarrow e^{-i\epsilon}\phi^*$$

$$\phi'^* = e^{-i\epsilon}\phi^* \cong \phi^* - i\epsilon\phi^*$$

$$\begin{pmatrix}\phi' \\ \phi'^*\end{pmatrix} = \begin{pmatrix}\phi \\ \phi^*\end{pmatrix} + \epsilon\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix}\begin{pmatrix}\phi \\ \phi^*\end{pmatrix}$$

$$\left[I_{(a)}, I_{(b)}\right] = 0 \longrightarrow f_{bc}^a = 0$$

Transformação Local

$$\delta A^a_\mu = \epsilon^b(x) f^a_{bc} A^c_\mu + \frac{1}{g}\partial_\mu \epsilon^a(x) \longrightarrow \delta A_\mu = \frac{1}{g}\partial_\mu \epsilon(x)$$

$$\delta \mathcal{F}^a_{\mu\nu} = \epsilon^b(x) f^a_{bc} \mathcal{F}^c_{\mu\nu} \longrightarrow \delta \mathcal{F}_{\mu\nu} = 0$$

$$\mathcal{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a_{bc} A^b_\mu A^c_\nu \longrightarrow \mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\nabla_\mu \Phi^A = \partial_\mu \Phi^A - g A^a_\mu I^A_{(a)B} \Phi^B$$

$$\nabla_\mu \phi = \partial_\mu \phi - igA_\mu \phi$$

$$\nabla_\mu \phi^* = \partial_\mu \phi^* + igA_\mu \phi^*$$

Campo de Stueckelberg

$$\mathcal{L}_m \longrightarrow \mathcal{L}_m[\mathcal{F}_{\mu\nu}, A_\mu]$$

$$\frac{\partial \mathcal{L}_m}{\partial \mathcal{F}_{\mu\nu}}(0) + \partial_\mu \epsilon(x) \frac{1}{g} \frac{\partial \mathcal{L}_m}{\partial A_\mu} = 0 \longrightarrow \frac{\partial \mathcal{L}_m}{\partial A_\mu} = 0$$

$$\delta B = m\epsilon(x) \longrightarrow \delta\left(\partial_\mu B\right) = \partial_\mu (\delta B)$$
$$= m\partial_\mu \epsilon(x)$$

$$\mathcal{L}_m \longrightarrow \mathcal{L}_m[\mathcal{F}_{\mu\nu}, A_\mu] \longrightarrow \mathcal{L}_S[\mathcal{F}_{\mu\nu}, A_\mu, \partial_\mu B, B]$$

$$\frac{\partial \mathcal{L}_S}{\partial \mathcal{F}_{\mu\nu}}(0) + \partial_\mu \epsilon(x) \frac{1}{g}\frac{\partial \mathcal{L}_S}{\partial A_\mu} + \frac{\partial \mathcal{L}_S}{\partial B} m\epsilon(x) + \frac{\partial \mathcal{L}_S}{\partial(\partial_\mu B)} m\partial_\mu \epsilon(x) = 0$$

$$\frac{\partial \mathcal{L}_S}{\partial B} m\epsilon(x) + \left[\frac{1}{g}\frac{\partial \mathcal{L}_S}{\partial A_\mu} + m\frac{\partial \mathcal{L}_S}{\partial(\partial_\mu B)}\right]\partial_\mu \epsilon(x) = 0$$

$$\frac{1}{g}\frac{\partial \mathcal{L}_S}{\partial A_\mu} + m\frac{\partial \mathcal{L}_S}{\partial(\partial_\mu B)} = 0$$

$$\Rightarrow \quad V_\mu = A_\mu - \frac{1}{mg}\partial_\mu B$$

$$\delta V_\mu = \delta A_\mu - \frac{1}{mg}\partial_\mu(\delta B) = \frac{1}{g}\partial_\mu \epsilon(x) - \frac{1}{mg}\left[m\partial_\mu \epsilon(x)\right] = 0$$

$$\frac{\partial \mathcal{L}_S}{\partial \mathcal{F}_{\mu\nu}}(0) + \frac{\partial \mathcal{L}_S}{\partial V_\mu}(0) = 0$$

$$\mathcal{L}_m \longrightarrow \mathcal{L}_m\left[\mathcal{F}_{\mu\nu}, A_\mu\right] \longrightarrow \mathcal{L}_S\left[\mathcal{F}_{\mu\nu}, V_\mu\right]$$

