Renormalização do Seesaw Tipo 1

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Introduction

The porpuse of this final seminar is to apply concepts we leared along the semester (ressomation of diagrams, renormalization schemes, unstable prticles decays, running constant etc) to Type 1 Seesaw.

- In the first part, I'll renormalize the Majorana neutrino self-energy diagram at 1-loop order and diagonalise the propagator; I'll then compute it's contribution to CP violation parameter.
- In the second part, I'll integrate out these heavy degrees of fredoom and find an effective non-renormalizable operator at low energies; I'll then compute it's coupling costant running.

Introduction

Type 1 seesaw is a popular extension to SM that explains the smallness of neutrino masses and breaks $U(1)_{B-L}$ symmetry, then providing a new source of CP violation.
Type 1 seesaw lagrangian contains 3 new Majorana fields N_i that couples to SM particles in a Yukawa Type interaction.

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2}\overline{N_i^c} M_i N_i - (\lambda_{\alpha i}\overline{l_\alpha}\tilde{H} P_R N_i + h.c.) \tag{1}$$

Where $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$ and $\psi^c = C\overline{\psi}^T$ is the C-conjugated. We also assume Majorana masses are big $M_i >> \langle H^0 \rangle$.
Majorana condition means:

$$N_i = \nu_R + \nu_R^c \tag{2}$$

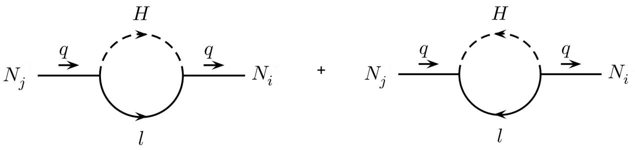
Spoiler: The P_R is the root of CP violation.

Introduction

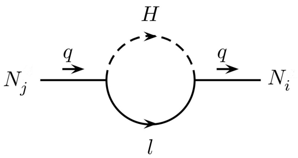
If there is something I learned from this semster is that the lagrangian is not the end of the story, and a ressumation is mandatory to fiind physical parameters and full propagators. I now turn to obtain

$$S_{(q)} = \frac{i}{\not{q} - M - \Sigma_{(q)}} \tag{3}$$

where $-i\Sigma_{(q)}$ equals, at leading order:



Calculating Chiral Self Energy Diagram

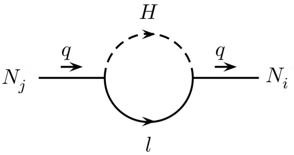


$$-i\hat{\Sigma}^{ji}_{R(q)} = \int \frac{d^4k}{(2\pi)^4}(-i\lambda^*_{\alpha i}\epsilon_{ab}P_L) \times \frac{i(\not{q} - \not{k})}{(q-k)^2} \times \frac{i}{k^2} \times (-i\lambda_{\alpha j}\epsilon_{ab}P_R) \tag{4}$$

$$= (h^\dagger_{i\alpha}h_{\alpha j}) \times \epsilon^2_{ab} \times \int \frac{d^4k}{(2\pi)^4}\frac{i(\not{q} - \not{k})}{(q-k)^2}\frac{1}{k^2} \times P_R \tag{5}$$

$$= K_{ij} \times 2 \times \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx \frac{1-x}{(l^2-\Delta^2)^2} \tag{6}$$

Calculating Chiral Self Energy Diagram



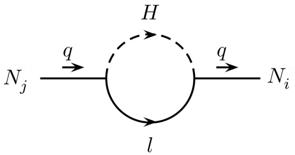
Where:

$$l^\mu = k^\mu - xq^\mu \tag{7}$$

$$\Delta^2 = -x(1-x)q^2 \tag{8}$$

$$K_{ij} = \sum_{\alpha=e,\mu,\tau} h^\dagger_{i\alpha} h_{\alpha j} \tag{9}$$

Calculating Chiral Self Energy Diagram



then:

$$-i\hat{\Sigma}^{ji}_{R(q)} = iK_{ij} \times 2 \times \int_0^1 \frac{dx}{16\pi^2}(1-x) \times \left(\frac{2}{\epsilon} + log\left(\frac{\tilde{\mu}^2}{\Delta^2}\right)\right) \times (\not{q} P_R) \tag{10}$$

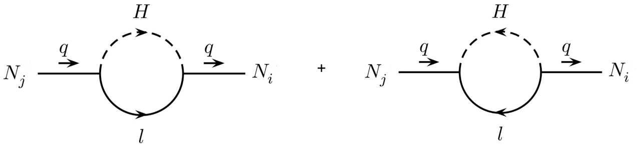
In $\overline{MS}$ scheme

$$\hat{\Sigma}_{R(q)} = K^T \times a_{(q^2)} \times (\not{q} P_R) \tag{11}$$

Where

$$a_{(q^2)} = \frac{1}{16\pi^2}\left(log\left(\frac{q^2}{\mu^2}\right) - 2 - i\pi\Theta_{(q^2)}\right) \tag{12}$$

