

$$\mathcal{L} = \bar{\Psi}_e (\not{\partial} - m) \Psi_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underbrace{C_1 \bar{\Psi}_e \Psi_e + C_2 (F^{\mu\nu} F_{\mu\nu})^2 + \dots}_{\text{WHERE?}}$$

QUANTUM FIELD THEORY

Lagrangians and Observables an invitation to renormalization



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\mathcal{L} = \bar{\Psi}_e (\not{\partial} - m) \Psi_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i e \bar{\Psi}_e \not{A} \Psi_e$$

$\begin{cases} m_e \\ m_f = 0 \end{cases}$

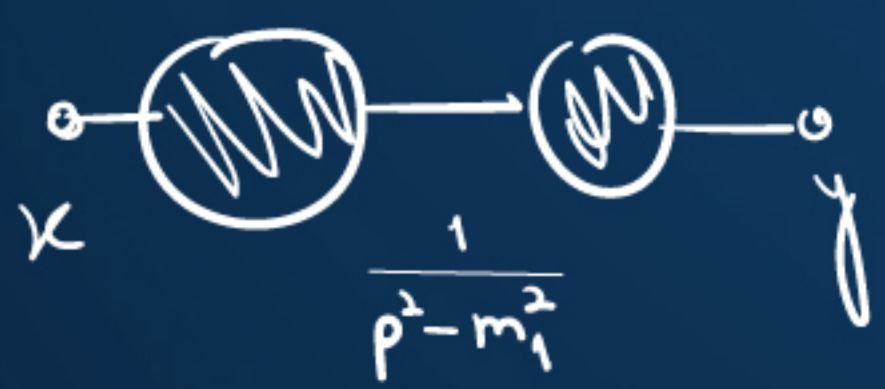
$\text{Im}[P^2]$



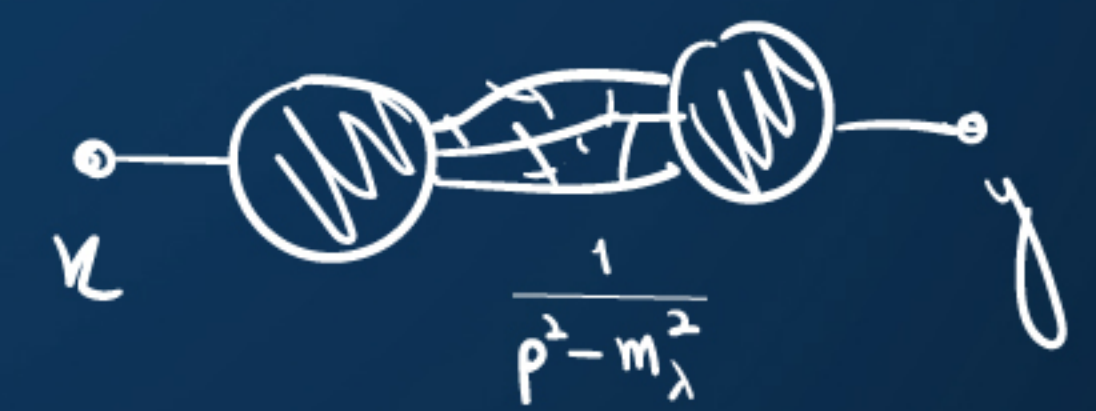
$$\langle \Omega | T \{ \phi(x) \phi(y) \} | \Omega \rangle = \int_0^\infty \frac{dM^2}{2\pi} \rho(M^2) D_F(x-y, M^2)$$

QUANTUM FIELD THEORY

Källén-Lehmann Spectral Representation



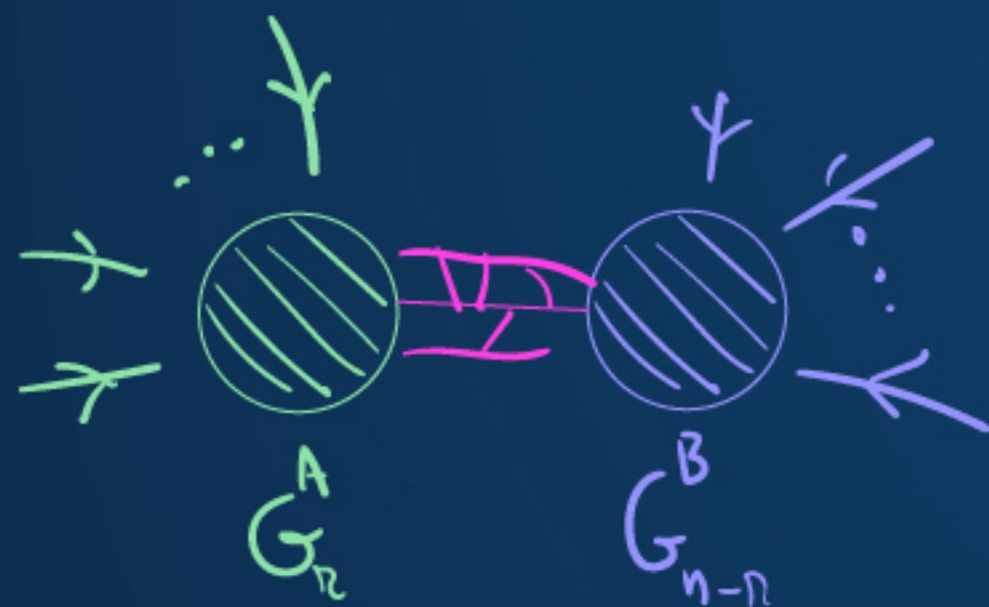
IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$G_n(p_1, \dots, p_n) = (2\pi)^4 \delta^4(p_1 + \dots + p_n) \sum_{\lambda} G_n^A(q, p_2, \dots, p_n) \frac{i}{q^2 - m_{\lambda}^2 + i\epsilon} G_{n-1}^B(q, p_{n+2}, \dots, p_n) + \text{D.T.}$$

QUANTUM FIELD THEORY

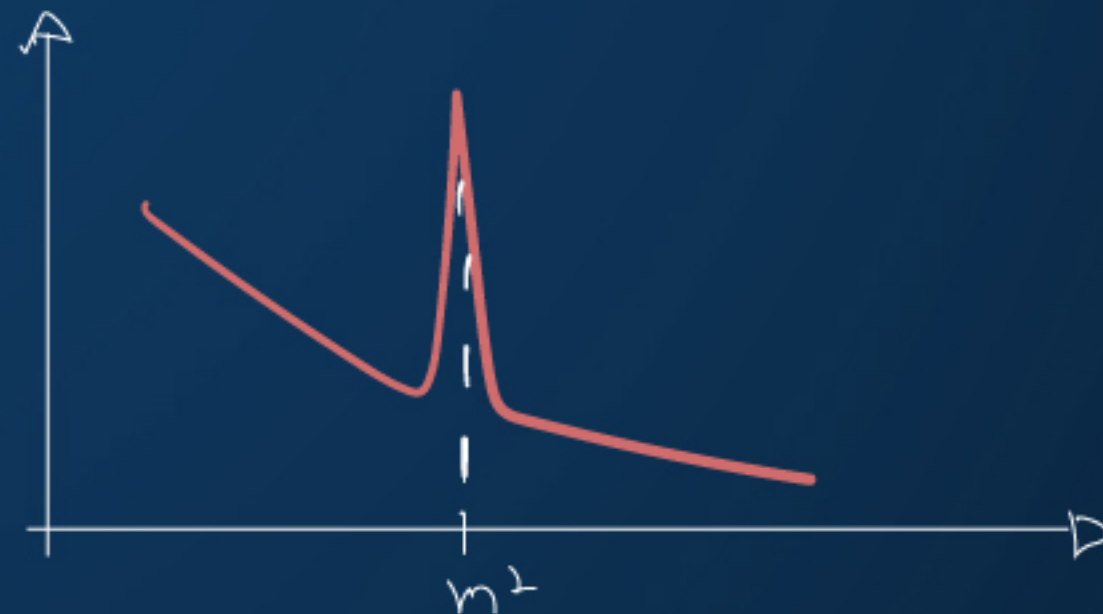
Polology



$$\langle \Omega | j^{\mu\nu}(x) | \pi^b(p) \rangle$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$\left(\prod_{i=1}^n \frac{\sqrt{z}^i}{p_i^2 - m^2 + i\epsilon} \right) \left(\prod_{j=1}^m \frac{\sqrt{z}^j}{k_j^2 - m^2 + i\epsilon} \right) \langle \vec{p}_1 \dots \vec{p}_n | S | \vec{k}_1 \dots \vec{k}_m \rangle$$

QUANTUM FIELD THEORY

The LSZ Reduction Formula (Lehmann, Symanzik and Zimmermann)

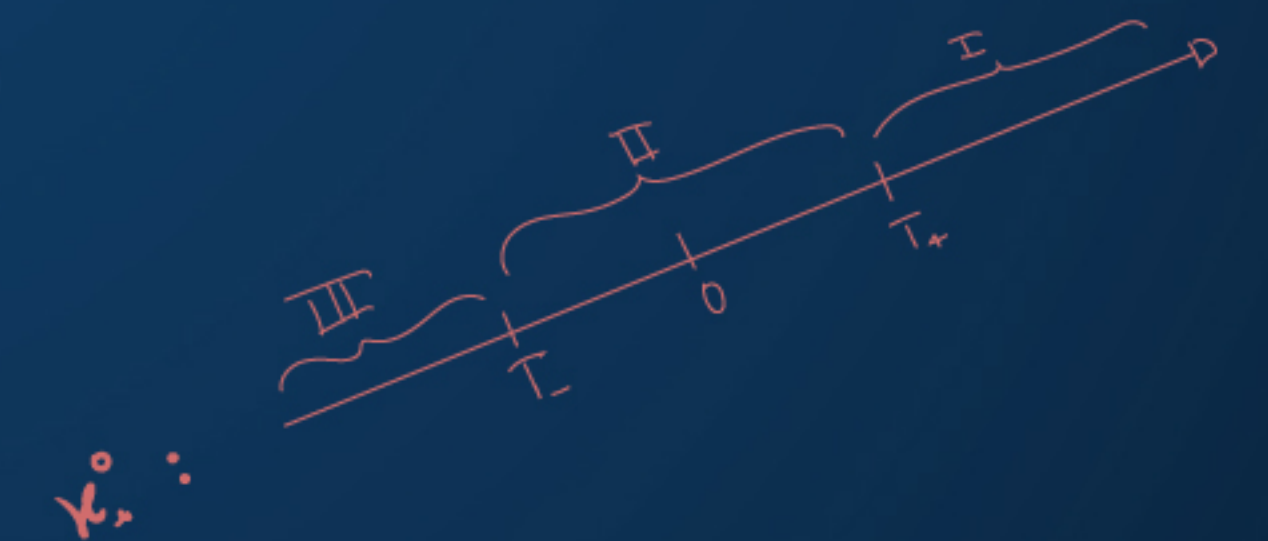


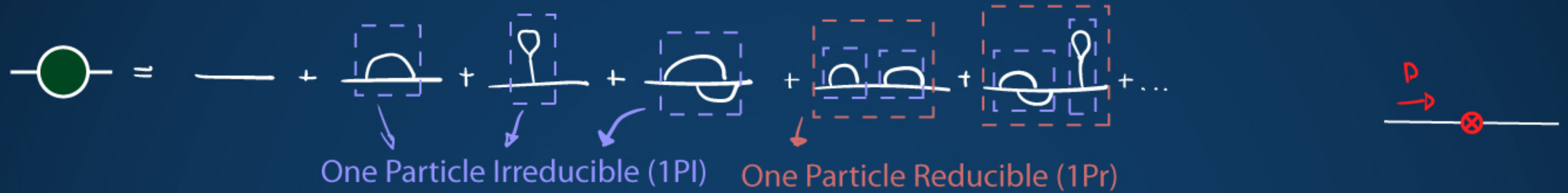
IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\langle \Omega | T \{ \phi(x_1) \dots \phi(x_n) \} | \Omega \rangle$$