$$\mathcal{F}_{\mu\nu}^V = \partial_\mu V_\nu - \partial_\nu V_\mu = \partial_\mu \left[A_\nu - \frac{1}{mg}\partial_\nu B\right] - \partial_\nu \left[A_\mu - \frac{1}{mg}\partial_\mu B\right]$$

$$\mathcal{F}_{\mu\nu}^V = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{mg}\partial_\mu \partial_\nu B + \frac{1}{mg}\partial_\nu \partial_\mu B = \mathcal{F}_{\mu\nu}$$

$$\mathcal{L}_m \longrightarrow \mathcal{L}_m[\mathcal{F}_{\mu\nu}, A_\mu] \longrightarrow \mathcal{L}_S[\mathcal{F}_{\mu\nu}, V_\mu] \longrightarrow \mathcal{L}_S[\mathcal{F}^V_{\mu\nu}, V_\mu]$$

Lagrangiano de um campo massivo:

$$\mathcal{L}_S[\mathcal{F}^V_{\mu\nu}, V_\mu] = -\frac{1}{4}\mathcal{F}^V_{\mu\nu}\mathcal{F}^{V\mu\nu} + \frac{1}{2}(mg)^2 V_\mu V^\mu$$

$$\partial_\nu \mathcal{F}^{V\nu\mu} + (mg)^2 V^\mu = 0 \longrightarrow \partial_\mu V^\mu = \partial_\mu \left(A^\mu - \frac{1}{mg}\partial^\mu B\right) = 0$$

$$\mathcal{L}_S[\mathcal{F}_{\mu\nu}^V, V_\mu] = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}(mg)^2\left(A_\mu - \frac{1}{mg}\partial_\mu B\right)\left(A^\mu - \frac{1}{mg}\partial^\mu B\right)$$

Gauge Hooft:

$$g = \partial_\mu A^\mu + \alpha m g B$$

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha}\left(\partial_\mu A^\mu + \alpha m g B\right)(\partial^\nu \mathrm{A}_\nu + \alpha m g B)$$

$$\partial_\mu A'^\mu + \alpha m B' = \partial_\mu\left(A^\mu + \frac{1}{g}\partial^\mu \epsilon(x\right)\Bigg) + \alpha m g\big(B + m\epsilon(x)\big)$$

$$\partial_\mu A'^\mu + \alpha m B' = \partial_\mu A^\mu + \alpha m g B + \left(\frac{1}{g}\partial_\mu \partial^\mu \epsilon(x) + \alpha m^2 g \epsilon(x\right)\Bigg)$$

$$\partial_\mu \partial^\mu \epsilon(x) + \alpha(mg)^2\epsilon(x) = 0$$

Gauge de Stueckelberg-Feynman:

$$\mathcal{L}_{gf} = -\frac{1}{2}\left(\partial_\mu A^\mu + mgB\right)(\partial^\nu A_\nu + mgB)$$

$$\left[\partial_\mu \partial^\mu + (mg)^2\right]\epsilon(x) = 0$$

Condição Gupta-Bleuler:

$$(\partial^\nu A_\nu)^{(-)} \parallel Phys\rangle = 0 \longrightarrow (\partial^\nu A_\nu + mgB)^{(-)} \parallel Phys\rangle = 0$$

$$\mathcal{L}_{Stu} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}(mg)^2\left(A_\mu - \frac{1}{mg}\partial_\mu B\right)\left(A^\mu - \frac{1}{mg}\partial^\mu B\right)$$

$$-\frac{1}{2}\left(\partial_\mu A^\mu + mgB\right)\left(\partial^\nu A_\nu + mgB\right)$$

$$\mathcal{L}_{Stu} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}(mg)^2\left(A_\mu A^\mu - \frac{1}{mg}\partial_\mu B A^\mu - \frac{1}{mg}A_\mu\partial^\mu B + \frac{1}{(mg)^2}\partial^\mu B\partial_\mu B\right)$$

$$-\frac{1}{2}\left(\partial_\mu A^\mu\partial^\nu A_\nu + mgB\partial^\nu A_\nu + mgB\partial_\mu A^\mu + (mg)^2B^2\right)$$

$$\mathcal{L}_{Stu} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}(mg)^2A_\mu A^\mu - \frac{1}{2}\partial_\mu A^\mu\partial^\nu A_\nu + \frac{1}{2}\partial_\mu B\partial^\mu B - \frac{1}{2}(mg)^2B^2$$

$$\partial_\mu\partial^\mu A^\nu + (mg)^2A^\nu = 0 \quad \wedge \quad \partial_\mu\partial^\mu B + (mg)^2B = 0$$

OBRIGADO