Calculating Chiral Self Energy Diagram



Therefore, the full propagator in $\overline{MS}$ scheme becomes:

$$S_{(q)} = \frac{i}{\not{q} - M - \hat{\Sigma}_{\overline{MS}(q)}} \tag{13}$$

Where

$$\hat{\Sigma}_{\overline{MS}(q)} = \Sigma_{R(q^2)} \times (\not{q} P_R) + \Sigma_{L(q^2)} \times (\not{q} P_L) \tag{14}$$

And

$$\Sigma_{L(q^2)} = (\Sigma_{R(q^2)})^T = K \times a_{(q^2)} \tag{15}$$

Calculating Chiral Self Energy Diagram

We then have:

$$\left(\not{q} - M - \hat{\Sigma}_{\overline{MS}(q)}\right) S_{(q)} = i \implies \quad (16)$$

$$\left(\not{q} - M - \Sigma_{R(q^2)} \times (\not{q} P_R) + \Sigma_{L(q^2)} \times (\not{q} P_L)\right) S_{(q)} = i \quad (17)$$

Making the decomposition

$$S_{(q)} = P_R \times S^{RR}_{(q^2)} + P_L \times S^{LL}_{(q^2)} + P_L \not{q} \times S^{LR}_{(q^2)} + P_R \not{q} \times S^{RL}_{(q^2)} \quad (18)$$

Calculating Chiral Self Energy Diagram

We then find:

$$S^{LR} = M^{-1}(1 - \Sigma_R)S^{RR} \tag{19}$$

$$S^{RL} = M^{-1}(1 - \Sigma_L)S^{LL} \tag{20}$$

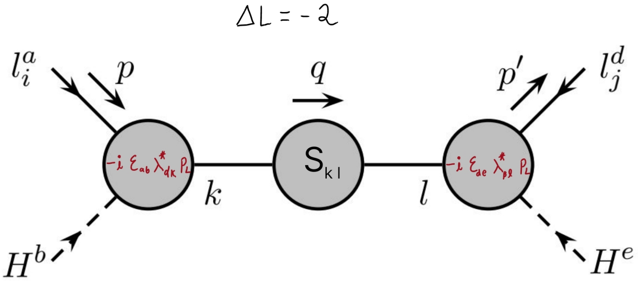
$$S^{RR} = \frac{i}{(1 - \Sigma_L)M^{-1}q^2(1 - \Sigma_R) - M} \tag{21}$$

$$S^{LL} = \frac{i}{(1 - \Sigma_R)M^{-1}q^2(1 - \Sigma_L) - M} \tag{22}$$

Here, we start to see that Majorana propagators provide a new source to distinguish matter from anti-matter, as matter fields couple to the " R " part of the propagator, and anti-matter couples to the " L " part.

Majorana Propagator Mediated $2 \to 2$ Scattering

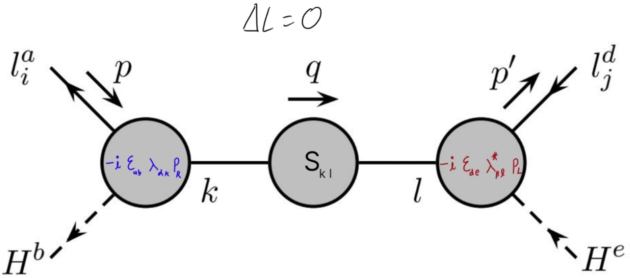
Each one of the propagator components accounts for a 2-body scattering processes. Below, we compute one lepton number violating interaction and remember the analogous happens with $\mathscr{M}_{RR}$



$$i\mathscr{M}_{LL} = \overline{v}_{(p')} \times (-i\epsilon_{de}\lambda^*_{\beta l}P_L) \times S^{LL}_{lk} \times (-i\epsilon_{ab}\lambda^*_{\alpha k}P_L) \times u_{(p)} \quad (23)$$

