$$\mathcal{L} = \bar{\psi}_e (\not\partial - m)\psi_e - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{c_1\, \bar{\psi}_e \psi_e \bar{\psi}_e \psi_e + c_2 \left(F^{\mu\nu}F_{\mu\nu}\right)^2 + \ldots}_{\text{WHERE?}}$$

QUANTUM FIELD THEORY

Lagrangians and Observables

an invitation to renormalization

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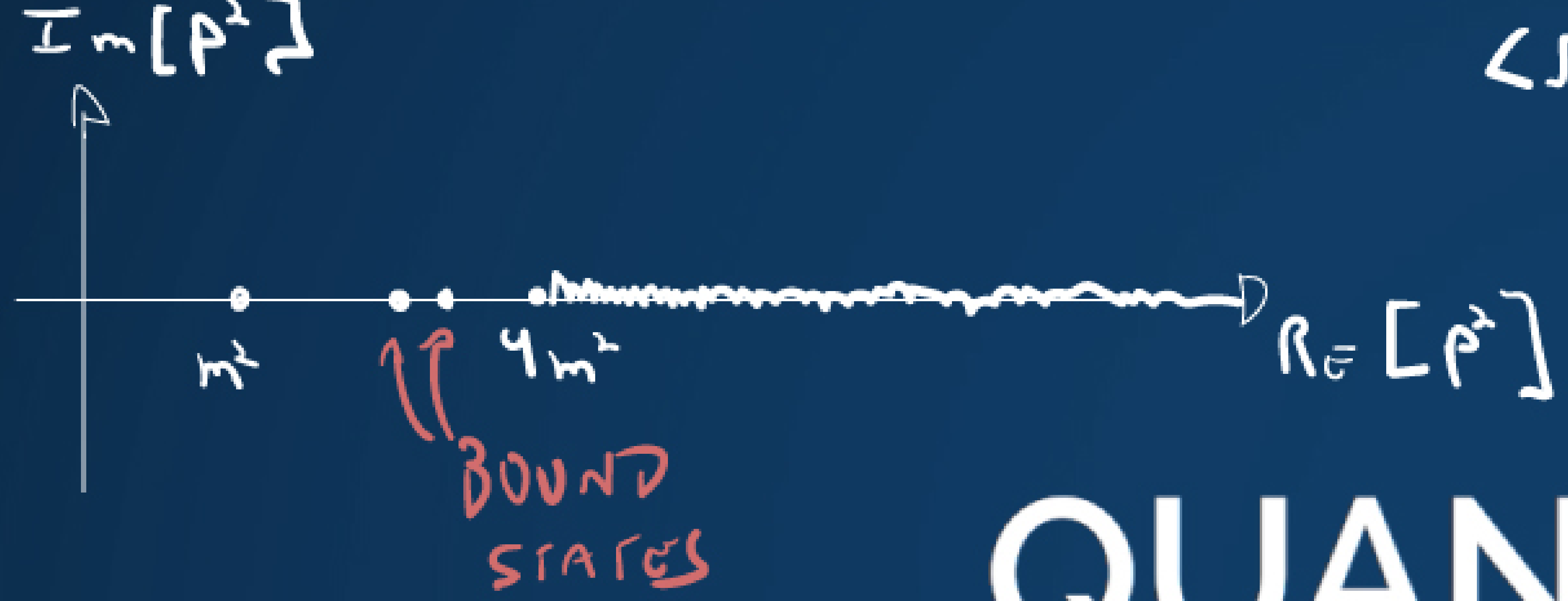
$$\mathcal{L} = \bar{\psi}_e (\not\partial - \underbrace{m}_{\to m_e,\ \to m_\gamma = 0})\overbrace{\psi_e}^{\to e^-,\, e^+} - \frac{1}{4}\overbrace{F_{\mu\nu}F^{\mu\nu}}^{\to \gamma} - i\underbrace{e}_{\to e}\bar{\psi}_e \not A \psi_e$$



$$\langle \Omega | T\{\phi(x)\phi(y)\} | \Omega \rangle = \int_0^\infty \frac{dM^2}{2\pi}\, \rho(M^2)\, D_F(x-y, M^2)$$

QUANTUM FIELD THEORY

Källén–Lehmann Spectral Represention

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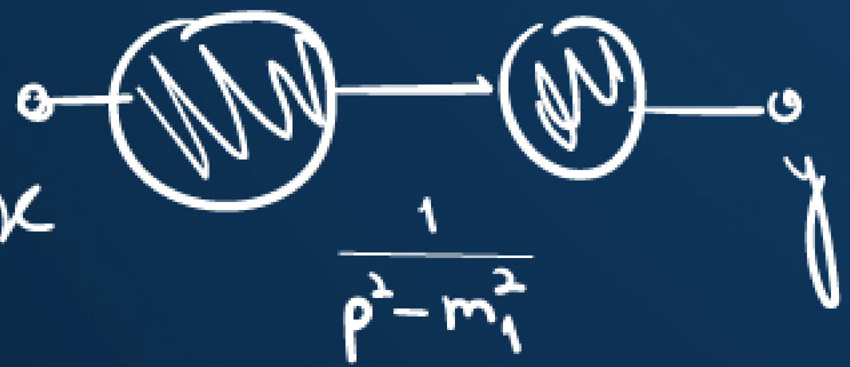
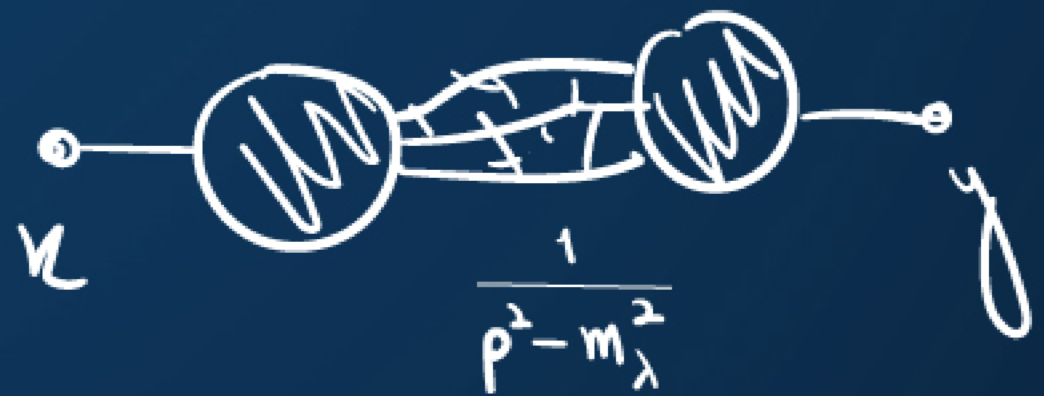
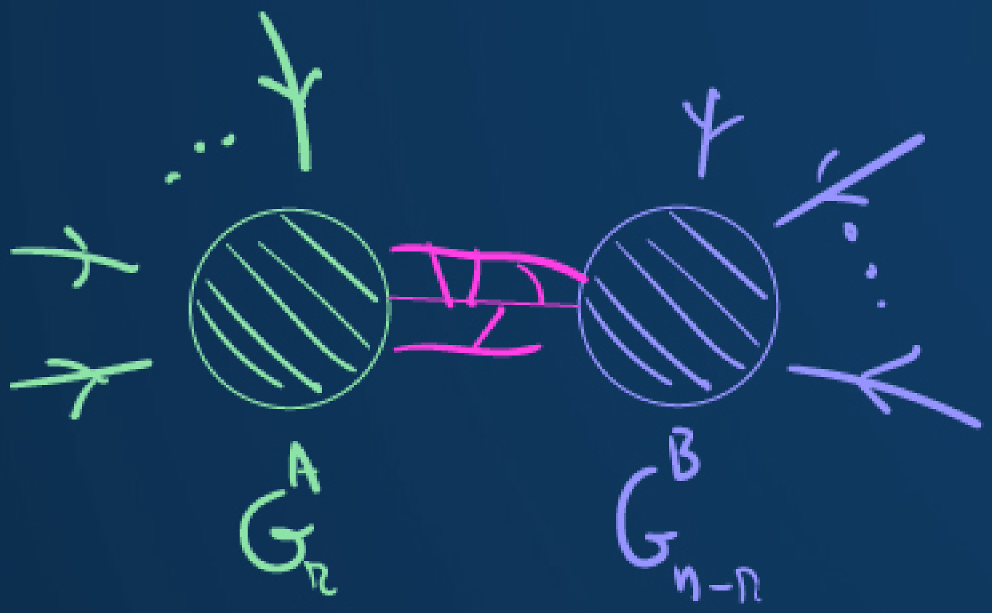




$$G_n(p_1,\ldots,p_n) = (2\pi)^4 \delta^4(p_1+\ldots+p_n) \sum_\lambda G^A_r(q,p_2,\ldots,p_r) \frac{i}{q^2-m_\lambda^2+i\epsilon} G^B_{n-r}(q,p_{r+2},\ldots,p_n) + \text{O.T.}$$

