Inflation

How quantum correlations are detected in the Universe

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Presentation for the Quantum Field Theory I course

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Inflation

People say that quantum physics describes small-scale systems and general relativity describes large-scale systems. Inflation is one counterexample, where quantum correlations generate the seeds of the perturbations in the Universe, forming the structure of galaxies and the Cosmic Microwave Background radiation.

Brief intro to Cosmology

Cosmology is the study of the Universe in its largest scales. We first assume all distributions are statistically homogeneous and isotropic (when averaged over sufficiently large scales). Under homogeneity, we work with the FRW spacetime:

$$\boxed{\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right]}$$

Brief intro to Cosmology

The scale factor a(t) determines the expansion of the Universe. Calculating the Einstein equations, we get to the Friedmann equations for the scale factor:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} ,$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) .$$

Brief intro to Cosmology

The evolution of scale factor a(t) depends on the contents of the Universe. Non-relativistic matter density scales with a^{-3}, radiation and ultra-relativistic matter energy density scales as a^{-4} and a cosmological constant energy density is constant. Thus, the first Friedmann equation can be written as:

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

with parameters H_0, Ω_Λ, Ω_m.

Brief intro to Cosmology

This model, called ΛCDM, describes all cosmological data (CMB, supernovae, galaxy position and shape distributions...) very well with the parameters:

$$|\Omega_k| \leq 0.01\,, \quad \Omega_r = 9.4 \times 10^{-5}\,, \quad \Omega_m = 0.32\,, \quad \Omega_\Lambda = 0.68$$

Brief intro to Cosmology

Since the CMB radiation is almost homogeneous, we assume the matter distribution in the Universe began very homogeneous. Small perturbations from homogeneity are described by cosmological perturbation theory. We now add small, position-dependent perturbations to the homogeneous Universe:

$$\mathrm{d}s^2 = a^2(\tau)\left[(1+2\Psi)\mathrm{d}\tau^2 - (1-2\Phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

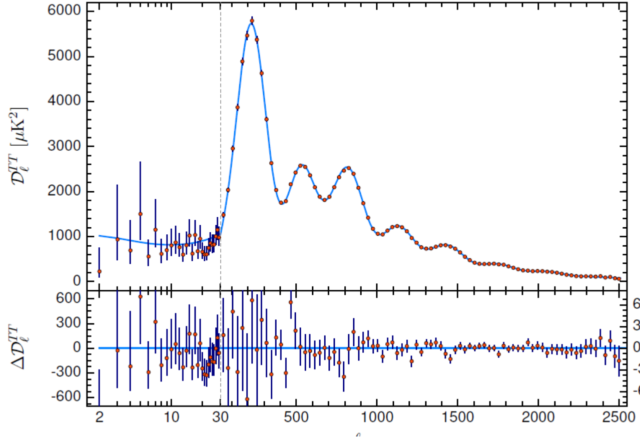
Brief intro to Cosmology

In the same way, we can get linear perturbation equations for the metric and matter perturbations:

$$
\begin{aligned}
\nabla^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) &= 4\pi G a^2\,\delta\rho\ , \\
\Phi' + \mathcal{H}\Phi &= -4\pi G a^2(\bar{\rho} + \bar{P})v \\
\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi &= 4\pi G a^2\,\delta P\ .
\end{aligned}
$$

These equations can be used to describe the CMB radiation anisotropies with remarkable fit.

Brief intro to Cosmology

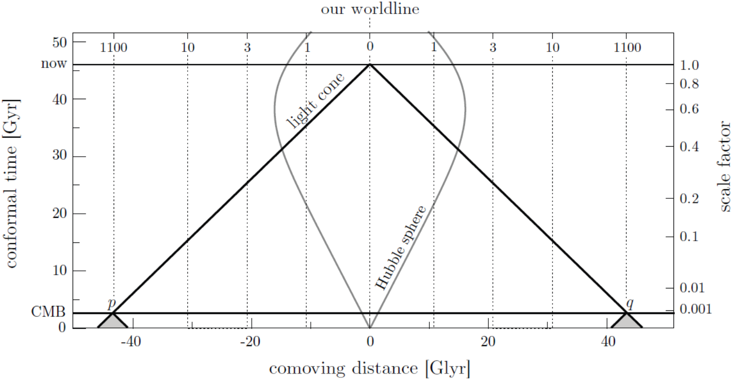


CMB Temperature Power Spectrum measured by the Planck Collaboration (arxiv:1807.06209)

Inflation

Inflation is a cosmological mechanism proposed to solve the horizon problem. Suppose we receive two CMB photons from opposite directions. The CMB is highly isotropic, so those photons will have almost the exact same temperature. However, the events for the emission of both photons are not causally connected!