$$\prod \left(\text{full propagators} \right) \langle S \rangle$$





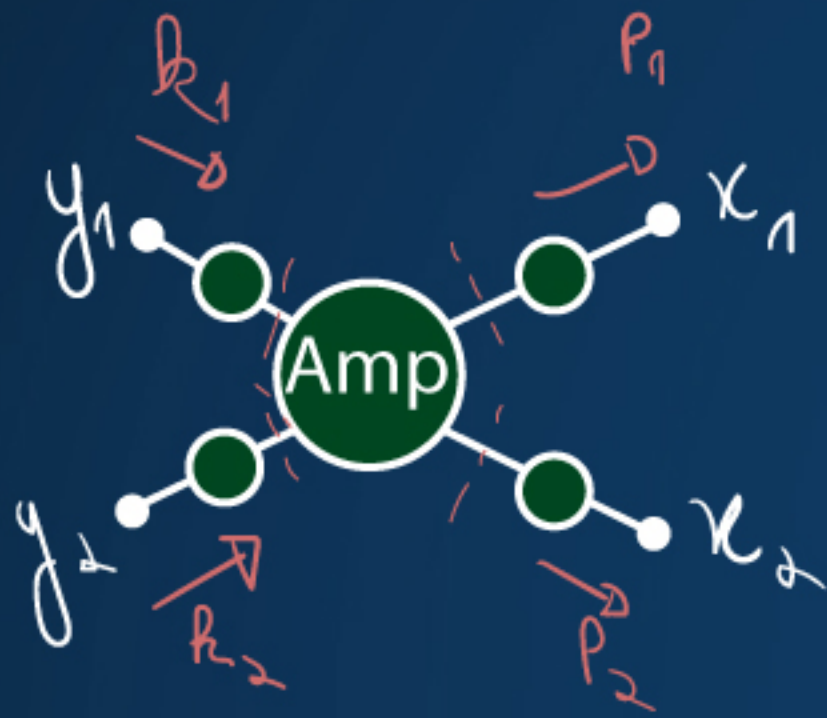
QUANTUM FIELD THEORY

Field and Mass Renormalization



Handwritten notes: $p^2 - m_0^2 - i\epsilon$ and $M^2(p^2)$ with a red arrow pointing to the denominator.



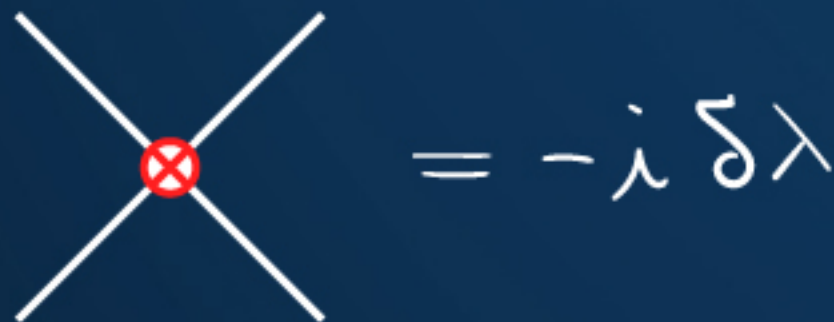


QUANTUM FIELD THEORY

Vertex Functions and Coupling Renormalization



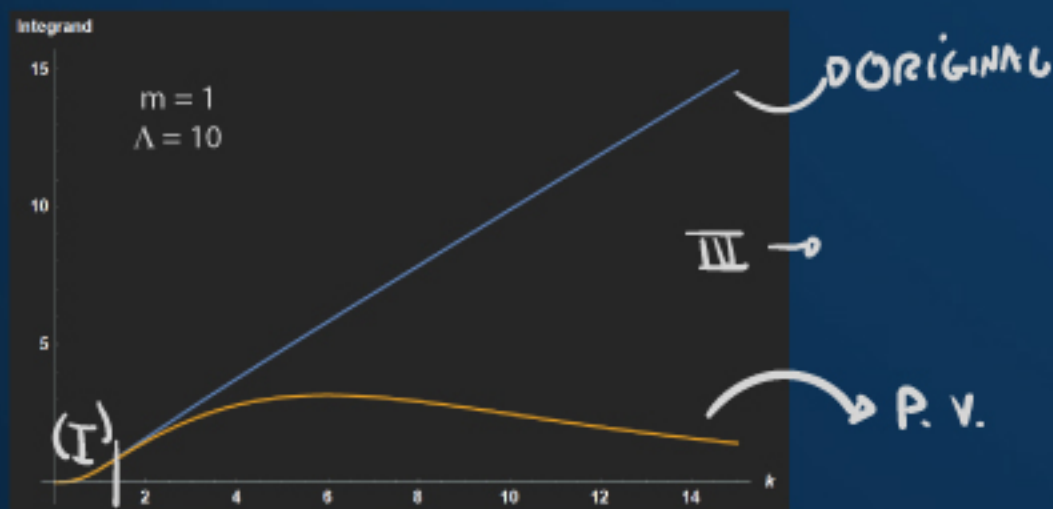
IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$i \Gamma^{(2)} = \frac{\lambda^2}{2} \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{(k^2 - m^2 + i\epsilon)} - \frac{1}{(k^2 - \Lambda^2 + i\epsilon)} \right] \frac{1}{(k+p)^2 - m^2 + i\epsilon}$$

QUANTUM FIELD THEORY

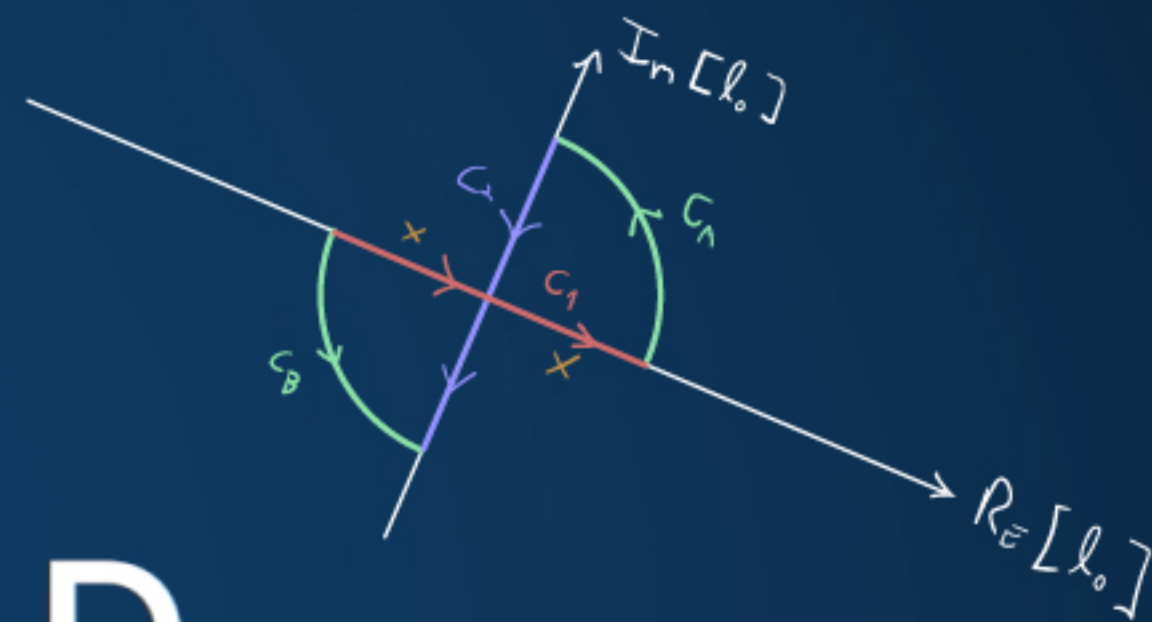
One-loop $\lambda\phi^4$ and Pauli-Villars Regularization



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$\frac{1}{[\not{x} A + \not{y} B]^2}$$



QUANTUM FIELD THEORY

Loop tools and tricks

Feynman Parametrization and Wick Rotation (in $\lambda\phi^4$)



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\mathcal{M}(s, t, u) = -\lambda + \left[\tilde{\Gamma}_q(s) + \tilde{\Gamma}_q(t) + \tilde{\Gamma}_q(u) - \tilde{\Gamma}_q(4m^2) \right] + \dots$$

$$\epsilon^{1/2} + L_N(N^2) + N \iff L_N(\Lambda^2)$$

QUANTUM FIELD THEORY

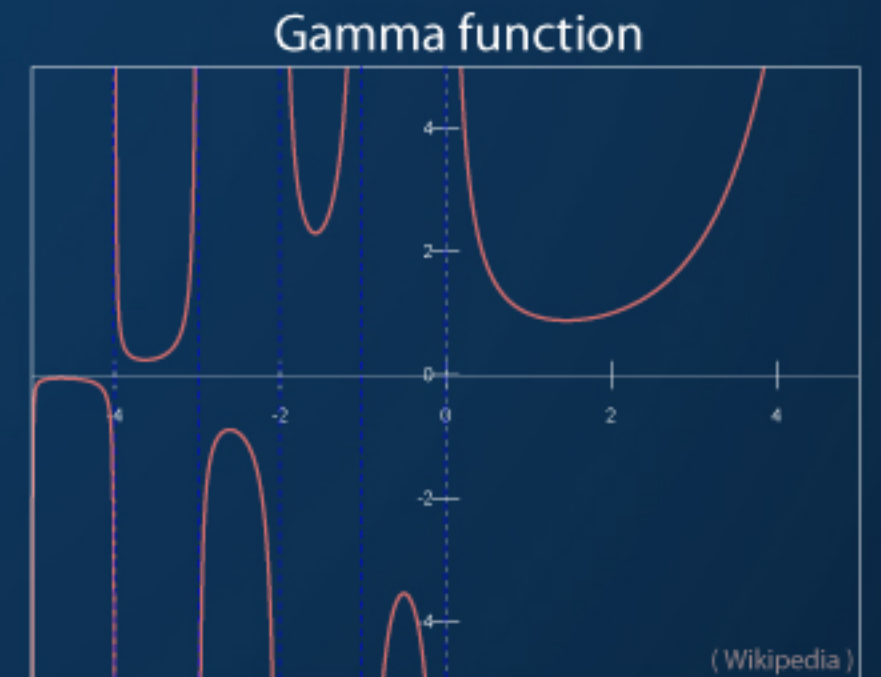
Dimensional Regularization

(in $\lambda\phi^4$)