QUANTUM FIELD THEORY

Pology



$$\langle \Omega | j^{\mu 5 a}(x) | \pi^b(p) \rangle$$

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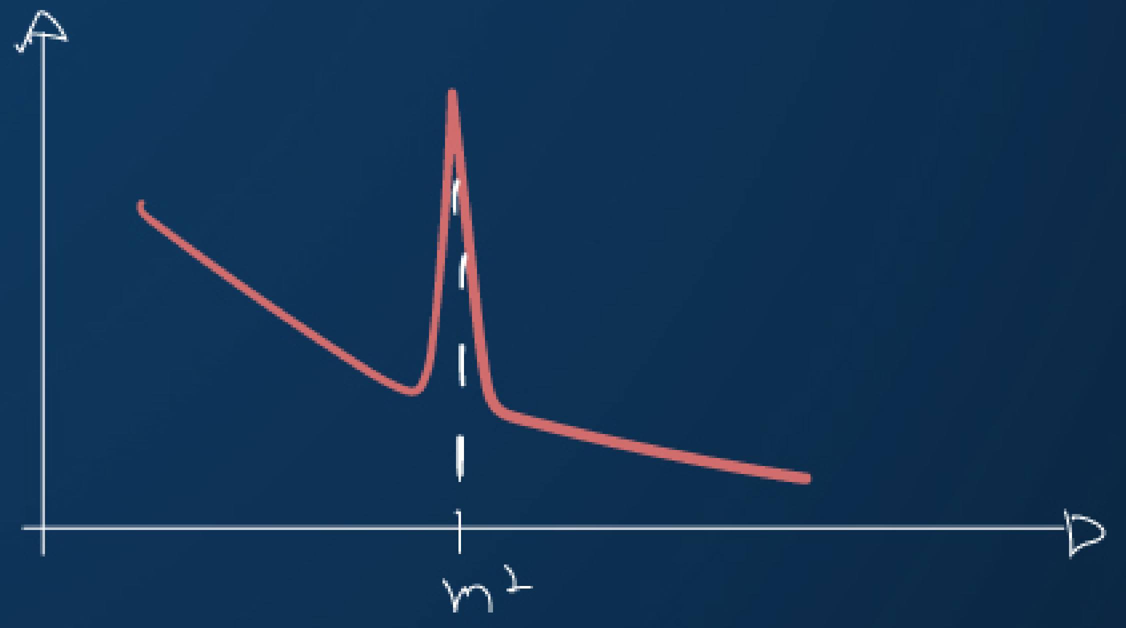

$$\left(\prod_{i=1}^{n} \frac{\sqrt{Z}\; i}{p_i^2 - m^2 + i\epsilon}\right)\left(\prod_{j=1}^{m} \frac{\sqrt{Z}\; i}{k_j^2 - m^2 + i\epsilon}\right)\langle \vec{p}_1 \dots \vec{p}_n | S | \vec{k}_1 \dots \vec{k}_n \rangle$$

QUANTUM FIELD THEORY

The LSZ Reduction Formula

(Lehmann, Symanzik and Zimmermann)

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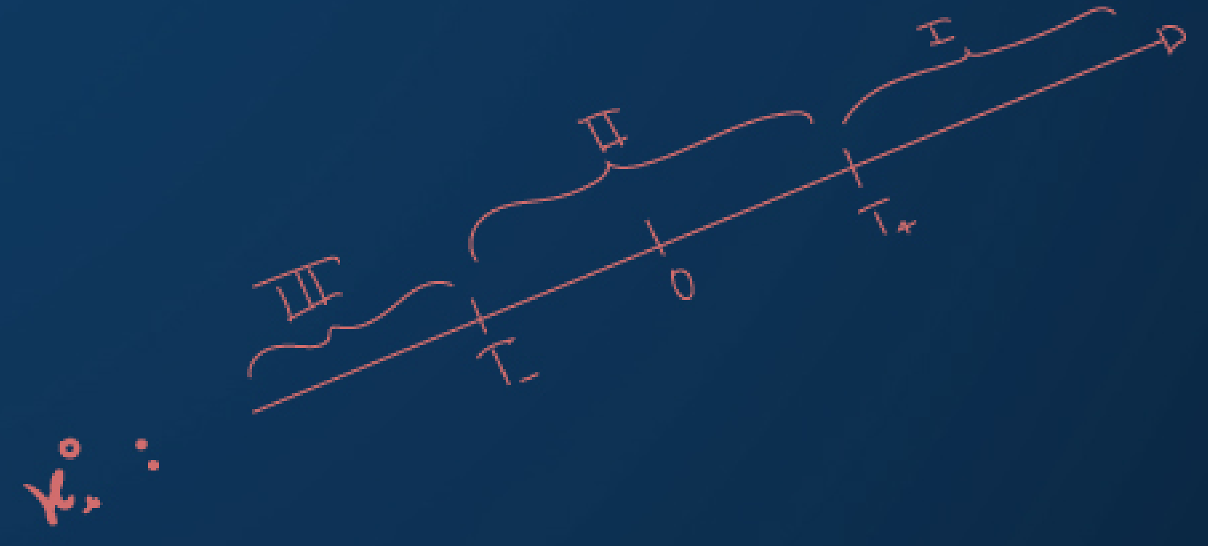
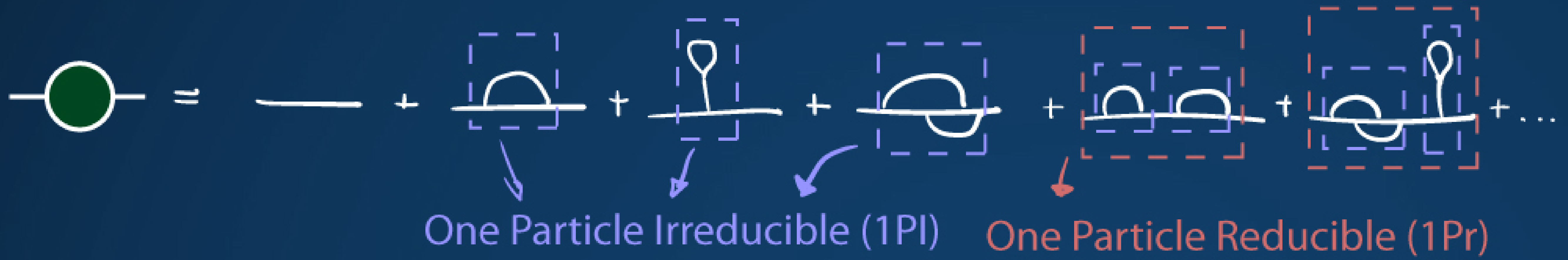
$$\langle \Omega | T\{\phi(x_1)\dots\phi(x_2)\} | \Omega \rangle$$

$$\prod \left(\begin{matrix}\text{full} \\ \text{propagators}\end{matrix} \right) \langle S \rangle$$





QUANTUM FIELD THEORY

Field and Mass Renormalization

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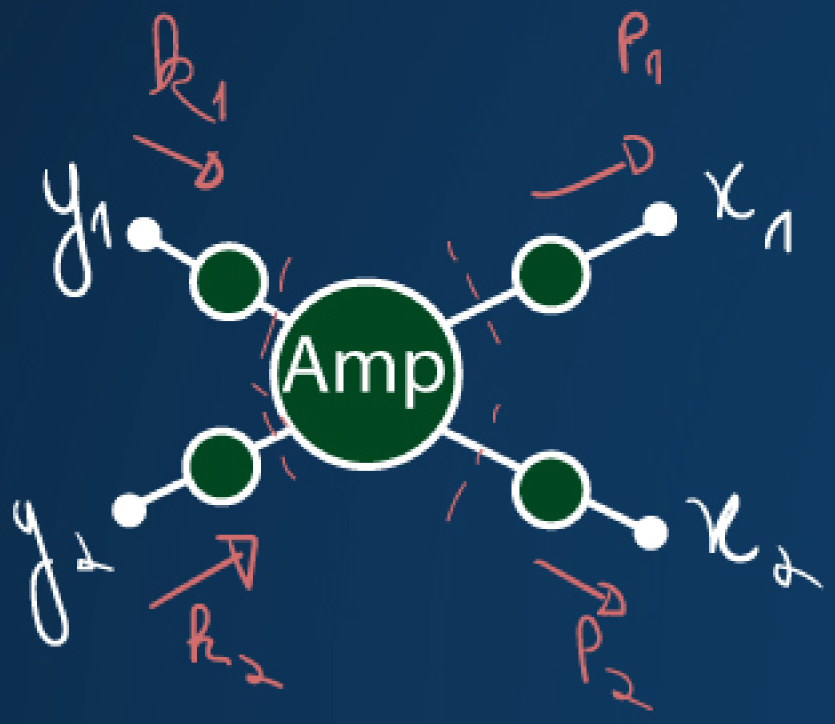
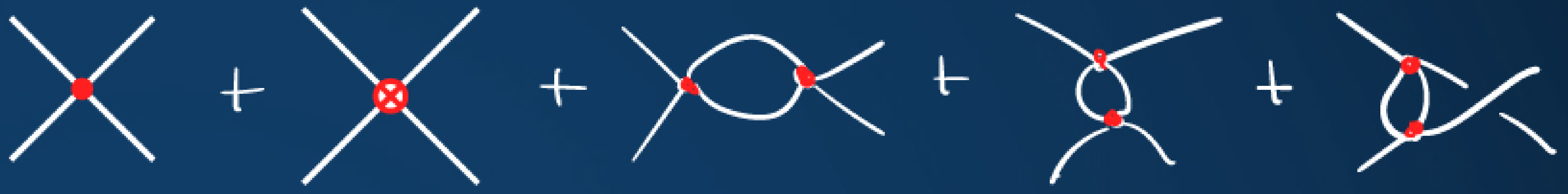
$$\frac{i}{p^2 - m_0^2 - M^2(p^2)}$$