Inflation



our worldline
50
now
40
30
20
10
CMB
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1100
10
3
1
0
1
3
10
1100
1.0
0.8
0.6
0.4
0.2
0.1
0.01
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light cone
Hubble sphere
p
q
-40
-20
0
20
40
conformal time [Gyr]
scale factor
comoving distance [Glyr]

Inflation

One solution to the horizon problem is to hypothesize an additional field to the Standard Model. The particle horizon for the event of CMB photon emission is given by[1]

$$\chi = \int_{t_i}^{t_*} \frac{dt}{a(t)} = \int_{\ln a_i}^{\ln a_*} \frac{d \ln a}{aH(a)} \tag{1}$$

where t_i is the Big Bang moment and t_* is the moment when the CMB photons were emitted (this moment is usually called recombination).

[1] Sometimes cosmologists refer to the horizon as $1/aH$. χ and $1/aH$ have the same order of magnitude

Inflation

Suppose that, just after the Big Bang, the Universe was dominated by a single type of particle (or a single field) whose energy density scales as $\rho \propto \mathrm{a}^{-3(1+\mathrm{w})}$, with $\mathrm{w} < -1/3$. Using the Friedmann equations, we can show that the lower bound of the integral

$$\chi = \int_{\mathrm{t_i}}^{\mathrm{t_*}} \frac{\mathrm{dt}}{\mathrm{a(t)}} \tag{2}$$

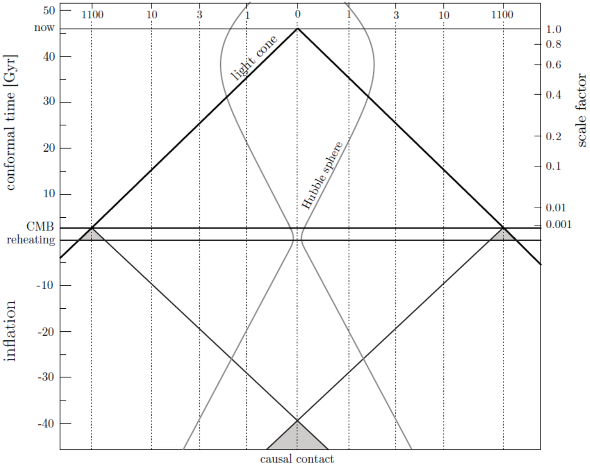
tends to

$$\lim_{\mathrm{a}\to 0} \frac{2}{\mathrm{H_0(1+3w)}} \mathrm{a}^{(1+3\mathrm{w})/2} = -\infty \tag{3}$$

Inflation

This makes the horizon become infinite: in the past, all particles would have causal contact with each other because the Universe would be expanding too quickly! In fact, using the Friedmann equations, $\mathrm{a} \propto \mathrm{t}^{\frac{2}{3(1+\mathrm{w})}} > \mathrm{t}$: the Universe must have an accelerated expansion.

Inflation



50
now
40
30
20
10
CMB
reheating
-10
-20
-30
-40
conformal time [Gyr]
inflation
1100
10
3
1
0
1
3
10
1100
1.0
0.8
0.6
0.4
0.2
0.1
0.01
0.001
scale factor
light cone
Hubble sphere
causal contact

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The Inflaton Field

The simplest field that could achieve this is a scalar field.
Suppose an action:

$$\mathrm{S} = \int \mathrm{d}^4\mathrm{x}\sqrt{-\mathrm{g}}\left(\frac{1}{2}\mathrm{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \mathrm{V}(\phi)\right) \tag{4}$$