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





$$\sim \frac{1}{\Lambda^n} \Lambda^D$$

$$\Delta_n > 0 \text{ 😄 } \Delta_n = 0$$

$$\Delta_n < 0 \text{ 😞 } ?$$

QUANTUM FIELD THEORY

Power Counting and Renormalizability I

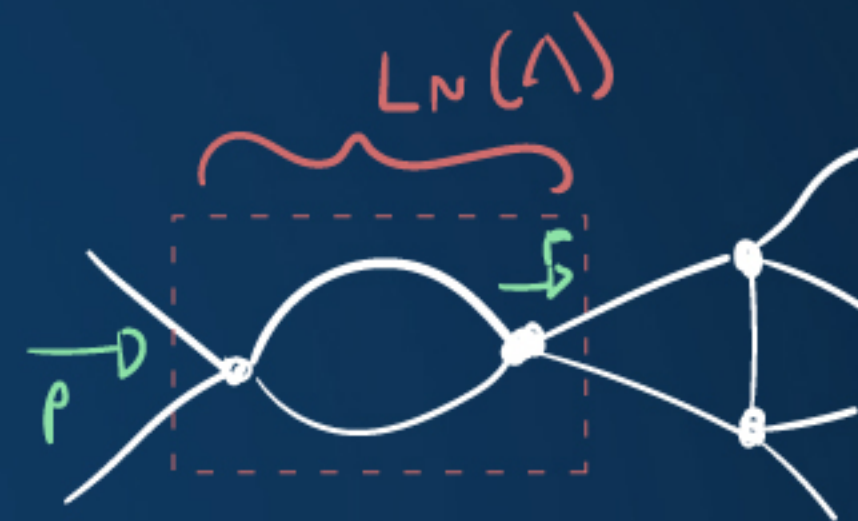
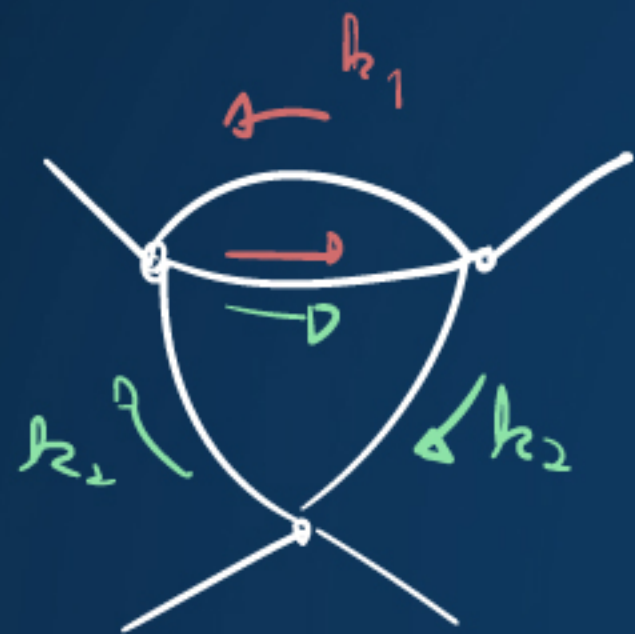
($\lambda\phi^n$ theories)



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\Delta_n \equiv d - n \left(\frac{d-2}{2} \right)$$

$$D = d - \Delta_n V - \left(\frac{d-2}{2} \right)^2 N$$



QUANTUM FIELD THEORY

Renormalization at 2 loop order (or more)



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\Delta_i \equiv \left[d - d_i - \sum_l n_{il} \left(s_l + \frac{d-2}{2} \right) \right]$$

$$\gamma \bar{\psi} \psi \phi$$

QUANTUM FIELD THEORY

Power Counting and Renormalizability II (spin 0, 1/2 and 1)

$$e A_\mu \bar{\psi} \gamma^\mu \psi$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$(\bar{\psi} \Gamma \psi) (\bar{\psi} \Gamma \psi)$$



$$G_F^0 (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi)$$

$$G_{F^1}^0 (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi)$$

$$G_{F^{11}}^0 (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi) (\bar{\Psi} \Gamma \Psi)$$



QUANTUM FIELD THEORY

Non-Renormalizable Theories

... are doing fine!



IFT - UNESP

INSTITUTO DE FÍSICA TEÓRICA

$$|\mathcal{M}(2 \rightarrow 2)|^2 \sim g_1^2 \frac{p^2}{M^4} + \dots + g_6^2 \frac{p^{10}}{M^{10}} + \dots + g_8^2 \frac{p^{16}}{M^{16}}$$

$$-ik^\nu \Gamma^3 = \left[\text{diagram} \right]^{-1} - \left[\text{diagram} \right]^{-1}$$

The diagram on the left shows a central green circle labeled Γ^3 . It has two incoming dashed lines from the bottom-left and bottom-right, each with an arrow pointing towards the center and labeled p_1 . It also has two outgoing dashed lines from the top-left and top-right, each with an arrow pointing away from the center and labeled k . To the left of this diagram is the expression $-ik^\nu$.

The two diagrams in the brackets on the right are identical in structure but differ in the internal shaded circle. The first diagram has a circle with diagonal hatching, and the second has a circle with horizontal hatching. In both, the incoming momentum is $p_1 + k$ and the outgoing momentum is p_1 .

QUANTUM FIELD THEORY

Ward-Takahashi Identities

$$\langle d_\nu j^\nu \rangle = 0$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$k_\nu M^\nu(k) = 0$$

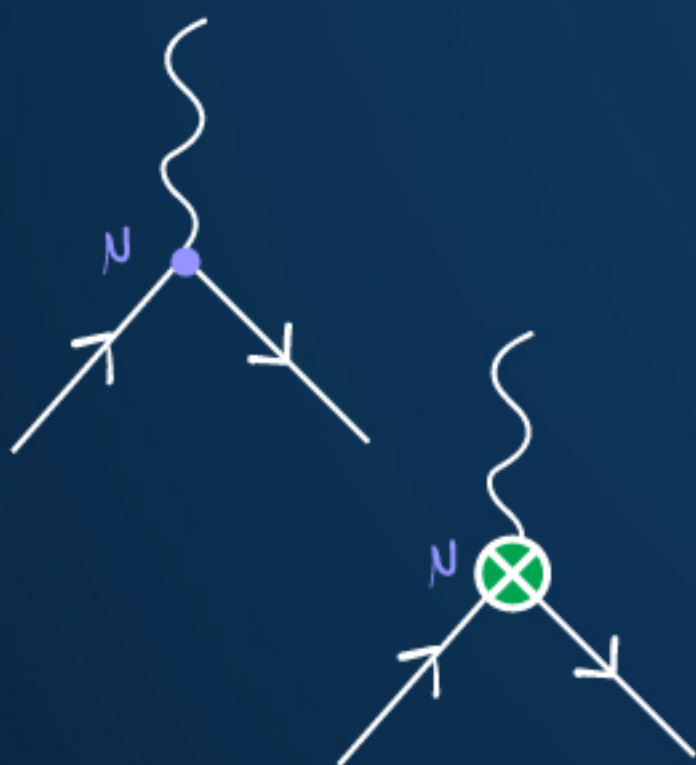


$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) \overline{\Pi}(q^2)$$

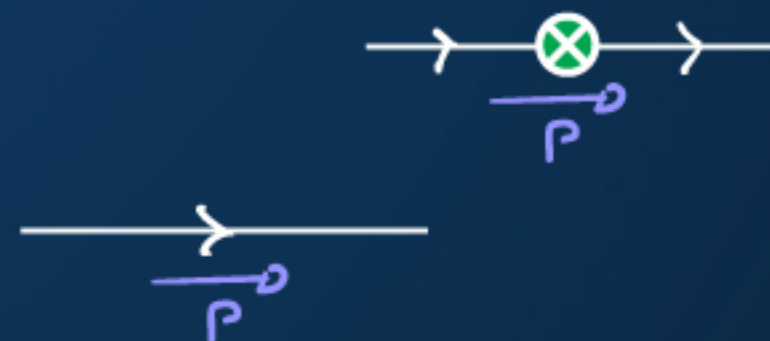
QUANTUM FIELD THEORY

Renormalization of QED I

Power Counting and Counterterms



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





QUANTUM FIELD THEORY

$g = e$

Renormalization of QED II

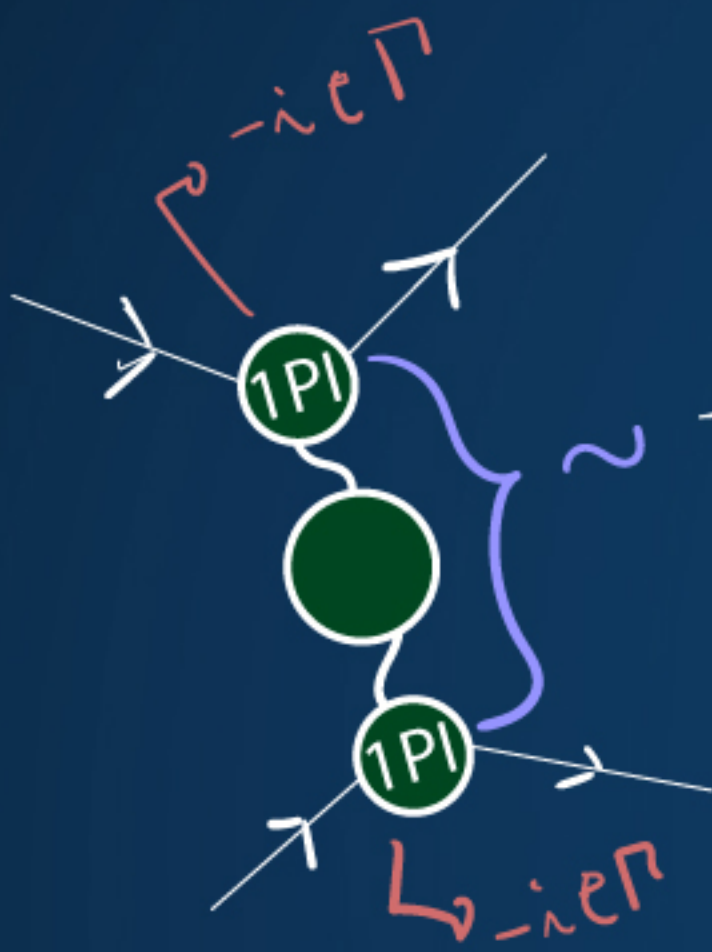
Formal Structure of the QED Vertex

$$\Gamma^{\mu} \equiv g \left(\frac{e}{2m} \right) \gamma^{\mu}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\Gamma^{\mu} \equiv \gamma^{\mu} F_1(q^2) + i \frac{\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2)$$