QUANTUM FIELD THEORY
Vertex Functions and Coupling Renormalization

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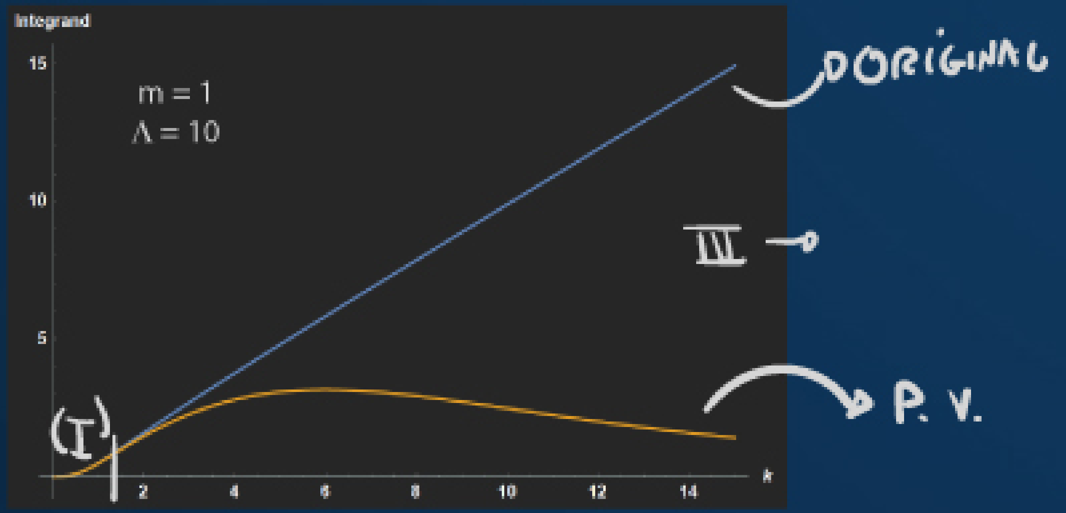
$= -i\,\delta\lambda$

$$i\,\Gamma^{(2)}_{l} = \frac{\lambda^2}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2-m^2+i\epsilon)} - \frac{1}{(k^2-\Lambda^2+i\epsilon)}\right]\frac{1}{((k+p)^2-m^2+i\epsilon)}$$

QUANTUM FIELD THEORY

One-loop $\lambda\phi^4$ and Pauli-Villars Regularization



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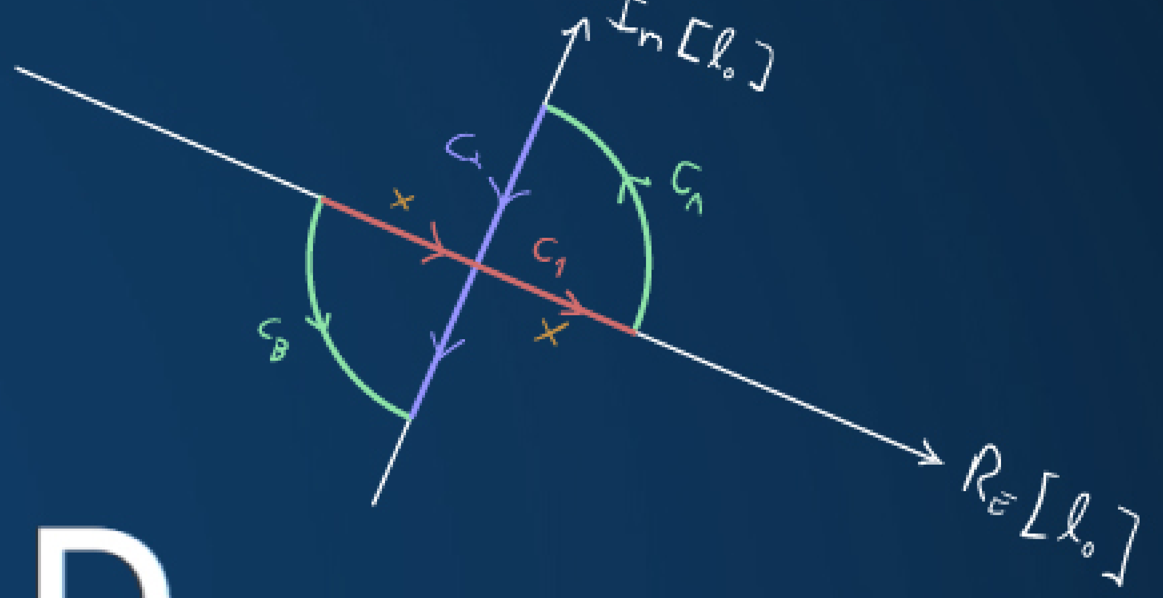
$$\frac{1}{\left[x A + y B\right]^2}$$



QUANTUM FIELD THEORY
Loop tools and tricks
Feynman Parametrization and Wick Rotation (in $\lambda\phi^4$)

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$$\mathcal{M}(s,t,u) = -\lambda + \left[\tilde{\Gamma}_\ell(s) + \tilde{\Gamma}_\ell(t) + \tilde{\Gamma}_\ell(u) - \tilde{\Gamma}_\ell(4m^2)\right] + \ldots$$

$$\frac{2}{\epsilon} + \mathrm{Ln}(\mu^2) + N \Longleftrightarrow \mathrm{Ln}(\Lambda^2)$$

QUANTUM FIELD THEORY

Dimensional Regularization (in $\lambda\phi^4$)

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

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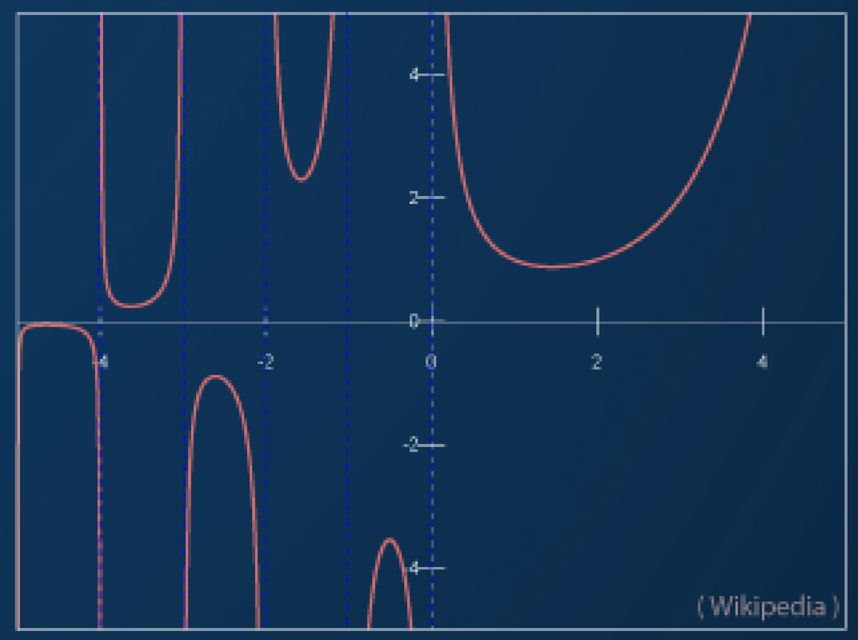
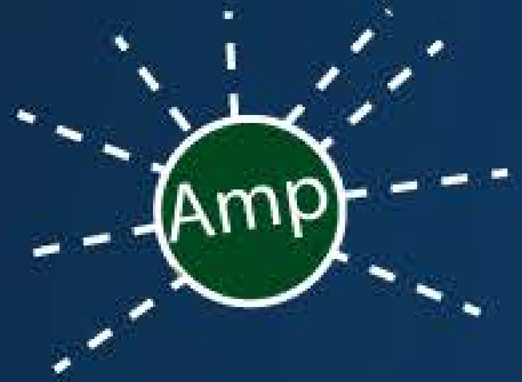
Gamma function