The Inflaton Field

Assuming a homogeneous field (i.e. $\nabla\phi = 0$), we have the action

$$\mathrm{S} = \int \mathrm{d}^4\mathrm{x}\, \mathrm{a}^3 \left(\frac{\dot{\phi}^2}{2} - \mathrm{V}(\phi) \right) \tag{5}$$

The Inflaton Field

The action can be used to derive the Klein-Gordon equations for a homogeneous field in the FRW metric:

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \frac{\mathrm{dV}}{\mathrm{d}\phi} = 0 \tag{6}$$

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The Inflaton Field

Just like the Klein-Gordon equation, we can have a classical intuition about this field: it is rolling down a potential ramp of shape $\mathrm{V}(\phi)$. Now, it has an extra "friction" term $3\mathrm{H}\dot{\phi}$: the Hubble friction

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \frac{\mathrm{dV}}{\mathrm{d}\phi} = 0 \tag{7}$$

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The Inflaton Field

The energy density and pressure (spatial part of the energy-momentum tensor) of the classical field is given by:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + \mathrm{V}(\phi); \quad \mathrm{P}_\phi = \frac{\dot{\phi}^2}{2} - \mathrm{V}(\phi) \tag{8}$$

The Inflaton Field

The energy scaling of the inflaton field is then determined by its equation of state:

$$\mathrm{w}_\phi = \frac{\mathrm{P}_\phi}{\rho_\phi} = \frac{\frac{\dot{\phi}^2}{2} - \mathrm{V}(\phi)}{\frac{\dot{\phi}^2}{2} + \mathrm{V}(\phi)} \tag{9}$$

The Inflaton Field

$$\mathrm{w}_\phi = \frac{\mathrm{P}_\phi}{\rho_\phi} = \frac{\frac{\dot{\phi}^2}{2} - \mathrm{V}(\phi)}{\frac{\dot{\phi}^2}{2} + \mathrm{V}(\phi)}$$

If the potential energy is much greater than the kinetic energy, the field will be almost frozen and $\mathrm{w}_\phi \approx -1 < -1/3$. A slowly-rolling scalar field can then realize inflation!

The Inflaton Field

For a slow-rolling field, with $\mathrm{w}_\phi \approx -1$, the scale factor evolves exponentially as

$$\mathrm{a} \propto \mathrm{e}^{\mathrm{Ht}}$$

and the Hubble factor H is constant. The metric becomes

$$\mathrm{ds}^2 = -\mathrm{dt}^2 + \mathrm{e}^{2\mathrm{Ht}}(\mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2) \tag{10}$$

we call this solution the de Sitter Universe.

Quantizing the Inflaton Field

Let's try to quantize the scalar field in our FRW metric. We will see that, even though the expected value of the field fluctuations is zero, it has fluctuations that can generate the perturbations in the Universe!

Quantizing the Inflaton Field

We will start by the scalar field action:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \tag{11}$$

actually, we will use the conformal time $d\tau = dt/a$, so that $d^4x = d\tau d^3x$ and $\sqrt{-g} = a^4$.

Quantizing the Inflaton Field

Expanding the summations (I denote $\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = \phi'$):

$$\mathrm{S} = \int \mathrm{d}\tau \mathrm{d}^3\mathrm{x} \left(\frac{\mathrm{a}^2 \phi'^2}{2} - \frac{\mathrm{a}^2 \nabla \phi^2}{2} - \mathrm{a}^4 \mathrm{V}(\phi) \right) \tag{12}$$

Quantizing the Inflaton Field

We can parametrize the scalar field as:

$$\phi(\tau, \vec{\mathrm{x}}) = \bar{\phi}(\tau) + \frac{\mathrm{f}(\tau, \vec{\mathrm{r}})}{\mathrm{a}(\tau)} \tag{13}$$

and again, assume that the perturbations are small so we can use perturbation theory, treating f as a first-order term.