$$-i g_{\mu\nu} e^2 / (q^2 (1 - \Pi(q^2)))$$



QUANTUM FIELD THEORY

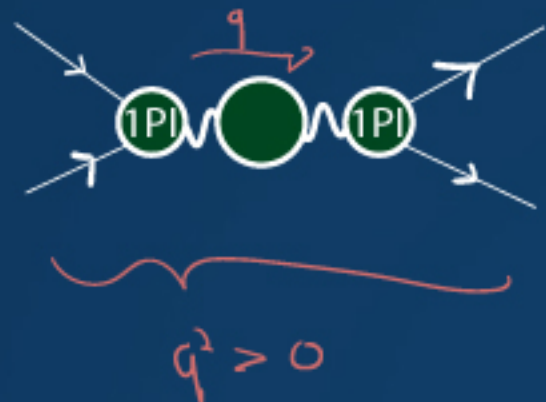
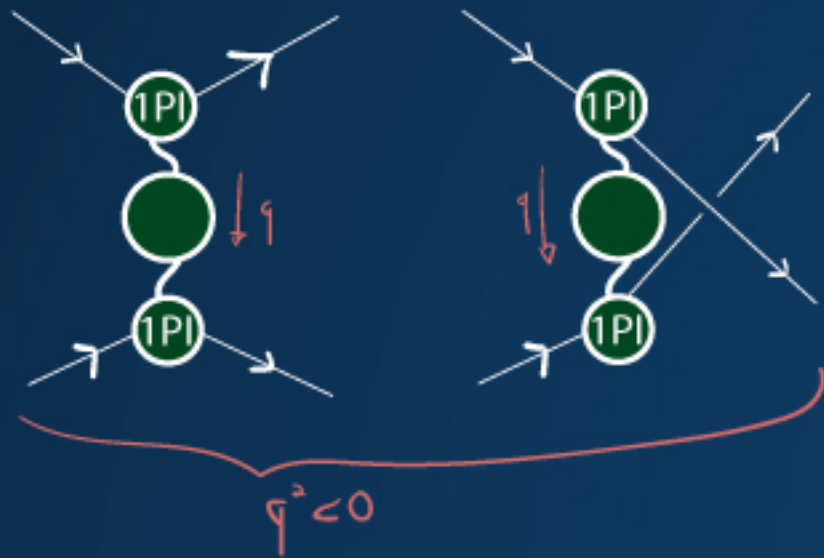
Renormalization of QED III

The Photon Self-Energy



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$e_{EFF}^2(q^2) \equiv \frac{e^2}{(1 - \Pi(q^2))}$$



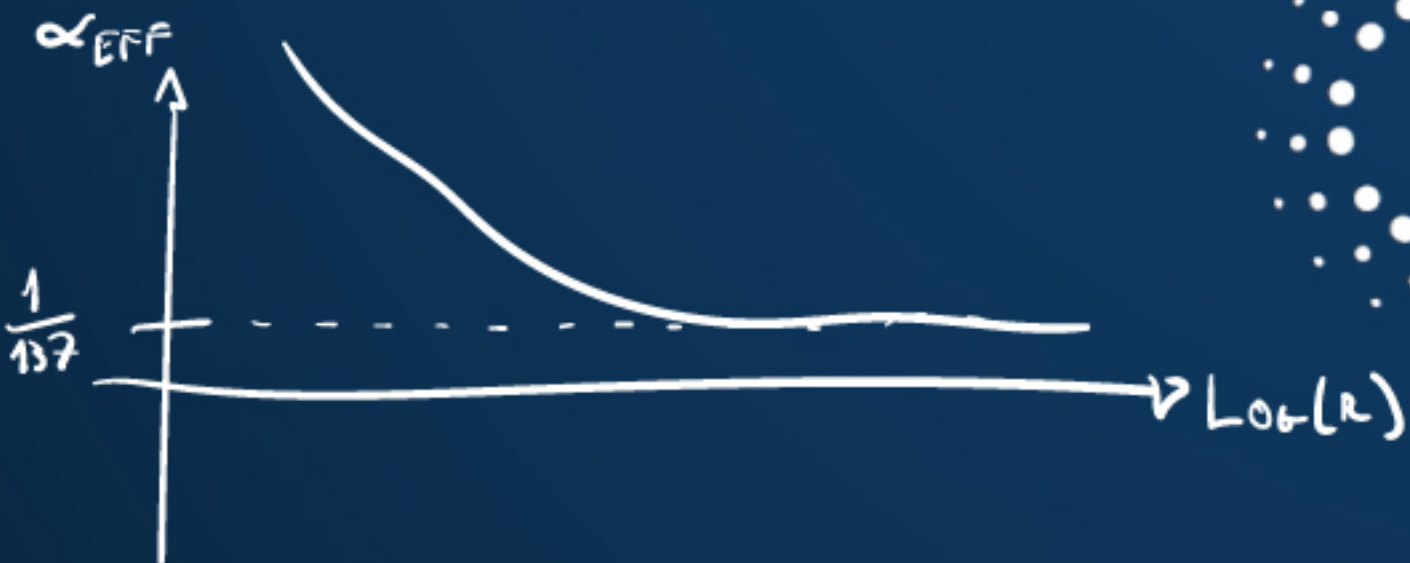
$$V(r) = -\frac{\alpha}{r} \left(1 + \underbrace{\frac{\alpha}{4\sqrt{\pi}} \frac{e^{-2mr}}{(mr)^{3/2}} + \dots}_{\text{Uehling Potential}} \right)$$

Uehling Potential

QUANTUM FIELD THEORY

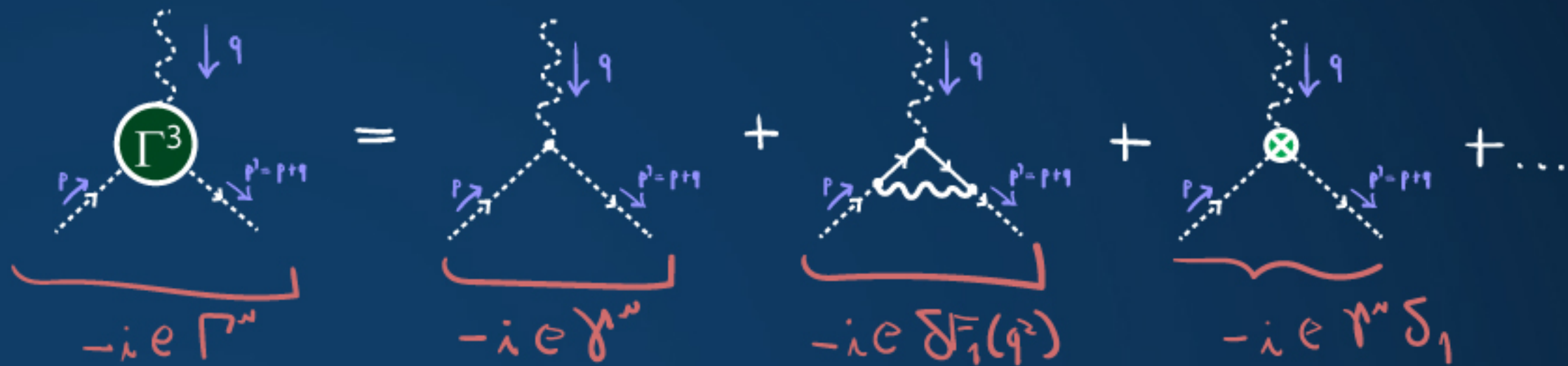
Renormalization of QED IV

Physics of $\Pi(q^2)$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\frac{\alpha}{2\pi}$$



QUANTUM FIELD THEORY

Renormalization of QED VI

Vertex Corrections and g-2



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\bar{F}_2(0) = \left(\frac{g-2}{2}\right)_{\text{exp}} = 0,00115965218073 (28)$$

(I) $E_\gamma \sim 0$ Soft Singularity

(II) $\cos(\theta) \sim 0$ Collinear Singularity



QUANTUM FIELD THEORY

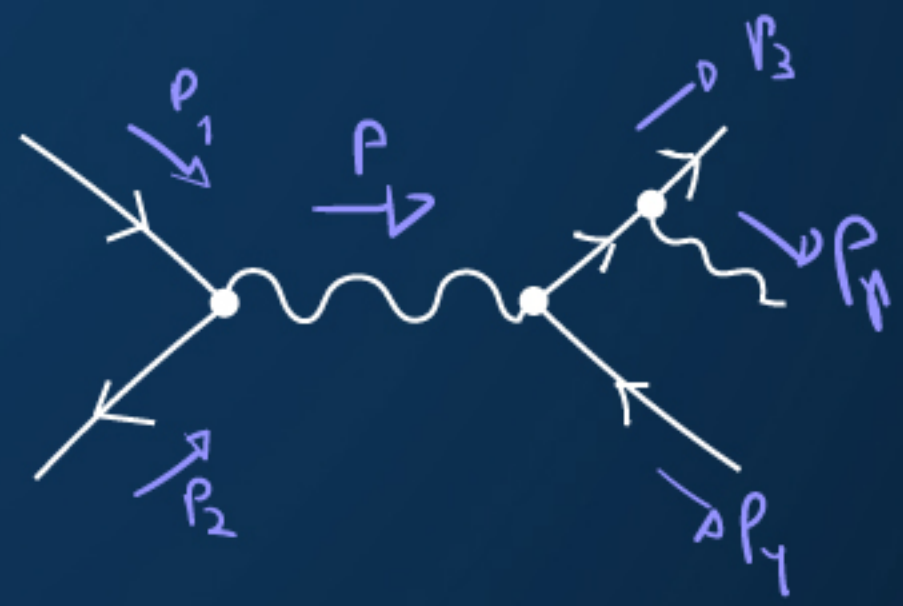
Renormalization of QED VII

Infrared Divergences

$$\tilde{\sigma}_{\text{TOT}} = \left(1 + \frac{3\alpha}{4\pi}\right) \tilde{\sigma}_{\text{LO}}(e^+e^- \rightarrow \nu\mu^+\mu^-)$$