$\sim \lambda_n^V \Lambda^D$

$\Delta_n > 0$ $\Delta_n = 0$

$\Delta_n < 0$

QUANTUM FIELD THEORY
Power Counting and Renormalizability I
($\lambda\phi^n$ theories)

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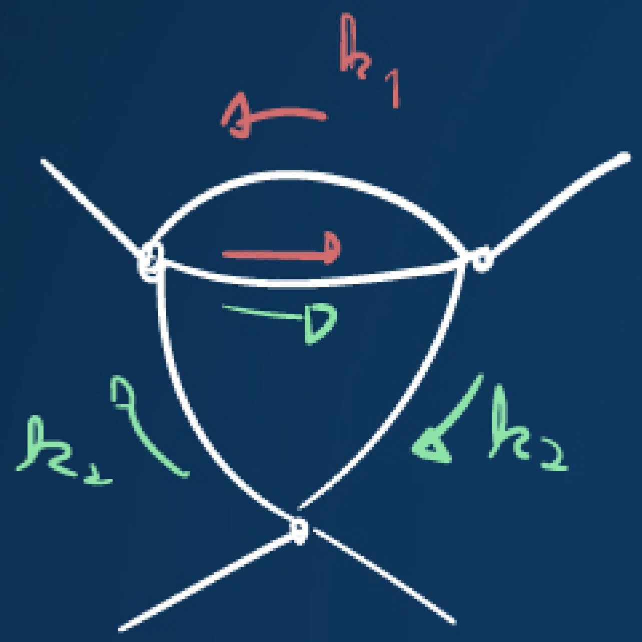
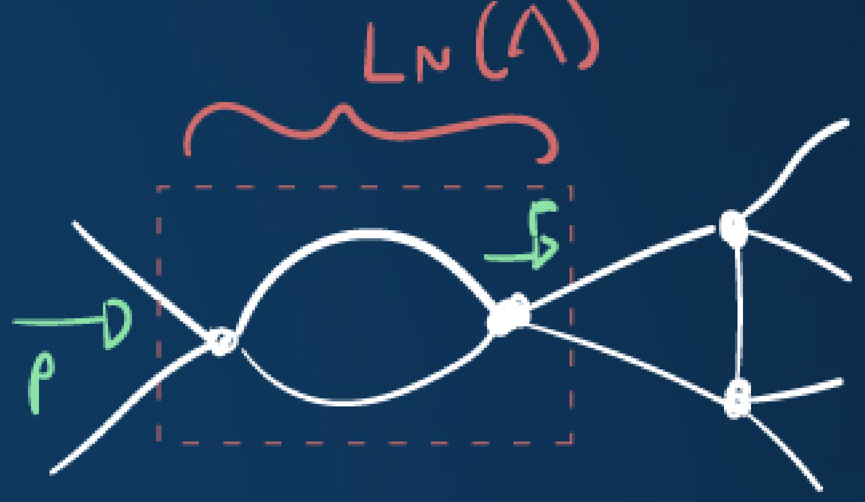
$$\Delta_n \equiv d - n\left(\frac{d-2}{2}\right)$$

$$D = d - \Delta_n V - \left(\frac{d-2}{2}\right) N$$





QUANTUM FIELD THEORY

Renormalization at 2 loop order

(or more)

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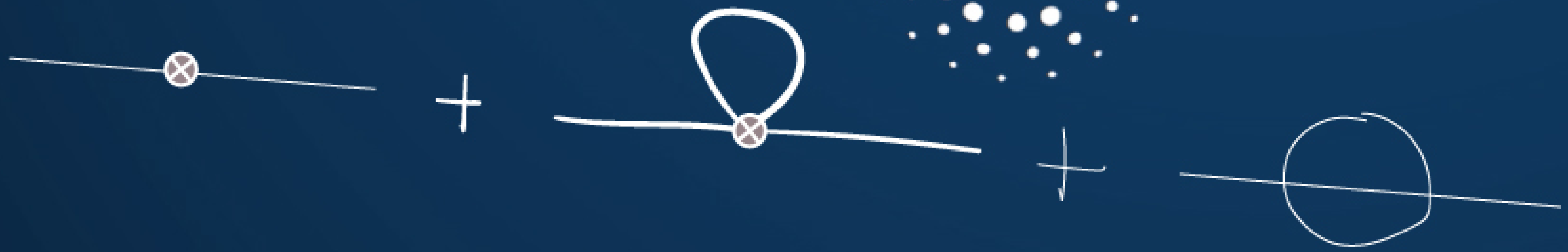

$$\Delta_i \equiv \left[d - d_i - \sum_\ell n_{i\ell}\left(s_\ell + \frac{d-2}{2}\right)\right]$$

$$\gamma\bar{\psi}\psi\phi$$

QUANTUM FIELD THEORY

Power Counting and Renormalizability II (spin 0, ½ and 1)

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$$eA_\mu\bar{\psi}\gamma^\mu\psi$$

$$(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)$$

$$G_F^0(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)$$
$$G_{F'}^0(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)$$
$$G_{F''}^0(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)(\bar{\psi}\,\Gamma\psi)$$

QUANTUM FIELD THEORY

Non-Renormalizable Theories

... are doing fine!

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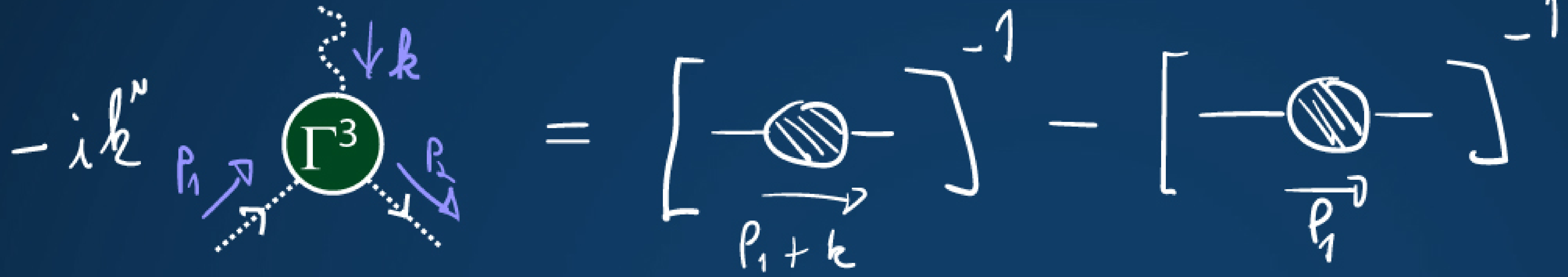
$$|\mathcal{M}(2\to 2)|^2 \sim g_4^2\,\frac{p^{4\Delta}}{M^4} + \dots + g_6^2\,\frac{p^{10}}{M^{10}} + \dots + g_8^2\,\frac{p^{16}}{M^{16}}$$



QUANTUM FIELD THEORY

Ward-Takahashi Identities

$$\langle \partial_\nu j^\nu \rangle = 0$$

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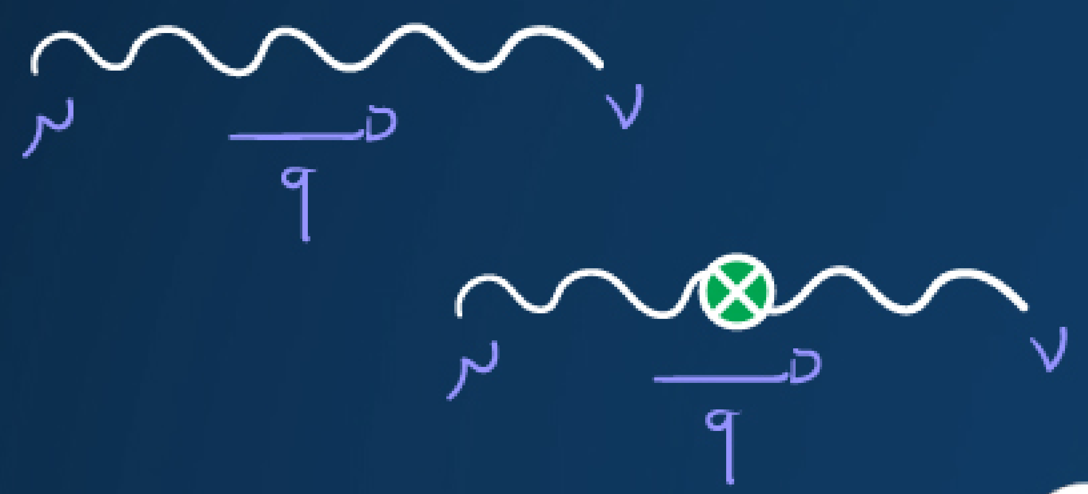
$$k_\nu \mathcal{M}^\nu(k) = 0$$