Quantizing the Inflaton Field

In order to find equations for the field perturbations f, we collect all quadratic terms on f^2:

$$S^{(2)} = \int d\tau d^3x \frac{1}{2}\left[(f')^2 - (\nabla f)^2 + \frac{a''}{a}f^2\right] \tag{14}$$

this is the action that we aim to quantize;

[2] the linear terms vanish due to the homogeneous part satisfying the KG equation

Quantizing the Inflaton Field

Before quantizing, let's obtain the classical field equation for the inflaton perturbation using Euler-Lagrange:

$$\mathrm{f}'' - \nabla^2 \mathrm{f} - \frac{\mathrm{a}''}{\mathrm{a}}\mathrm{f} = 0 \tag{15}$$

this is the Mukhanov-Sasaki equation. We can decompose it into Fourier modes:

$$\mathrm{f}'' + \left(\mathrm{k}^2 - \frac{\mathrm{a}''}{\mathrm{a}}\right)\mathrm{f} = 0 \tag{16}$$

this is like an harmonic oscillator with frequency $\omega_{\mathrm{k}}^2 = \mathrm{k}^2 - \frac{\mathrm{a}''}{\mathrm{a}}$

Quantizing the Inflaton Field

The canonical conjugate momentum to f is:

$$\pi = \frac{\partial \mathcal{L}}{\partial \mathrm{f}'} = \mathrm{f}' \tag{17}$$

and we now can make the canonical quantization of the field f by promoting f and π to operators satisfying

$$[\mathrm{f}(\tau, \vec{\mathrm{r}}), \pi(\tau, \vec{\mathrm{r}'})] = i\delta^3(\vec{\mathrm{r}} - \vec{\mathrm{r}'}) \tag{18}$$

Quantizing the Inflaton Field

We can decompose the operators $\mathrm{f}(\tau,\vec{\mathrm{r}})$ and $\pi(\tau,\vec{\mathrm{r}})$ in Fourier modes, obtaining

$$[\mathrm{f}(\tau,\vec{\mathrm{k}}),\pi(\tau,\vec{\mathrm{k}'})] = \mathrm{i}\delta^3(\vec{\mathrm{k}}+\vec{\mathrm{k}'}) \tag{19}$$

Quantizing the Inflaton Field

Defining the creation and annihilation operators is a bit more difficult. Let's go back to the quantum harmonic oscillator, whose solution is (page 38 in the lecture notes):

$$\mathrm{q(t)} = \frac{1}{\sqrt{2\omega}}\left(\hat{\mathrm{a}}\mathrm{e}^{-\mathrm{i}\omega \mathrm{t}} + \hat{\mathrm{a}}^{\dagger}\mathrm{e}^{\mathrm{i}\omega \mathrm{t}}\right) \tag{20}$$

but this solution is contingent in the boundary conditions, or the vacuum choice.

Quantizing the Inflaton Field

Notice that

$$\frac{1}{\sqrt{2\omega}} e^{\pm i\omega t}$$

are solutions to the classical equations $\ddot{q} + \omega^2 q = 0$ and $\ddot{q}^* + \omega^2 q^* = 0$. We chose the boundary conditions:

$$q(t \to -\infty) = Ae^{i\omega t}$$
$$q^*(t \to \infty) = Be^{-i\omega t}$$

which are equations 32.1 in the lecture notes. We must choose these boundary conditions: they correspond to the vacuum choice, the Wick rotation and the Feynman prescription.

Quantizing the Inflaton Field

We must adapt our quantization prescription, because now the frequency $\omega^2 = \mathrm{k}^2 + \mathrm{a}''/\mathrm{a}$ is a function of time. We will define our creation and annihilation operators as[3].:

$$\mathrm{f}(\vec{\mathrm{k}}, \tau) = \mathrm{f}_{\mathrm{cl}}(\vec{\mathrm{k}}, \tau)\hat{\mathrm{a}} + \mathrm{f}^*_{\mathrm{cl}}(\vec{\mathrm{k}}, \tau)\hat{\mathrm{a}}^\dagger \tag{21}$$

[3] I'm using the hat notation to differentiate the annihilation operator $\hat{\mathrm{a}}$ from the scale factor a

Quantizing the Inflaton Field

In analogy to the quantum harmonic oscillator, $\mathrm{f_{cl}}(\mathrm{k}, \tau)$ are the solutions of the classical field equation, which now is Mukhanov-Sasaki equation.

$$\mathrm{f}_\mathrm{k}'' + \left(\mathrm{k}^2 - \frac{\mathrm{a}''}{\mathrm{a}}\right)\mathrm{f}_\mathrm{k} = 0 \tag{22}$$

For $\tau \to -\infty$, the inflaton field will be slowly rolling with $\mathrm{w}_\phi \approx -1$ and, using the Friedmann equation, $\mathrm{a}''/\mathrm{a} \approx 2/\tau^2$.