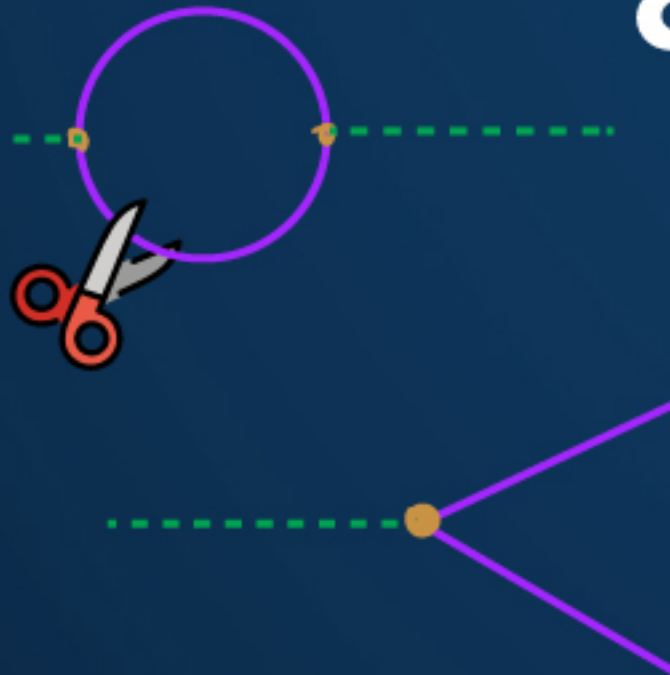
IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$2\text{Im} \left[\text{Diagram with a central green circle and four external lines labeled } h_1, h_2, k_1, k_2 \right] = \sum_x \int d\pi_x \left(\text{Diagram with a central green circle and four external lines labeled } h_1, h_2, k_1, k_2 \right) \times \left(\text{Diagram with a central green circle and four external lines labeled } h_1, h_2, k_1, k_2 \right)$$

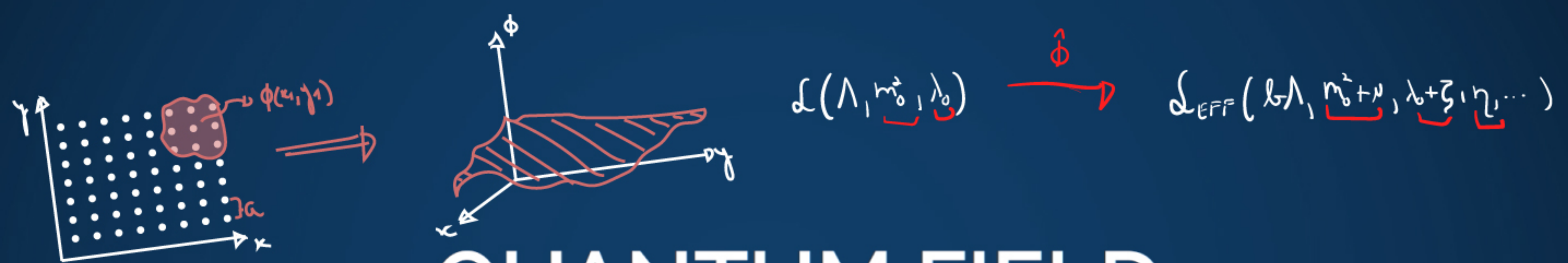
QUANTUM FIELD THEORY

The Optical Theorem and Unstable Particles



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\frac{1}{(s-m^2)^2 + m^2 \Gamma_{\text{Tot}}^2} \sim \frac{\Gamma}{m \Gamma} \delta(s-m^2)$$



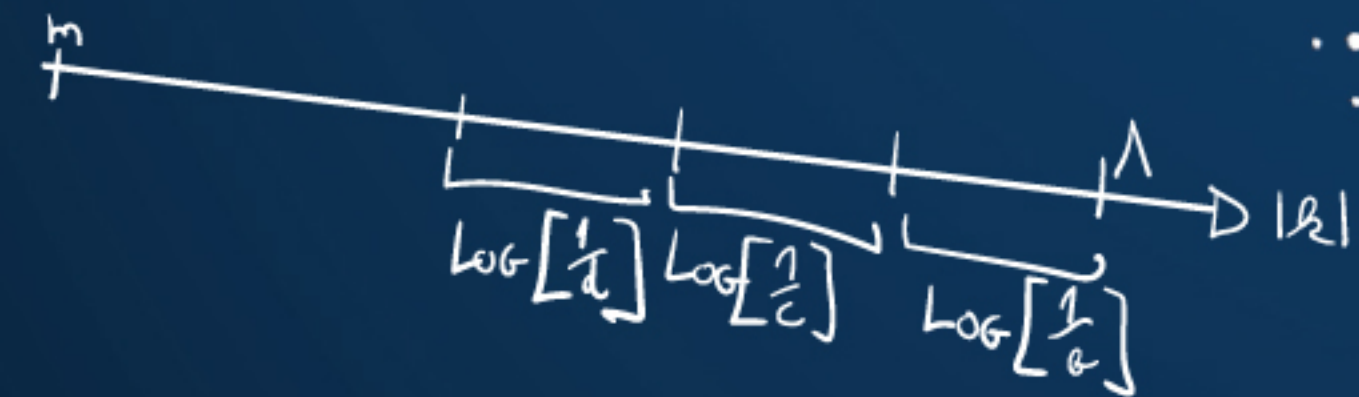
QUANTUM FIELD THEORY

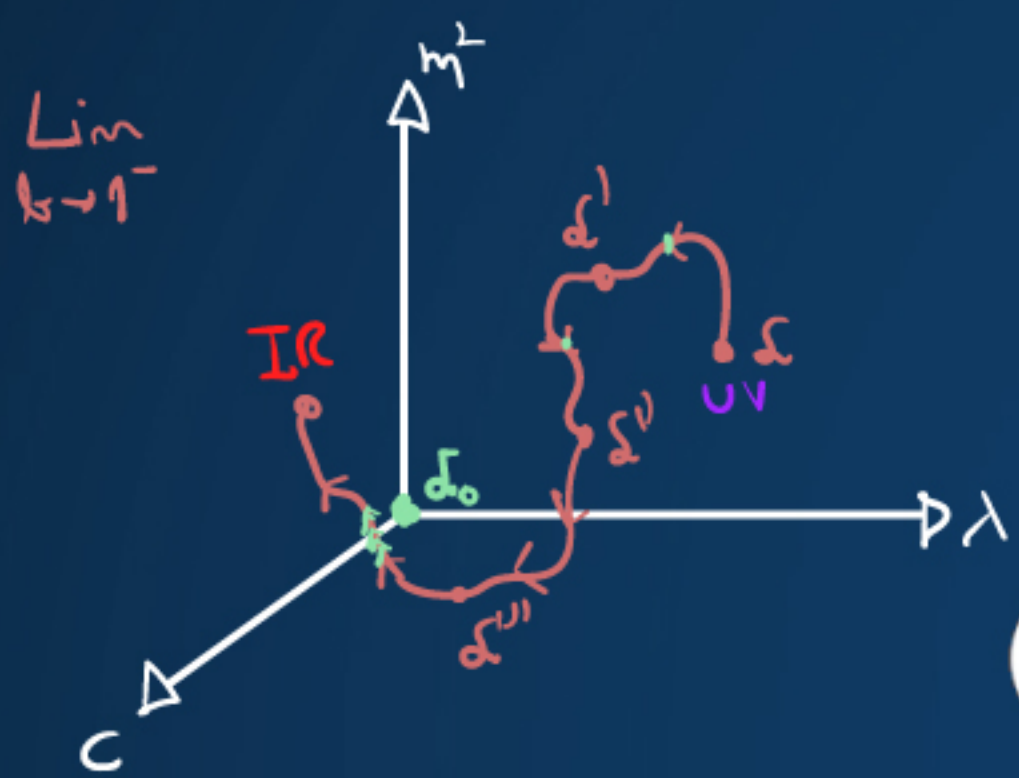
Wilsonian Renormalization I

Coarse Graining and Momentum Shell Integration



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





$$\mathcal{L}(\phi) = \mathcal{L}(\phi, \hat{\phi}) \xrightarrow[\text{SCALE } \int k' = \frac{k}{b}, x' = xb]{\text{INTEGRATE OUT } \hat{\phi}} \mathcal{L}_{\text{EFF}}(\phi'(x))$$

QUANTUM FIELD THEORY

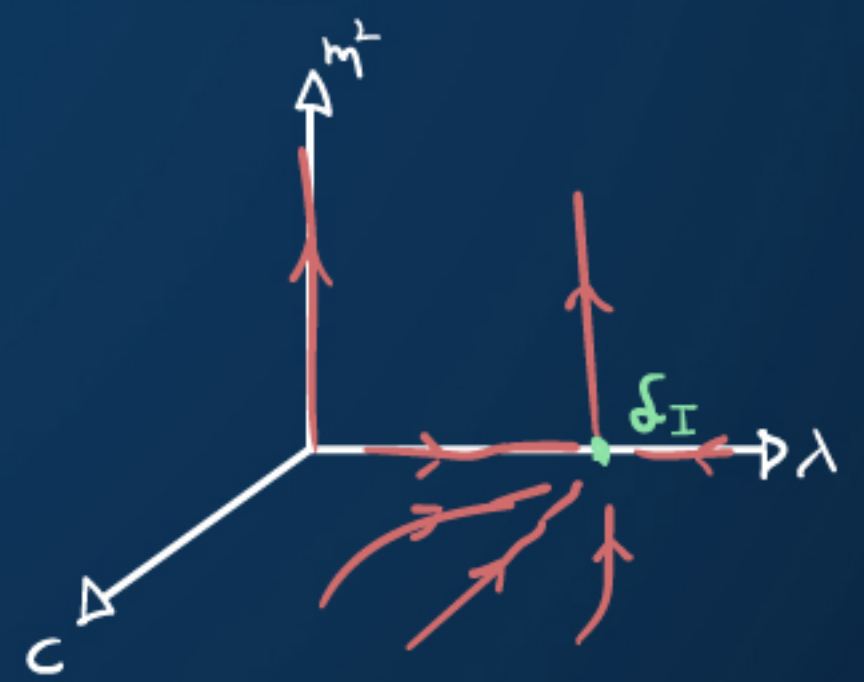
Wilsonian Renormalization II

Renormalization Group Transformations

$$1' \approx 1 - \frac{3\lambda^2}{16\pi^2} L_N \begin{pmatrix} 1 \\ 1 \\ 6 \end{pmatrix}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$\left[M \frac{d}{dM} + \beta(\lambda) \frac{d}{d\lambda} + n \gamma(\lambda) \right] G^{(n)}(\chi_1, \dots, \chi_n; M, \lambda) = 0$$

QUANTUM FIELD THEORY

Renormalization Group Equations (RGE)


The Callan-Symanzik Equation



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$\beta = m \frac{d\lambda(m)}{dm}$$

$$\frac{e^3}{12\pi^2} \quad \frac{3\lambda^2}{(4\pi)^2} \quad \frac{e^2}{12\pi^2} \quad \frac{\lambda^2}{12(4\pi)^2}$$


The diagrams show a starburst labeled with the Greek letter beta (β) and another labeled with the Greek letter gamma (γ). The beta diagram is associated with the coefficient $\frac{e^3}{12\pi^2}$ and the gamma diagram with $\frac{e^2}{12\pi^2}$. The coupling constant λ is also shown with its associated coefficients $\frac{3\lambda^2}{(4\pi)^2}$ and $\frac{\lambda^2}{12(4\pi)^2}$.