$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$

QUANTUM FIELD THEORY

Renormalization of QED I

Power Counting and Counterterms

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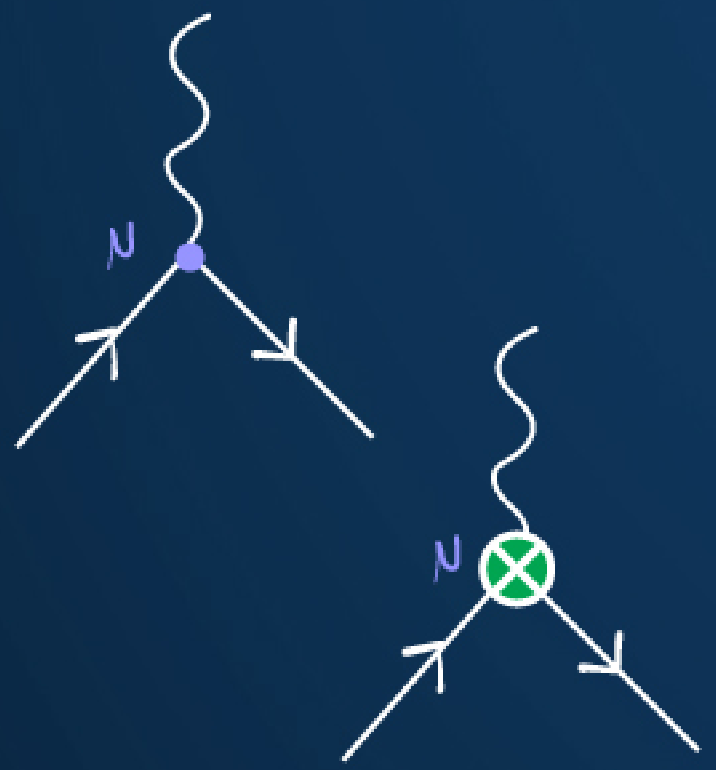

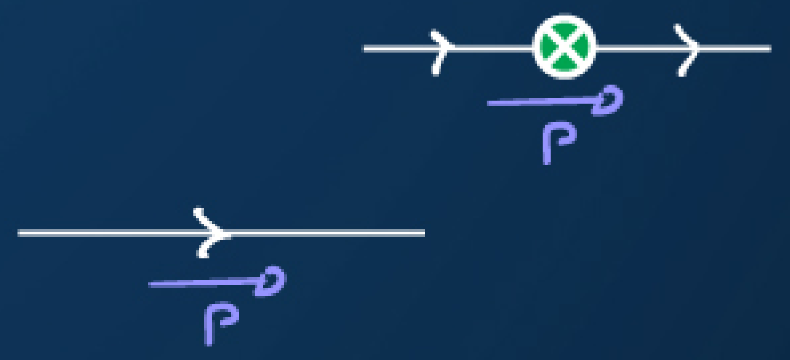

$g - 2$

QUANTUM FIELD THEORY

Renormalization of QED II

Formal Structure of the QED Vertex

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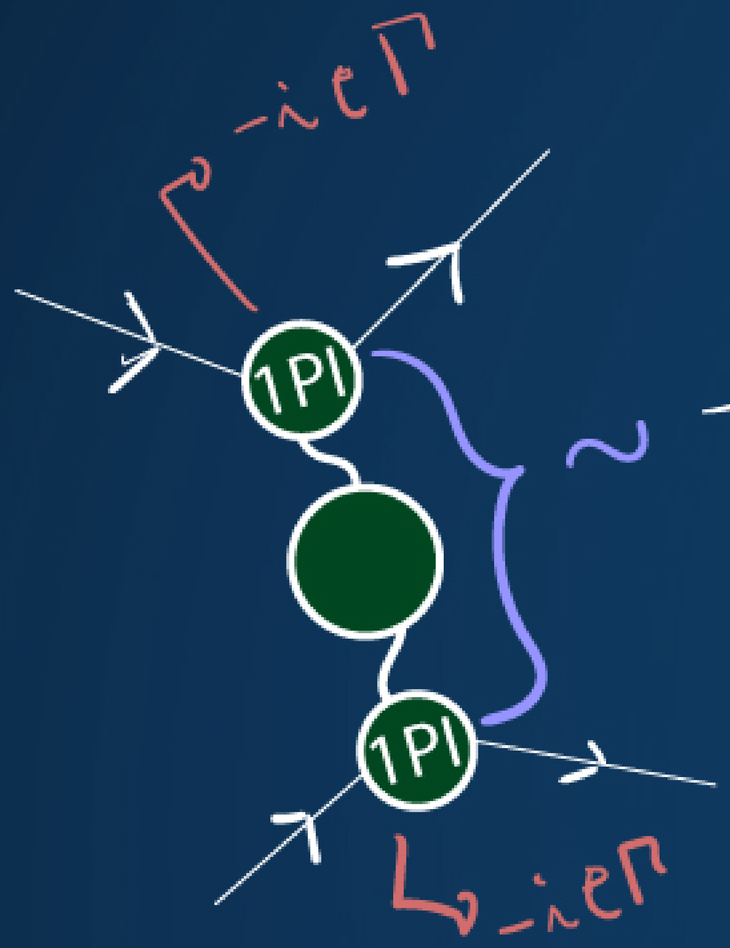
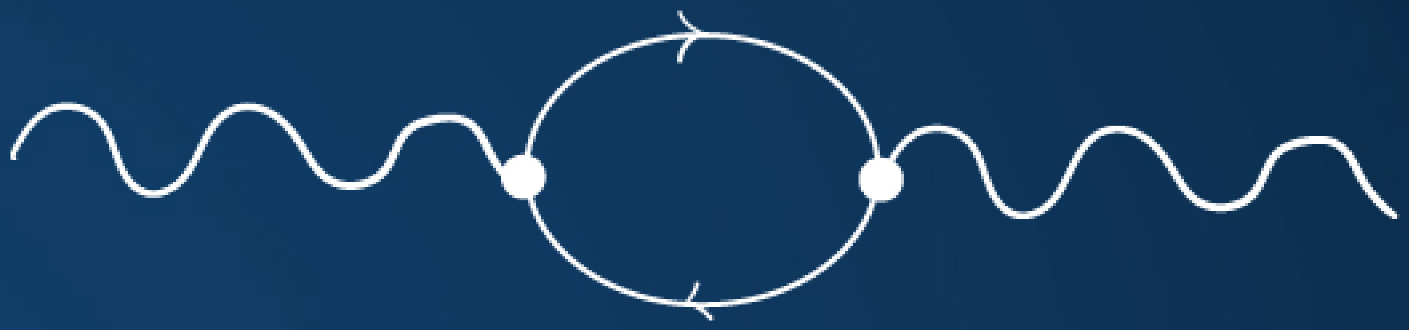
$$\vec{\mu} \equiv g\left(\frac{e}{2m}\right)\vec{S}$$

$$\Gamma^{\mu} \equiv \gamma^{\mu} F_1(q^2) + i\frac{\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2)$$