Quantizing the Inflaton Field

This way, in the infinite past, $\tau \to -\infty$, we had:

$$f_k'' + k^2 f_k = 0 \tag{23}$$

which is the harmonic oscillator equation! Thus, assuming slow-roll inflation, we can still use our Minkowski boundary condition $f_k(\tau \to -\infty) = \frac{1}{\sqrt{2k}} e^{ik\tau}$.

Quantizing the Inflaton Field

Interestingly, in an Universe dominated by dark energy in the far future $\tau \to +\infty$, we can also reduce the Mukhanov-Sasaki equation to an harmonic oscillator

$$\mathrm{f}_{\mathrm{k}}^{*\prime\prime} + \mathrm{k}^2 \mathrm{f}_{\mathrm{k}}^{*} = 0 \tag{24}$$

we can also use our Minkowski boundary condition $\mathrm{f}_{\mathrm{k}}^{*}(\tau \to +\infty) = \frac{1}{\sqrt{2\mathrm{k}}} \mathrm{e}^{-\mathrm{ik}\tau}$.

Quantizing the Inflaton Field

This choice of boundary conditions leads to an unique vacuum state, the Bunch-Davies vacuum. This was studied in ”Quantum field theory in de Sitter space: renormalization by point-splitting” https://inspirehep.net/literature/134659

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Quantizing the Inflaton Field

For a de Sitter spacetime, $a''/a = 2/\tau$. It's not hard to show that the solutions of the Mukhanov-Sasaki equation are:

$$f_{cl,k}(\tau) = A\frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + B\frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right) \qquad (25)$$

Quantizing the Inflaton Field

By the initial condition in the infinite past, we must impose

$$f_{cl,k}(\tau) = \frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right) \tag{26}$$

This is the last piece for the quantization.

Quantizing the Inflaton Field

Inserting the field operator solution in the canonical commutation relations, we obtain, as expected:

$$[\hat{a}_{\mathrm{k}}, \hat{a}^{\dagger}_{\mathrm{k}'}] = \delta^{(3)}(\vec{\mathrm{k}} - \vec{\mathrm{k}'}) \tag{27}$$

Quantizing the Inflaton Field

The scalar field operator is

$$f(\tau, \vec{r}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[f_{cl,k}(\tau)\hat{a}_k + f^*_{cl,k}(\tau)\hat{a}^\dagger_k \right] e^{i\vec{k}\cdot\vec{r}} \tag{28}$$

The creation and annihilation operators satisfy $[\hat{a}_k, \hat{a}^\dagger_{k'}] = \delta^{(3)}(\vec{k} - \vec{k'})$. The Hilbert space is the Fock space.

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Quantum Fluctuations

We are now ready to calculate vacuum expected values. It's easy to see that the expected value of the field in vacuum is zero:

$$\langle 0|\, \mathrm{f}(\tau, \vec{\mathrm{r}})\, |0\rangle = \int \frac{\mathrm{d}^3\mathrm{k}}{(2\pi)^{3/2}} \left[\mathrm{f}_{\mathrm{cl,k}}(\tau) \langle 0|\, \hat{\mathrm{a}}_{\mathrm{k}}\, |0\rangle + \mathrm{f}^*_{\mathrm{cl,k}}(\tau) \langle 0|\, \hat{\mathrm{a}}^\dagger_{\mathrm{k}}\, |0\rangle \right] \mathrm{e}^{\mathrm{i}\vec{\mathrm{k}}\cdot\vec{\mathrm{r}}} = 0 \tag{29}$$