QUANTUM FIELD THEORY

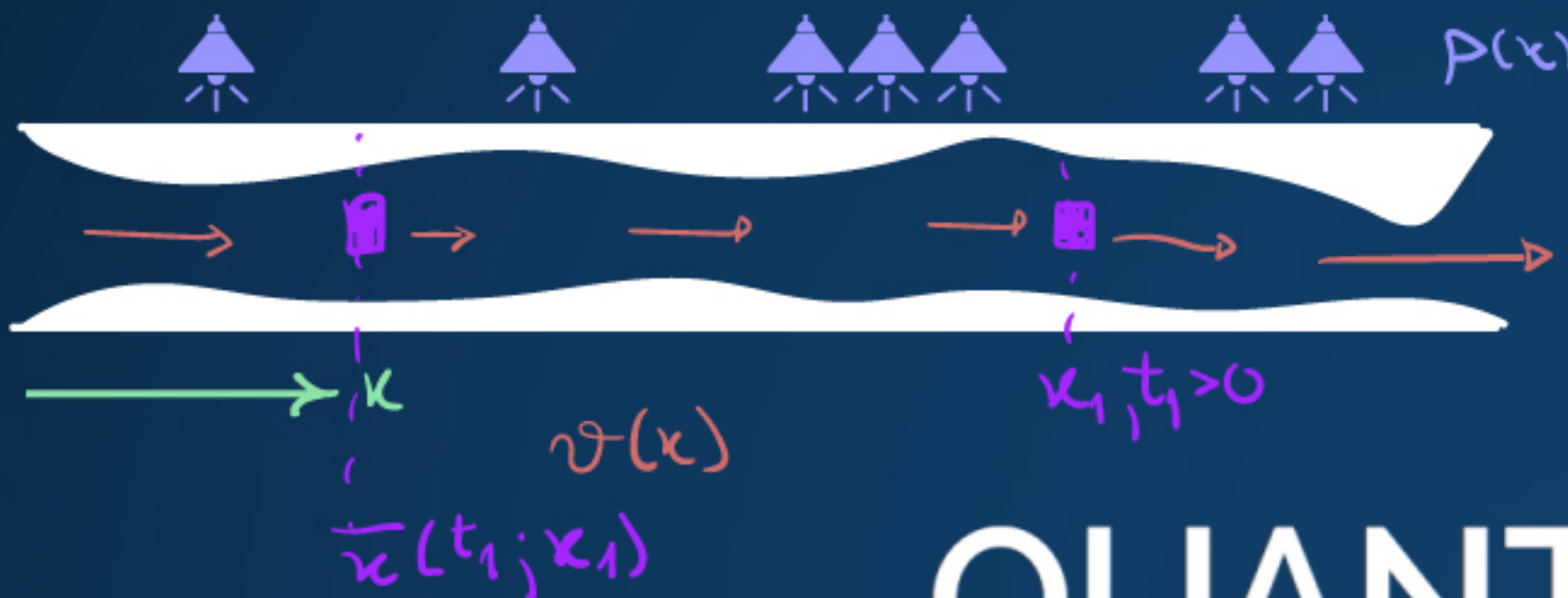
Beta (β) and Gamma (γ) Functions

$$\gamma = \frac{1}{\lambda} \frac{d\lambda}{dM}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





$$\frac{d}{d[\ln(\frac{k}{m})]} \bar{\lambda}(k; \lambda) = \beta(\bar{\lambda}(k; \lambda))$$

QUANTUM FIELD THEORY

Solving the Callan-Symanzik Equation



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\left[k \frac{d}{dk} + 2 - \beta(\lambda) \frac{d}{d\lambda} - 2\delta \right] G^{(2)} = 0$$

$$\left[q \frac{d}{dq} - \beta(e_n) \frac{d}{de_n} + 2 \right] V(q; M, e_n) = 0$$

$$\bar{e}(M; e_n) = e_n$$

QUANTUM FIELD THEORY



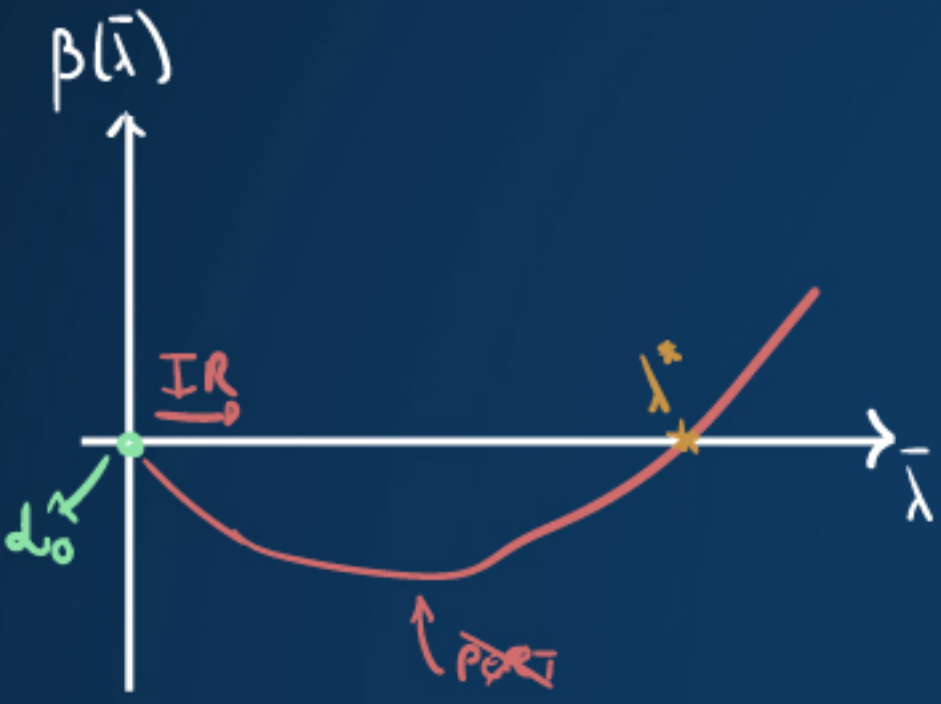
The Running of the Electric Charge

$$\bar{e}^2(q) = \frac{e_n^2}{1 - \left(\frac{e_n^2}{12\pi^2} \right) \ln\left(\frac{q^2}{M^2} \right)}$$

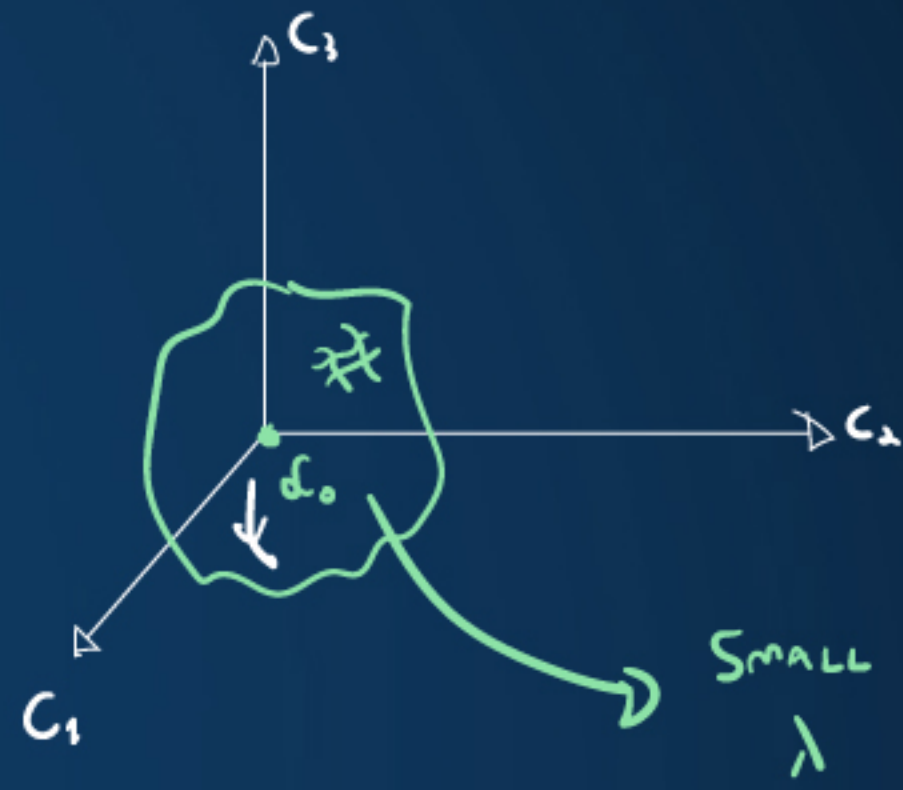


IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





QUANTUM FIELD THEORY

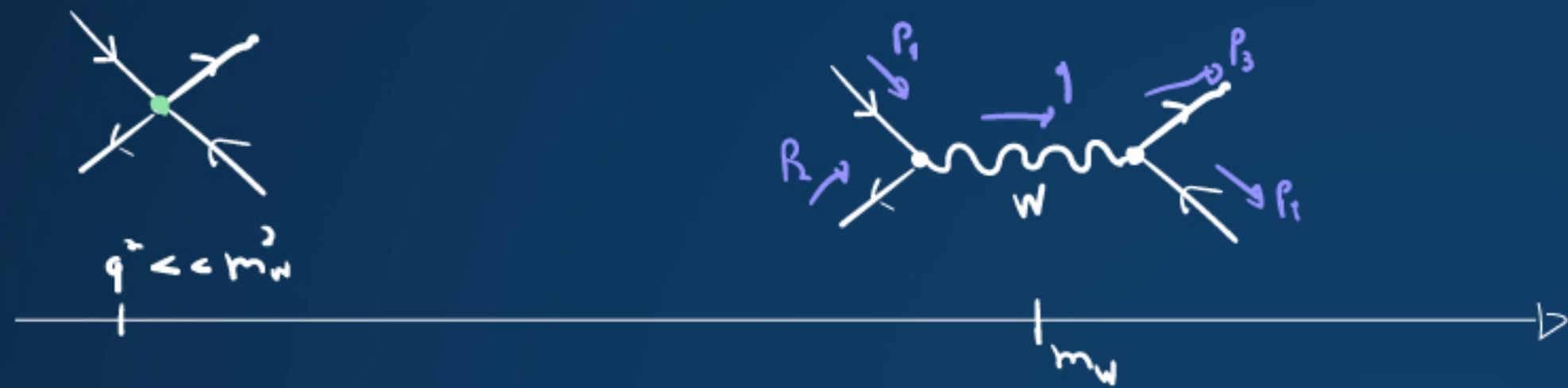


Fixed Points and Anomalous Dimensions



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$G^{(2)}(p, \lambda) = C \left(\frac{\lambda}{p^2} \right)^{1 - \gamma(\lambda^*)}$$