$$\sim \frac{-i\, g_{\mu\nu}\, e^2}{q^2\left(1-\Pi(q^2)\right)}$$



QUANTUM FIELD THEORY

Renormalization of QED III

The Photon Self-Energy

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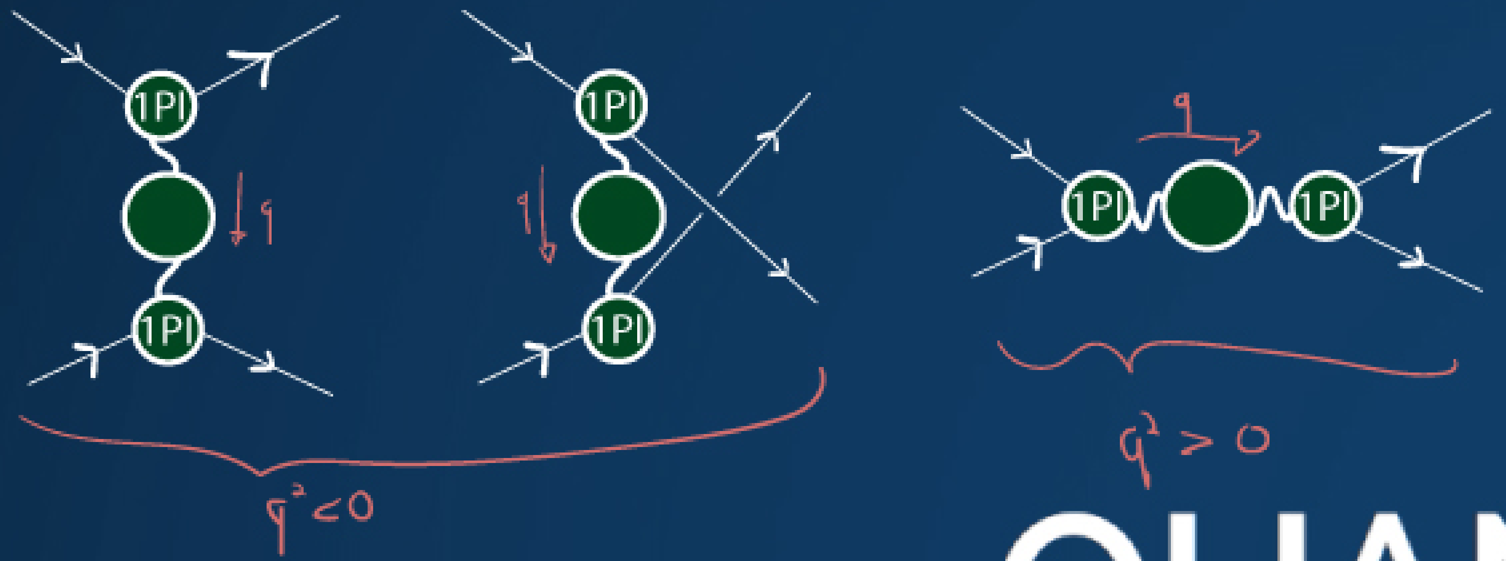
$$e^2_{EFF}(q^2) \equiv \frac{e^2}{\left(1-\Pi(q^2)\right)}$$



$$V(r) = -\frac{\alpha}{r}\left(1 + \underbrace{\frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2mr}}{(mr)^{3/2}}}_{\text{Uehling Potential}} + \cdots\right)$$

QUANTUM FIELD THEORY

Renormalization of QED IV

Physics of $\Pi(q^2)$

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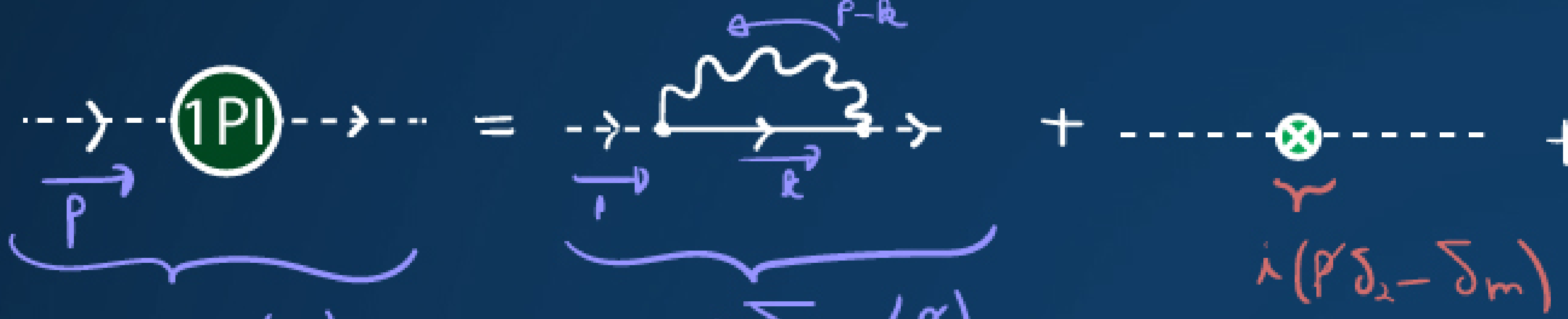

$$-i\Sigma(p) = -i\Sigma_2(p) + i(\not p\,\delta_2 - \delta_m) + \mathcal{O}(e^4 \sim \alpha^2)$$

QUANTUM FIELD THEORY

Renormalization of QED V

Electron Self Energy & the MS Scheme

$$m_{\overline{MS}}(\mu) = m - \Sigma_{\overline{MS}}(m, \mu)$$

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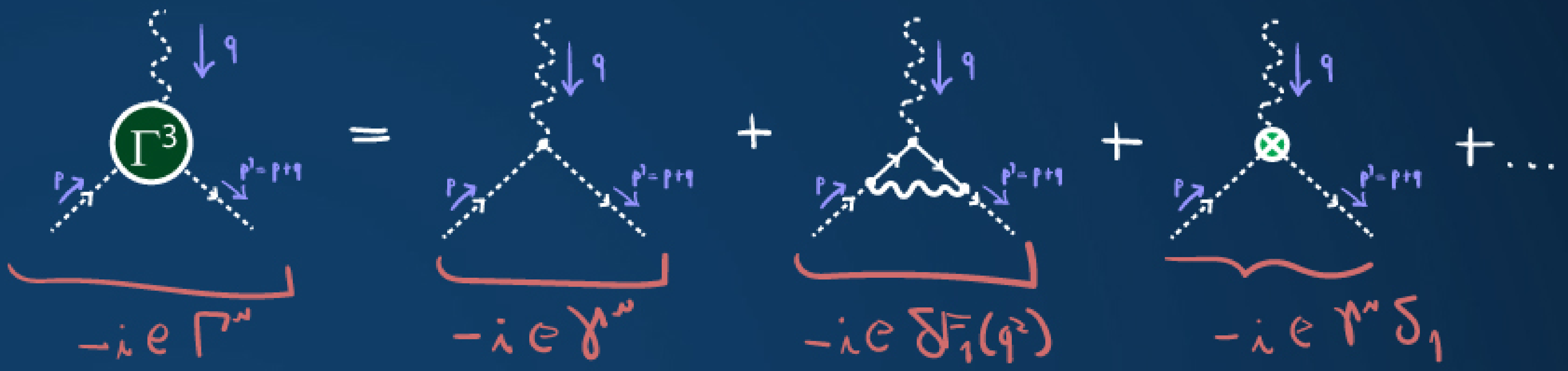
$$\delta_m^{MS} = \frac{-4\alpha\, m_{MS}}{2\pi\epsilon}$$

$$\delta_2^{MS} = -\frac{\alpha}{2\pi\epsilon}$$

$$\frac{\alpha}{2\pi}$$



QUANTUM FIELD THEORY

Renormalization of QED VI

Vertex Corrections and g-2

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$$F_2(0) = \left(\frac{g-2}{2}\right)_{EXP} = 0{,}00115965218073(28)$$