Majorana Propagator Mediated $2 \to 2$ Scattering

Also, below there is a lepton number conserving diagrams and the analogous happens for $\mathscr{M}_{RL}$



$$i\mathscr{M}_{LR} = \overline{v}_{(p')} \times (-i\epsilon_{de}\lambda^*_{\beta l}P_L) \times \not{q}S^{LR}_{lk} \times (-i\epsilon_{ab}\lambda_{\alpha k}P_R) \times v_{(p)} \tag{24}$$

$$= \overline{v}_{(p')} \times (-i\epsilon_{de}\lambda^*_{\beta l}P_L) \times \not{q}[M^{-1}(1-\Sigma_R)S^{RR}]_{lk} \times (-i\epsilon_{ab}\lambda_{\alpha k}P_R) \tag{25}$$

Majorana Neutrino Decay

The consistent definition of an on-shell contribution of a single heavy Majorana neutrino to the two-body scattering amplitudes requires that the transition amplitudes extracted from lepton-number conserving and lepton-number violating processes are the compatible.
Therefore, in order to be able to talk about decays, we have to diagonalize the S_{kl} matrix when the momenta are on-shell.

Majorana Neutrino Decay

S^{RR} and S^{LL} are symmetric complex matrices, since $\Sigma_R = (\Sigma_L)^T$. Therefore, they can be diagonalized by complex and orthogonal matrices.

$$S^{LL}_{(q^2)} = V^T_{(q^2)} \times M \times D_{(q^2)} \times V_{(q^2)} \tag{26}$$

$$S^{RR}_{(q^2)} = U^T_{(q^2)} \times M \times D_{(q^2)} \times U_{(q^2)} \tag{27}$$

Then, assuming diagonal elemts of $\Sigma_L = \Sigma_{D(q^2)} + \Sigma_{ND(q^2)}$ are much bigger then the non diagonal

$$D_{(q^2)} = M^{-1} \times V_{(q^2)} \times S^{LL}_{(q^2)} \times V^T_{(q^2)} = M^{-1} \times U_{(q^2)} \times S^{RR}_{(q^2)} \times U^T_{(q^2)} \tag{28}$$

$$= \frac{i}{q^2(1 - \Sigma_{D(q^2)})^2 - M^2} + O(\Sigma^2_{ND}) \tag{29}$$

Majorana Neutrino Decay

Expanding for $q^2 = M^2_{ph\,i}$

$$D_i = \frac{i(1-\Sigma_{D(M^2_{ph\,i})ii})^{-2}}{q^2 - M_i^2(1-\Sigma_{D(M^2_{ph\,i})ii})^{-2}} \tag{30}$$

We have

$$(1-\Sigma_{D(M^2_{ph\,i})ii})^{-2} \approx 1 + 2\Sigma_{ii} = 1 + 2K_{ii} \times a_{(M_{ph}{}_i^2)} \tag{31}$$

$$= 1 + \frac{K_{ii}}{8\pi^2}\left(log\left(\frac{M^2_{ph\,i}}{\mu^2}\right) - 2 - i\pi\right) \tag{32}$$

Letting $Z_i = 1 + \frac{K_{ii}}{8\pi^2}\left(log\left(\frac{M^2_{ph\,i}}{\mu^2}\right) - 2\right)$ we have

$$D_{i(q^2)} = \frac{iZ_i}{q^2 - M_i^2 Z_i + iM_i^2 \frac{K_{ii}}{8\pi}} + Finite Terms \tag{33}$$

Majorana Neutrino Decay

The Majorana Neutrino eigenstates have Masses and Decay Width (at tree level)

$$M_{ph\,i}^2 = M_i^2 \times Z_{i_{(M_{ph\,i}^2)}} \tag{34}$$

$$\Gamma_i = \frac{K_{ii} M_i}{8\pi} \tag{35}$$

What remains is to take out the diagonalizing matrices U and V.

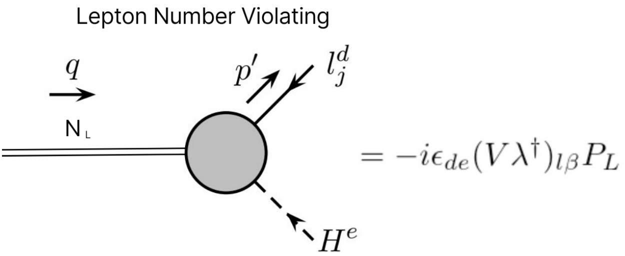
Majorana Neutrino Decay

If we substitute the diagonalization of neutrino propagator back in the scattering amplitude, we find:

$$i\mathscr{M}_{LL} = \overline{v}_{(p')} \times (-i\epsilon_{de}\lambda^{*}_{\beta l}P_L) \times (V^{T}_{(q^2)} \times M \times D_{(q^2)} \times V_{(q^2)})_{lk} \times (-i\epsilon_{ab}\lambda \quad (36)$$

$$= \overline{v}_{(p')} \times (-i\epsilon_{de}(V\lambda^{\dagger})_{l\beta}P_L) \times M_l D_l \times (-i\epsilon_{ab}(V\lambda^{\dagger})_{l\alpha}P_L)u_{(p)} \quad (37)$$