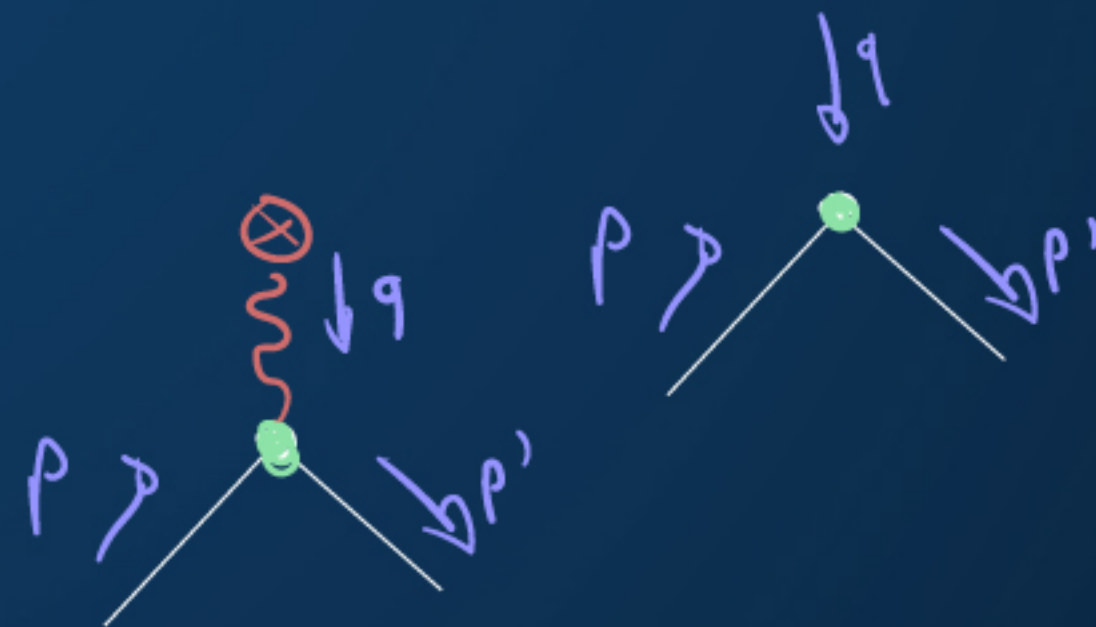
QUANTUM FIELD THEORY

Renormalization of Local Operators

$$\gamma_0(l) = \frac{M}{Z} \frac{\partial Z_0}{\partial M} = M \frac{\partial}{\partial M} \ln[Z_0(M)]$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$G^{(n)} = G^{(n;0)} + m^2 G^{(n;1)} + (m^2)^2 G^{(n;2)} + \dots + (m^2)^l G^{(n;l)}$$

QUANTUM FIELD THEORY

Evolution of Mass Parameters

$$\overline{P}_m = P_m \left(\frac{P}{M} \right)^{-2}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\left[m \frac{d}{dm} + \beta(\lambda) \frac{d}{d\lambda} + n \gamma(\lambda) + \gamma_{\phi^2}(\lambda) m^2 \frac{d}{dm^2} \right] G^{(n)} = 0$$

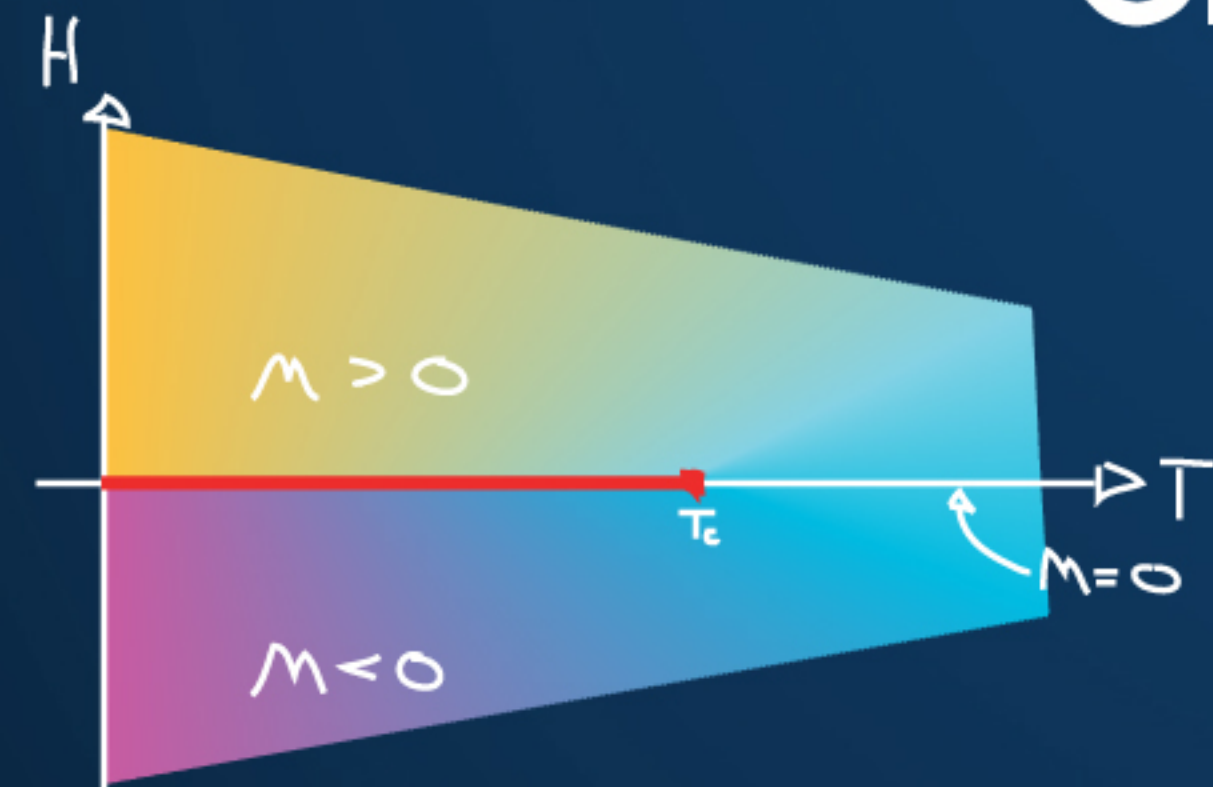
$$\rho_m \left(\frac{M}{\rho_0} \right)^2 - \gamma \phi^2(\lambda^*)$$

$$\xi \sim (T - T_c)^{-\nu/2}$$

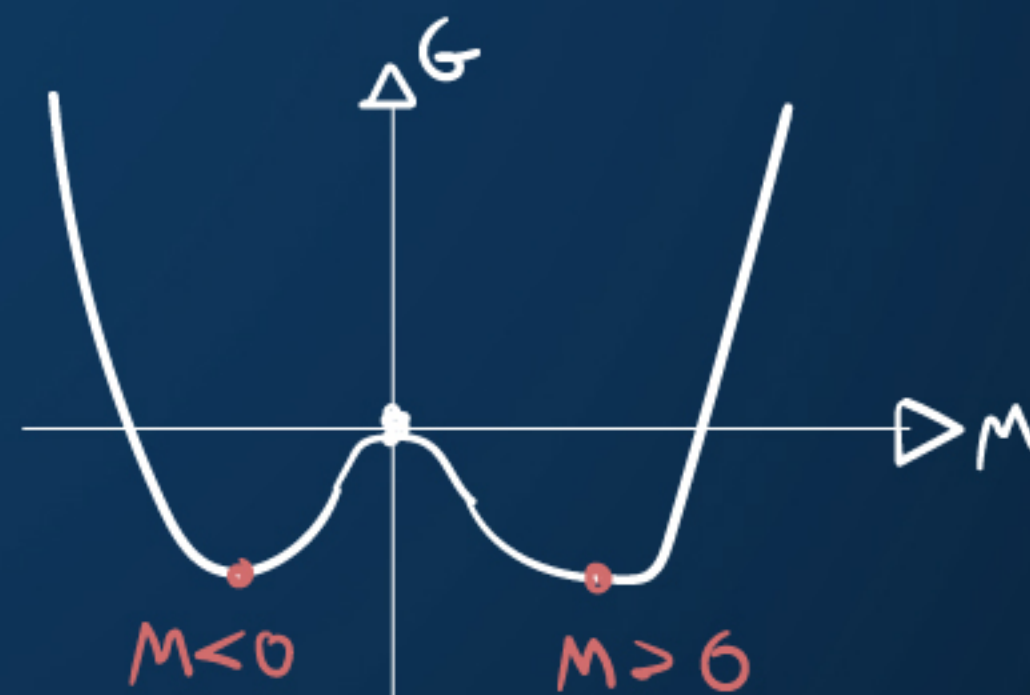
$$\xi \sim (T - T_c)^{-\nu}$$

QUANTUM FIELD THEORY

Critical Exponents



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA



$$f^{ade} f^{bcd} + f^{bde} f^{cad} + f^{cde} f^{abd} = 0$$

SU(N)

$$U_R^A = \begin{pmatrix} U_R^R \\ U_R^G \\ U_R^B \end{pmatrix}$$

QUANTUM FIELD THEORY

Lie Algebras, the quick and dirty

Non-Abelian Theories, Part I

$$(T^a)_{bc} \equiv i f^{abc}$$



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$[T^a, T^b] \equiv i f^{abc} T^c$$

$$\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}(F_{\mu\nu}^a)^2$$

QUANTUM FIELD THEORY

Yang-Mills Theory

Non-Abelian Theories, Part II

$$A_\mu^3 \rightarrow V(x) \left[A_\mu^3 + \frac{i}{g} \not{\partial}_\mu \right] V^\dagger(x)$$

$$\frac{1}{2} \begin{pmatrix} A_\mu^3 & A_\mu^1 - i A_\mu^2 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 \end{pmatrix}$$