(I) $E_\gamma \sim 0$ Soft Singularity

(II) $\cos(\theta) \sim 0$ Collinear Singularity

QUANTUM FIELD THEORY

Renormalization of QED VII

Infrared Divergences

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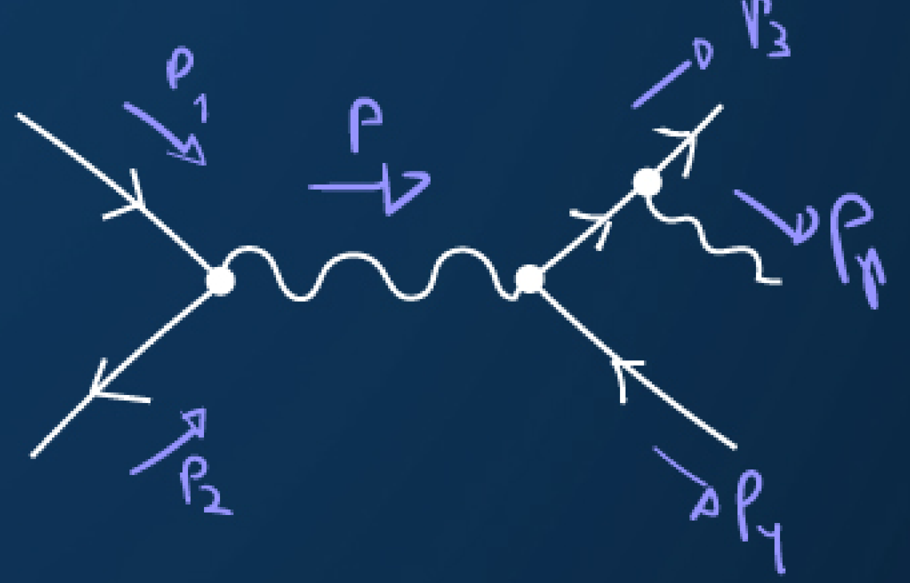
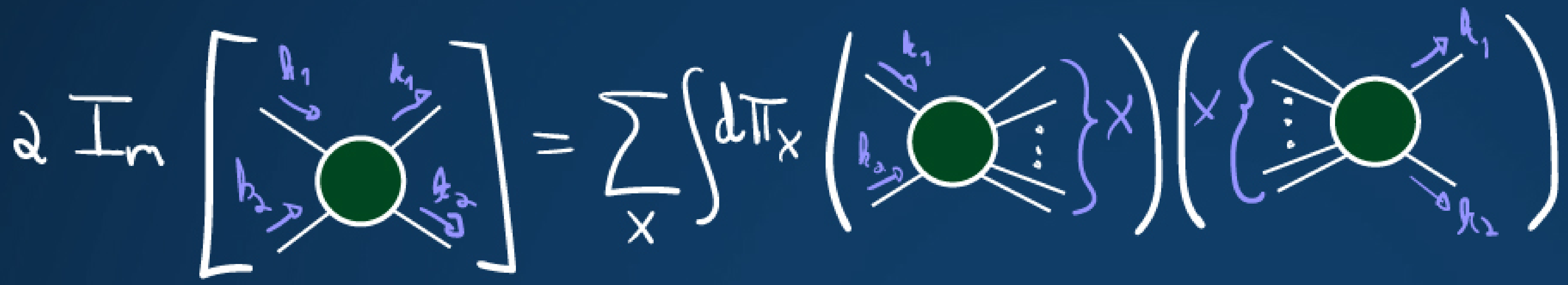
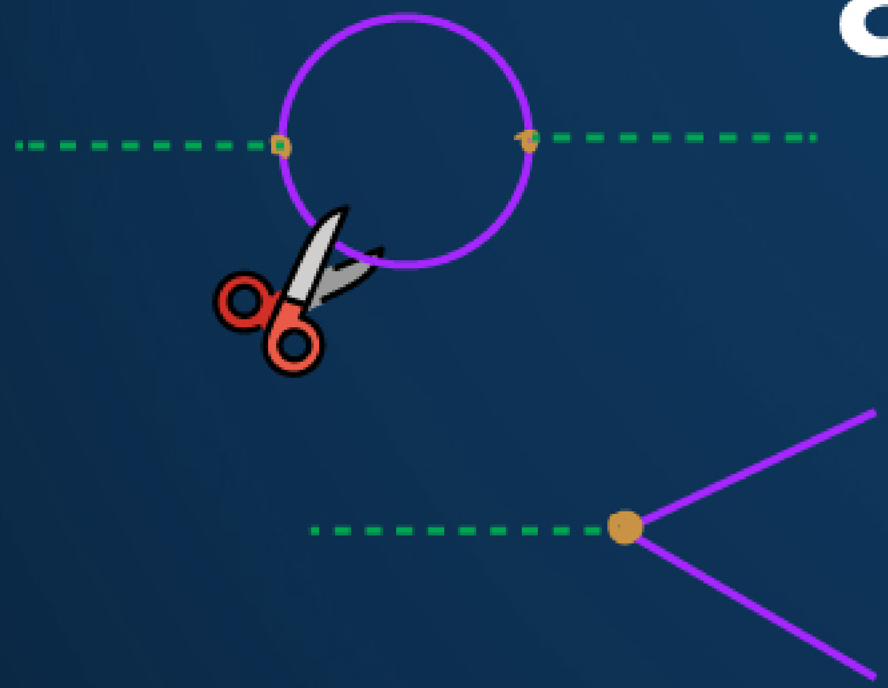
$$\sigma_{TOT} = \left(1 + \frac{3\alpha}{4\pi}\right)\sigma_{LO}\left(e^+e^- \to \mu^+\mu^-\right)$$





QUANTUM FIELD THEORY

The Optical Theorem and Unstable Particles



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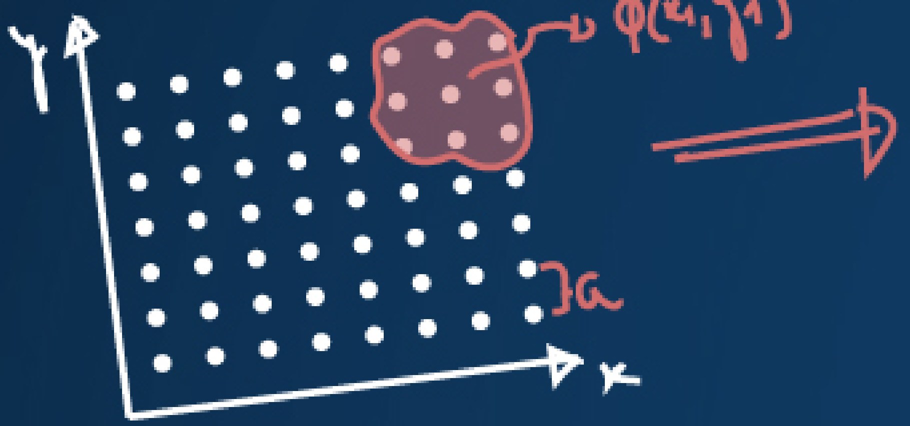
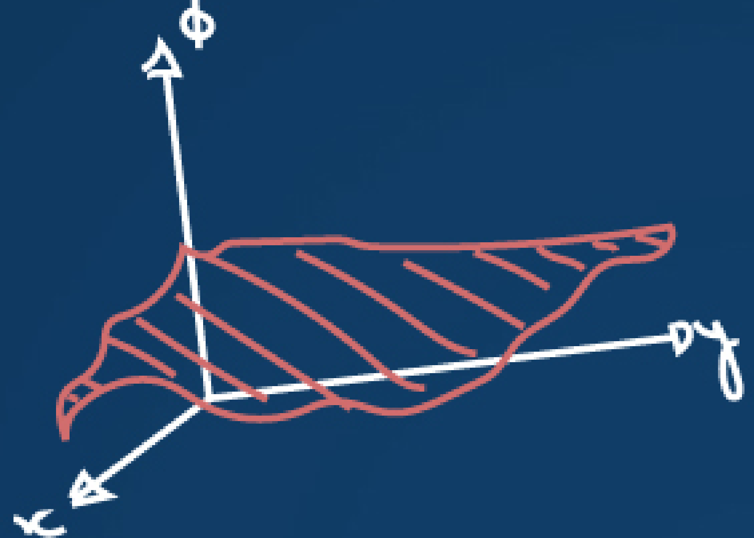
$$\frac{1}{(s-m^2)^2 + m^2\Gamma_{TOT}^2} \sim \frac{\pi}{m\Gamma}\delta(s-m^2)$$





$$\mathcal{L}(\Lambda, \underline{m_0^2}, \underline{\lambda_0}) \xrightarrow{\hat{\phi}} \mathcal{L}_{EFF}(b\Lambda, \underline{m_0^2+\mu}, \underline{\lambda_0+\zeta}, \underline{\eta}, \cdots)$$

QUANTUM FIELD THEORY

Wilsonian Renormalization I

Coarse Graining and Momentum Shell Integration

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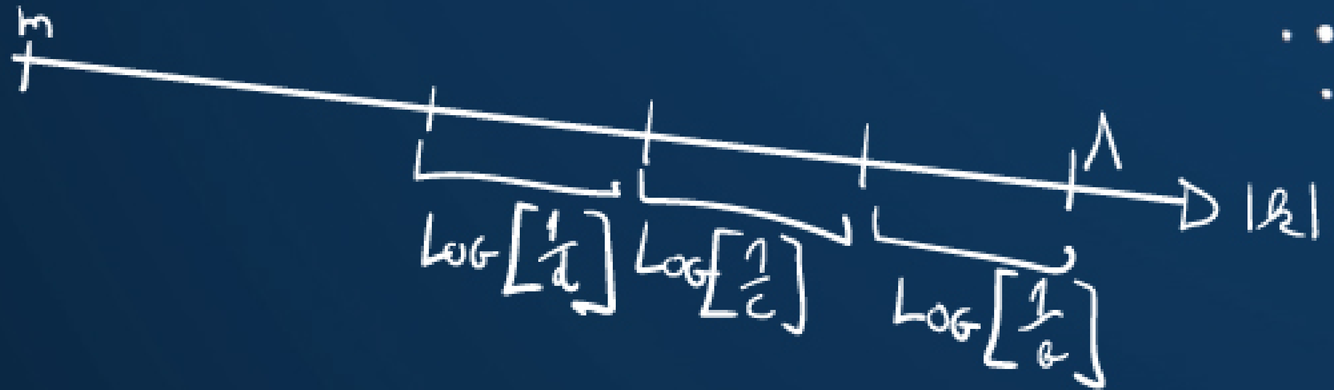
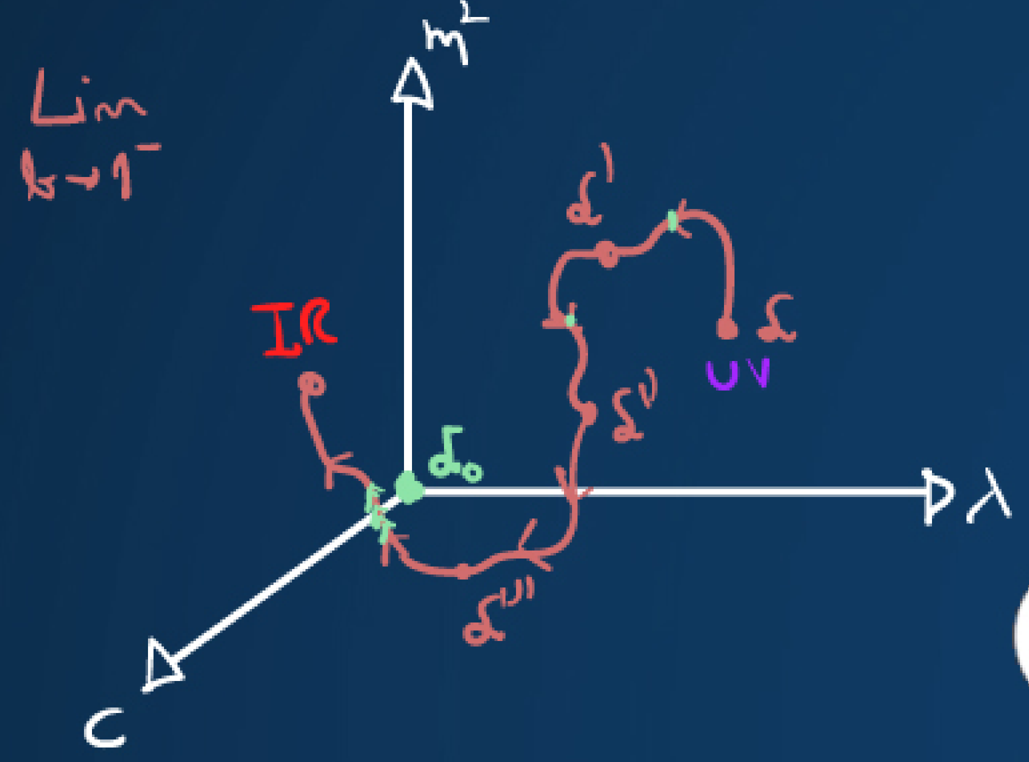
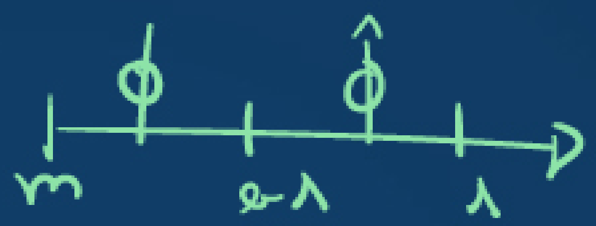