Quantum Fluctuations

However, the field variance in the vacuum is not zero[4]:

$$\langle 0| \, |f(\tau, \vec{0})|^2 \, |0\rangle = \int \frac{d^3k}{(2\pi)^{3/2}} \int \frac{d^3k'}{(2\pi)^{3/2}}$$

$$\langle 0| \left[f_{cl,k}(\tau)\hat{a}_k + f^*_{cl,k}(\tau)\hat{a}^\dagger_k \right] \left[f_{cl,k'}(\tau)\hat{a}_{k'} + f^*_{cl,k'}(\tau)\hat{a}^\dagger_{k'} \right] |0\rangle$$

[4] I'm choosing the "origin" just to show that this is not zero

Quantum Fluctuations

However, the field variance in the vacuum is not zero:

$$\langle 0| \, |f(\tau, \vec{0})|^2 \, |0\rangle = \int \frac{d^3k}{(2\pi)^3} |f_{cl,k}(\tau)|^2$$

$$\langle 0| \, |f(\tau, \vec{0})|^2 \, |0\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} \left(1 + \frac{1}{k^2\tau^2}\right)$$

Quantum Fluctuations

In cosmology, correlations of statistically isotropic distributions are encoded by the power spectrum. We can define the dimensionless power spectrum by:

$$\Delta_f^2(k,\tau) = \frac{d\,\langle |f(\tau,\vec{0})|^2 \rangle}{d\ln k}$$

$$\Delta_f^2(k,\tau) = \frac{k^3}{2\pi^2}\frac{1}{2k}\left(1+\frac{1}{k^2\tau^2}\right)$$

Quantum Fluctuations

We can finally relate back to the field perturbations

$$\Delta^2_{\delta\phi}(\mathrm{k},\tau) = \Delta_{\mathrm{f}}(\mathrm{k},\tau)/\mathrm{a}^2$$

$$\Delta^2_{\delta\phi}(\mathrm{k},\tau) = \left(\frac{\mathrm{H}}{2\pi}\right)^2\left[1+\left(\frac{\mathrm{k}}{\mathrm{aH}}\right)^2\right]$$

The power spectrum will depend on the ratio between the mode scale k^{-1} and the "horizon" scale $1/\mathrm{aH}$!

Quantum Fluctuations

Due to inflation, the horizon scale diverges and the ratio becomes negligible - we refer to this as the superhorizon regime. Thus, the power spectrum becomes:

$$\Delta^2_{\delta\phi}(\mathrm{k},\tau) \rightarrow \left(\frac{\mathrm{H}}{2\pi}\right)^2 \Bigg|_{\mathrm{k=aH}}$$

After the horizon crossing moment, $\mathrm{k} = \mathrm{aH}$, the amplitudes of the Fourier modes freeze

$$\mathrm{f}''_{\mathrm{k}} + \left(\mathrm{k}^2 - \frac{\mathrm{a}''}{\mathrm{a}}\right)\mathrm{f}_{\mathrm{k}} = 0 \quad \mathrm{k}^2 \gg \frac{\mathrm{a}''}{\mathrm{a}}$$

Quantum Fluctuations

The inflaton perturbations generate curvature perturbations $\mathcal{R}$ through the perturbed Einstein equations! Their power spectrum is related by

$$\Delta_{\mathcal{R}}^2 = \frac{1}{2\epsilon}\frac{\Delta_{\delta\phi}^2}{\mathrm{M}_{\mathrm{Pl}}^2} \equiv \mathrm{A}_{\mathrm{s}} \tag{30}$$

where $\epsilon = \frac{\dot{\phi}^2}{2\mathrm{H}^2\mathrm{M}_{\mathrm{Pl}}^2}$. The curvature power spectrum does not depend on k (just through the horizon crossing moment)!

Quantum Fluctuations

The ϵ quantity can be expressed in terms of the scalar field potential

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 \tag{31}$$

if we can measure the power spectrum, we can have information about the inflaton potential!