$$G_\mu^a = (G_\mu^1, \dots, G_\mu^8)$$



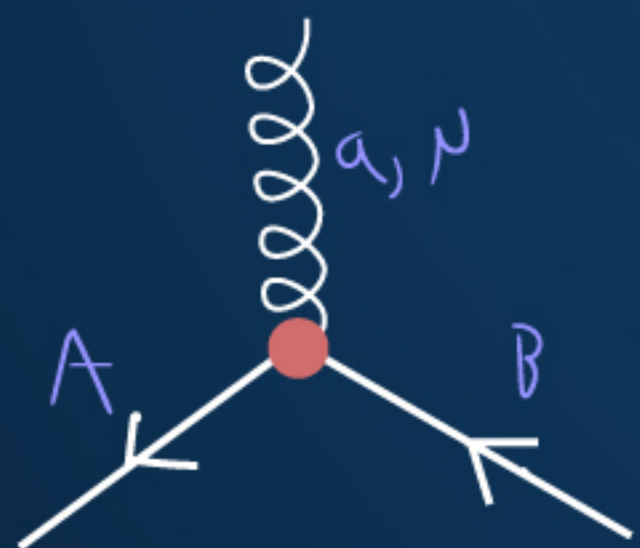
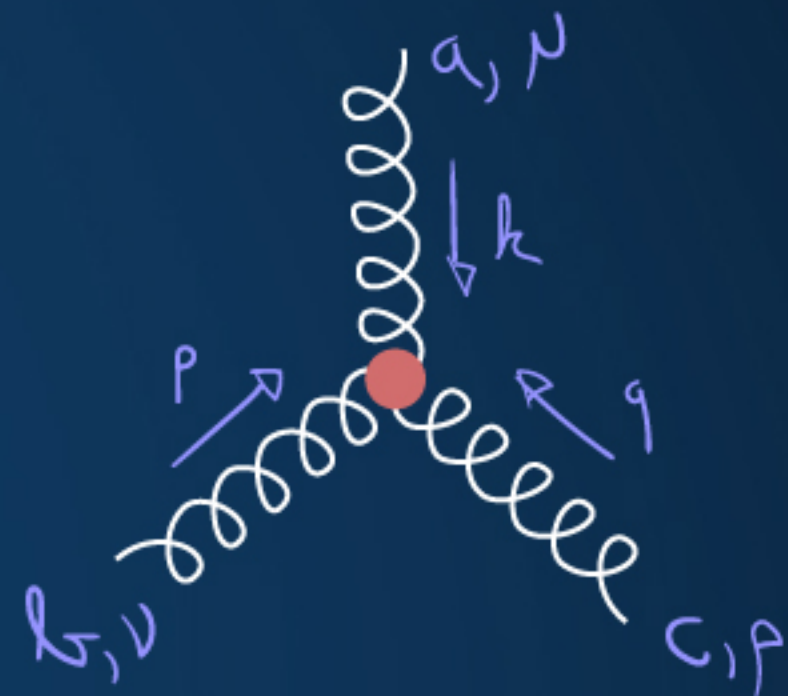
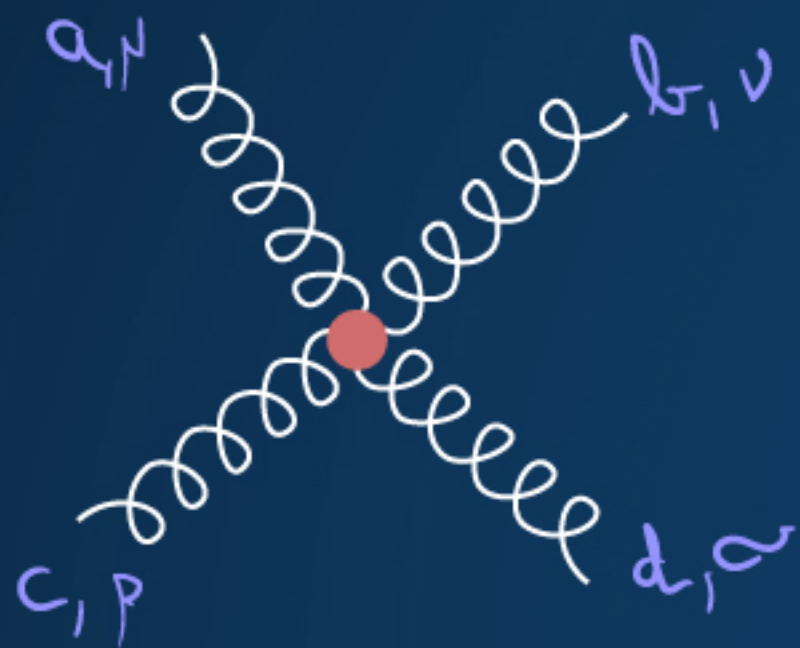
IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$A_\mu^a = (A_\mu^1, A_\mu^2, A_\mu^3)$$

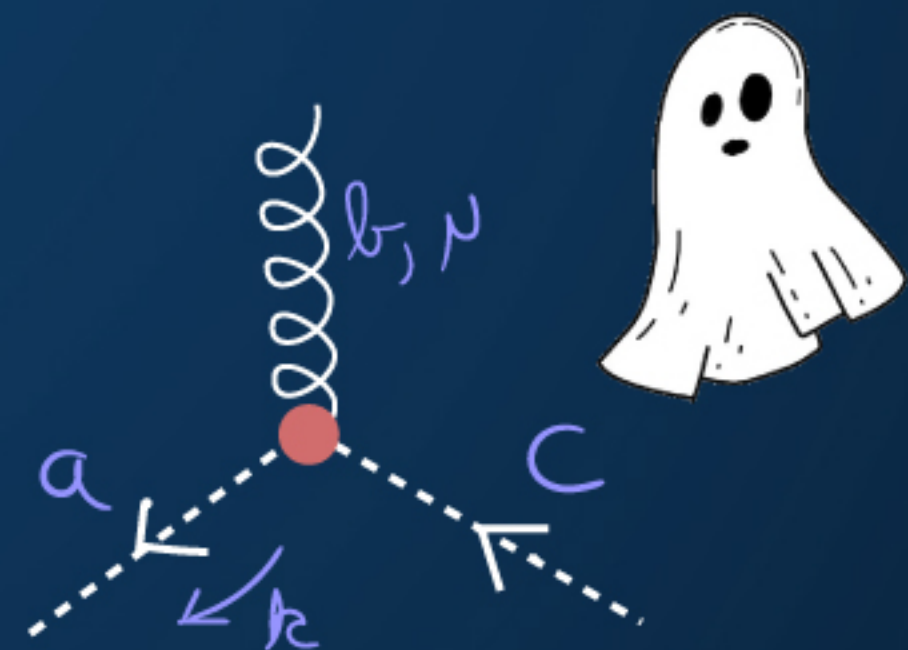
QUANTUM FIELD THEORY

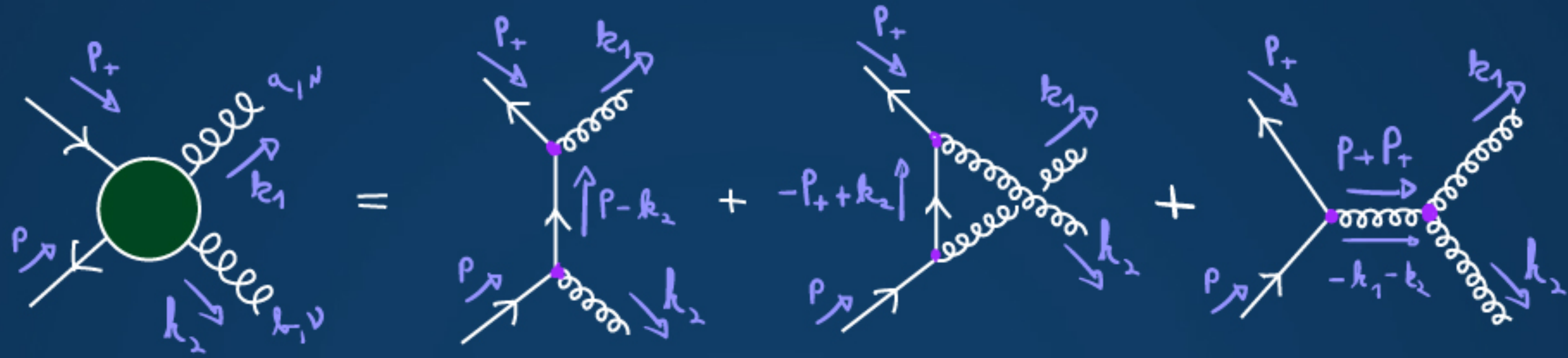
Quantization of a Non-Abelian Gauge Field

Non-Abelian Theories, Part III



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





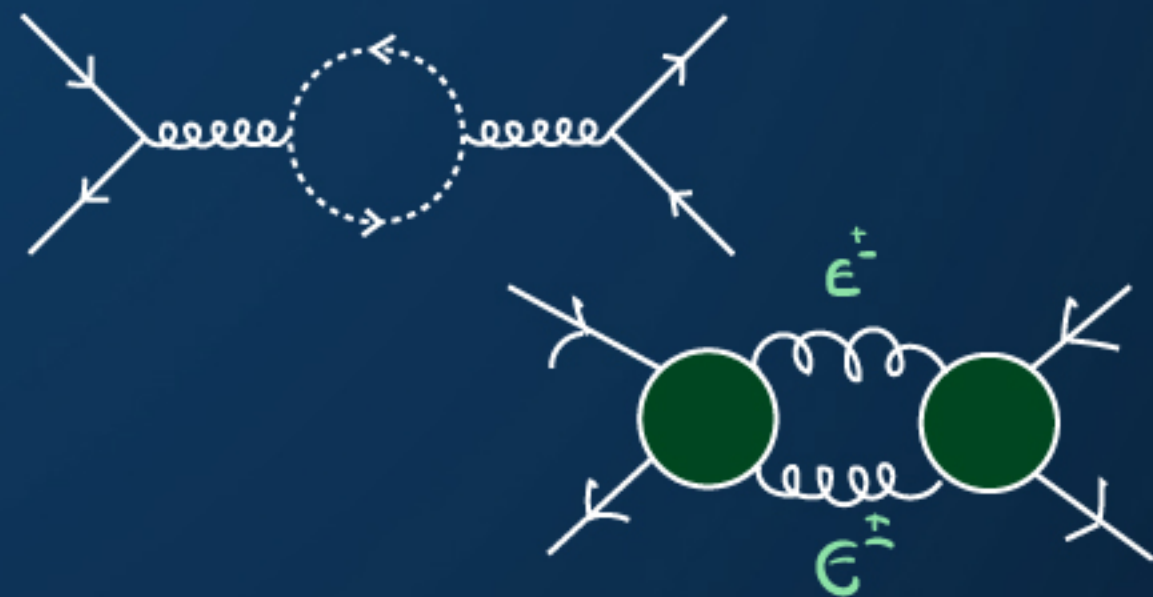
QUANTUM FIELD THEORY

Faddeev-Popov Ghosts and Unitarity

Non-Abelian Theories, Part IV



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA





QUANTUM FIELD THEORY

Asymptotic Freedom (in Gauge Theories)

Non-Abelian Theories, Part V



IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} n_f C_2(R) \right]$$