Therefore, in Lepton Number Violating scatterings, the eigenstate N_l (at tree level)



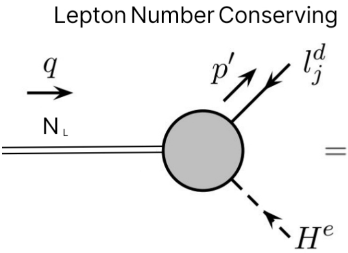
Majorana Neutrino Decay

The Lepton Number Conserving scatterings provides the kinect part of neutrino eigenstate propagator

$$i\mathscr{M}_{LR} = \overline{v}_{(p')} \times (-i\epsilon_{de}\lambda^*_{\beta l}P_L) \times \not{q}[M^{-1}(1-\Sigma_R)U^T_{(q^2)}MD_{(q^2)}U_{(q^2)}]_{lk} \times (- \tag{38}$$

$$= \overline{v}_{(p')} \times [-i\epsilon_{de}(MU(1-\Sigma_L)M^{-1}\lambda^\dagger)_{l\beta}P_L] \times \not{q}D_l \times (-i\epsilon_{ab}(U^T)_{l\alpha} \tag{39}$$

Therefore, in Lepton Number Conserving scatterings, the eigenstate N_l (at tree level)



$$= -i\epsilon_{de}(MU(1-\Sigma_L)M^{-1}\lambda^\dagger)_{l\beta}P_L$$

Majorana neutrino Decay

The analogous also happens to the second lepton number conserving amplitude. From which we obtain:

$$U = M \times V \times (1 - \Sigma_R) \times M^{-1} \tag{40}$$

$$V = M \times U \times (1 - \Sigma_L) \times M^{-1} \tag{41}$$

This calculation is not iluminating, but it's solution is

$$U_{(q^2)} = 1 + u_{(q^2)} \tag{42}$$

$$V_{(q^2)} = 1 + v_{(q^2)} \tag{43}$$

Majorana Neutrino Decay

where, at $O(\Sigma_{ND})$

$$v_{ij} = w_{ij}(M_i \Sigma_{ND\,ji} + M_j \Sigma_{ND\,ij}) \quad (44)$$

$$u_{ij} = w_{ij}(M_i \Sigma_{ND\,ij} + M_j \Sigma_{ND\,ji}) \quad (45)$$

And

$$w_{ij}^{-1}{}_{(q^2)} = (M_i - M_j)(1 + \frac{M_i M_j}{q^2}) - 2a_{(q^2)}(M_i K_{jj} - M_j K_{ii}) \quad (46)$$

CP Asymetry

The diagonalization provided us with a eigenstate for the Majorana degrees of freedom, such that it is possible to talk about decay of a unstable particle. we are intersted in the parameter

$$\epsilon_{CP} = \frac{\Gamma_{(N_1 \to l+H)} - \Gamma_{(N_1 \to \bar{l}+H^*)}}{\Gamma_{(N_1 \to l+H)} + \Gamma_{(N_1 \to \bar{l}+H^*)}} \tag{47}$$

Decays width are extracted from the diagonalized vertices

$$\Gamma_{(N_1 \to l+H)} \propto \sum_{\beta} |(U_{(M_i^2)} \lambda^T)_{1\beta}|^2 \tag{48}$$

$$\Gamma_{(N_1 \to \bar{l}+H^*)} \propto \sum_{\beta} |(V_{(M_i^2)} \lambda^\dagger)_{1\beta}|^2 \tag{49}$$

CP Asymetry

Then

$$\epsilon_{CP} = \frac{1}{K_{11}} Re[(u_{(M_1^2)} K)_{11} - (v_{(M_1^2)} K^T)_{11}] \implies \tag{50}$$

$$\epsilon_{CP} = \frac{-1}{8\pi} \sum_j \frac{Im[K_{1j}^2]}{K_{11}} \frac{M_1 M_j}{M_1^2 - M_j^2} \tag{51}$$

Low Energy EFT

Seesaw Type 1 theory is the simplest extension of SM wich accounts for lepton number violation interaction from a renormalizable term $\lambda_{\alpha i}\overline{l_\alpha}\tilde{H}P_R N_i$.

However, at energies far below the RH neutrino mass, we may prefer to work with the effective local and non-renormalizable operator, in much the same way we passed from Weak Interaction to Fermi Theory.