Quantum Fluctuations

The calculations were made assuming a perfect inflation. This means that $w_\phi = -1$ - the field is frozen until it eventually decays into the Standard Model fields. If we make corrections accounting for the slow rolling of the field, we get a more general power spectrum, parametrized by:

$$\Delta^2_{\mathcal{R}} = A_s \left(\frac{k}{k_p}\right)^{n_s - 1} \tag{32}$$

Quantum Fluctuations

Just like A_s, the parameter n_s also has information about the shape of the potential:

$$n_s - 1 = -2\epsilon - M_{Pl}^2 \frac{V''}{V} \tag{33}$$

Quantum Fluctuations

Today, using data from CMB, baryonic acoustic oscillations, galaxy position and shape distributions, redshift space distortions, luminosity distances from Type-IA supernovae... we constrain $\mathrm{A_s}$ and $\mathrm{n_s}$

$$\mathrm{A_s} \approx 2.1 \times 10^{-9}; \quad \mathrm{n_s} \approx 0.96 \tag{34}$$

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Tensor Quantum Fluctuations

Gravitational waves can also be quantized! We consider the tensor perturbations in the metric:

$$\mathrm{ds}^2 = \mathrm{a}^2[\mathrm{d}\tau^2 - (\delta_{\mathrm{j}} - 2\mathrm{E}_{\mathrm{ij}})\mathrm{dx}^{\mathrm{i}}\mathrm{dx}^{\mathrm{j}}] \tag{35}$$

where E_{ij} is a transverse ($\nabla_{\mathrm{i}}\mathrm{E}^{\mathrm{ij}} = 0$) and traceless 3-tensor, and therefore has 2 free components.

Tensor Quantum Fluctuations

If we calculate the Einstein equations for E, we get two copies of the same inflaton action! Defining:

$$\frac{M_{Pl}}{2} aE = \frac{1}{\sqrt{2}} \begin{pmatrix} f_{+} & f_{\times} & 0 \\ f_{\times} & -f_{+} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{36}$$

we get

$$S = \frac{1}{2} \sum_{i=+,\times} \int d\tau d^3x \left[(f_i')^2 - (\nabla f_i)^2 + \frac{a''}{a} f_i^2 \right] \tag{37}$$

Tensor Quantum Fluctuations

And we can use the same steps to calculate the primordial gravitational wave power spectrum:

$$\Delta_t^2 = \frac{2}{\pi^2} \frac{H^2}{M_{Pl}^2} \tag{38}$$

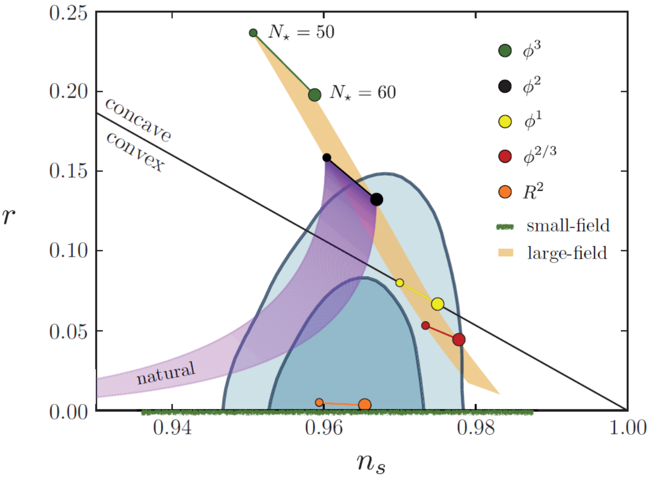
and again, we generalize to a slow-roll inflation and parametrize this by:

$$\Delta_t^2 = A_t \left(\frac{k}{k_p} \right)^{n_t} \tag{39}$$

Tensor Quantum Fluctuations

Primordial gravitational waves have not been detected so far. Their detection would be a direct evidence of the quantum inflation theory!

Tensor Quantum Fluctuations



Conclusions

- Inflation is a mechanism that solves the horizon problem and also generates perturbations from homogeneity
- These perturbations are detected today in the CMB radiation and in galaxy position and shape correlations
- These perturbations can be described by a quantum field
- In the scalar field inflation, the shape of the potential is not yet constrained
- Primordial gravitational waves should also obey the same correlations as the quantum field but, as of today, they haven't been detected

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Thanks, please make any questions or comments!