We proceed to integrate out the heavy neutrino degrees of freedom.

From now on, I'll use a simplified model: suppose we work with only one generation of lepton doublet $l = (\nu_e, e)^T$ and one of N.

Low Energy EFT

$$Z = \tilde{N}^{-1} \int DlDH \exp(-S_0) \int D\nu_R \exp(-S_{ss}) \tag{52}$$

$$= N^{-1} \int DlDH \exp(-S_0) \times \tag{53}$$

$$\int D\nu_R \langle 1 - (\int \lambda \bar{l} \tilde{H} \nu_R + h.c.) + \frac{1}{2} \int \int (\lambda \bar{l} \tilde{H} \nu_R \overline{\nu_R^c} H l \lambda^* + h.c.) + \ldots \tag{54}$$

We take the propagator evaluated at small p^2

$$\langle \nu_R \overline{\nu_R^c} \rangle = \frac{1}{M} C (2\pi)^4 \delta^{(4)}(p) \tag{55}$$

Low Energy EFT

Then

$$Z = N^{-1} \int DlDH \exp(-S_0)\langle 1 + \int \frac{\lambda M^{-1}\lambda^\dagger}{2}(\overline{l^c}H)(\tilde{H}^\dagger l) + ...\rangle \tag{56}$$

$$= \int DlDH \exp(-S_0 - \int \frac{-\lambda M^{-1}\lambda^\dagger}{2}(\overline{l^c}H)(\tilde{H}^\dagger l)) \tag{57}$$

Finally

$$\mathcal{L}_{eff} = \mathcal{L}_0 - \frac{\lambda M^{-1}\lambda^\dagger}{2}(\overline{l^c}H)(\tilde{H}^\dagger l) \tag{58}$$

$$= \mathcal{L}_0 - c_w O_w \tag{59}$$

Where O_w is the dim-5 Weinberg operator

Low Energy EFT

I now turn to calculate the anomalous dimension for the dim-5 Weinberg operator.
The 1-loop contribution to O_w correction is then

$$= i\frac{c_w^2}{16\pi^2}\int_0^1 x(\frac{2}{\epsilon} - log(\frac{\Delta^2}{\mu^2}))\, dx\not{p} \tag{60}$$

$$= i\frac{c_w^2}{16\pi^2}(... - \frac{1}{2}log(\frac{-p^2}{\mu^2}))\not{p} \tag{61}$$

then, at $p^2 = -\Lambda^2$

$$i\delta_w\not{p} = -i\frac{c_w^2}{16\pi^2}(... - \frac{1}{2}log(\frac{\Lambda^2}{\mu^2}))\not{p} \tag{62}$$

Low Energy EFT

Therefore

$$\Lambda \frac{\partial \delta_w}{\partial \Lambda} = \frac{c_w^2}{16\pi^2} \tag{63}$$

To proceed to calculate the anomalous dimension, we have to obtain the δ_ν and δ_{H^0} wich involves a two loop calculation. Once both contribute with 2 external legs, the anomalous dimension is given by:

$$\gamma_w = \Lambda \frac{\partial}{\partial \Lambda}(-\delta_w + \delta_\nu + \delta_{H^0}) \tag{64}$$

The solution I found in the literature was

$$\gamma_w = \frac{-3c_w^2}{16\pi^2} \tag{65}$$

A result that makes me suspect that maybe $\delta_\nu = \delta_{H^0} = \frac{-c_w^2}{16\pi^2}$

Low Energy EFT

It is now a matter of solving the running of c_w as

$$\Lambda \frac{\partial c_w}{\partial \Lambda} = \gamma_w c_w \tag{66}$$

$$= \frac{-3c_w^3}{16\pi^2} \tag{67}$$

And therefore

$$c_w{}^2_{eff} = \frac{c_w^2}{1 + \frac{3c_w^2}{16\pi^2} log(\frac{\Lambda^2)}{\mu^2}} \tag{68}$$

For the Weinberg operator, contrary to QED and $\lambda\phi^4$, the anomalous dimensions is negative, meaning that the interaction becomes stronger as external momenta are more energetic. It seems to me negative anomalous dimension happens for non-renormalizable operators.

References

1. Plumacher, M. (1998, July 30). Baryon Asymmetry, Neutrino Mixing and Supersymmetric SO(10) Unification. arXiv.org. https://arxiv.org/abs/hep-ph/9807557v1
2. R.D. Matheus' lecture notes "Teoria Quântica de Campos II"
3. Coy, R., Frigerio, M. (2019, May 31). Effective approach to lepton observables: The seesaw case. Physical Review; American Physical Society. https://doi.org/10.1103/physrevd.99.095040