$\mathcal{L}(\phi) = \mathcal{L}(\phi, \hat{\phi}) \xrightarrow{\text{INTEGRATE OUT } \hat{\phi}} \mathcal{L}_{EFF}(\phi'(x'))$

SCALE $\begin{cases} k' = k/b \\ x' = x\,b \end{cases}$



QUANTUM FIELD THEORY

Wilsonian Renormalization II

Renormalization Group Transformations

$$\lambda' \simeq \lambda - \frac{3\lambda^2}{16\pi^2} \ln\left(\frac{1}{b}\right)$$

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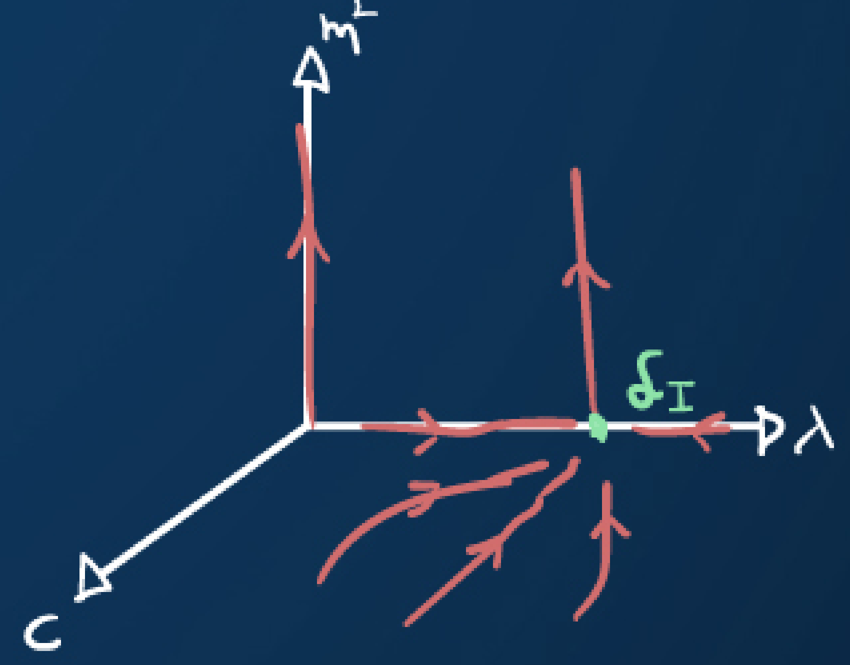

$$\left[M\frac{\partial}{\partial M}+\beta(\lambda)\frac{\partial}{\partial\lambda}+n\gamma(\lambda)\right]G^{(n)}(x_1,\ldots,x_n;M,\lambda)=0$$

QUANTUM FIELD THEORY

Renormalization Group Equations (RGE)

The Callan-Symanzik Equation

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$$\beta = \mu \frac{d\lambda(\mu)}{d\mu}$$

$$\frac{e^3}{12\pi^2} \qquad \beta \qquad \frac{3\lambda^2}{(4\pi)^2}$$

$$\gamma$$

$$\frac{e^2}{12\pi^2} \qquad \frac{\lambda^2}{12(4\pi)^2}$$

QUANTUM FIELD THEORY

Beta (β) and Gamma (γ) Functions

$$\gamma = \frac{1}{2}\frac{\mu}{Z}\frac{dZ}{d\mu}$$

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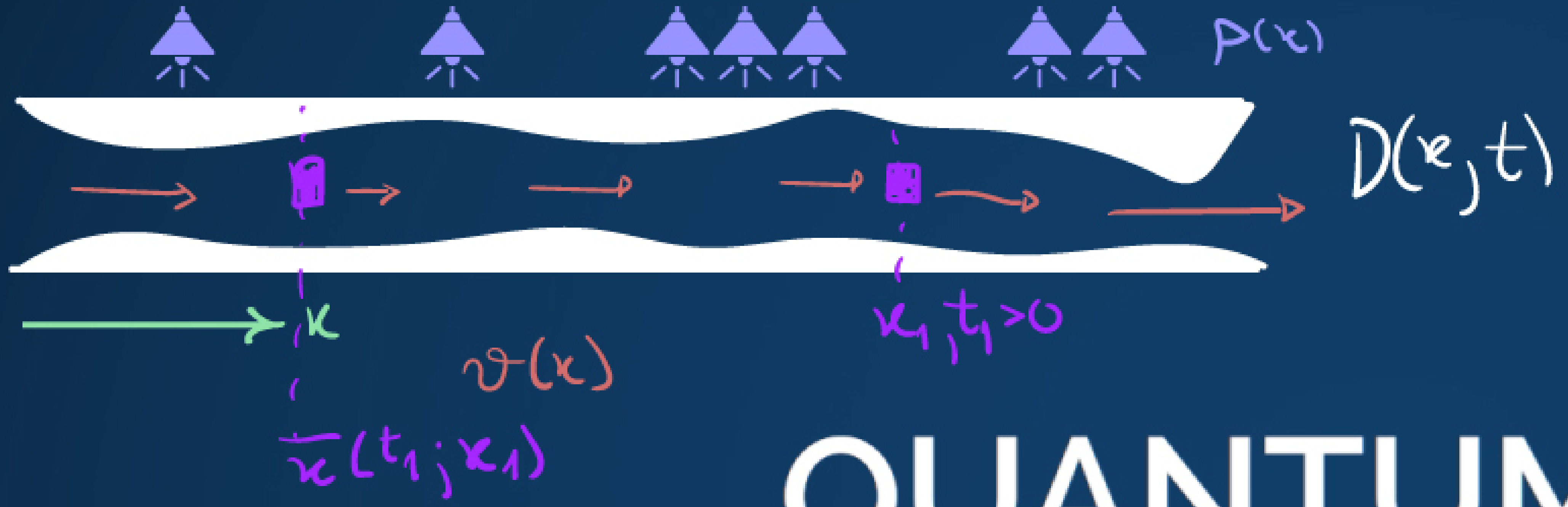

$$\frac{\partial}{\partial\left[\ln\left(\frac{k}{m}\right)\right]}\bar{\lambda}(k;\lambda)=\beta\left(\bar{\lambda}(k;\lambda)\right)$$

QUANTUM FIELD THEORY

Solving the Callan-Symanzik Equation

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$$\left[k\frac{\partial}{\partial k}+2-\beta(\lambda)\frac{\partial}{\partial\lambda}-2\gamma\right]G^{(2)}=0$$

$$\left[q\frac{\partial}{\partial q} - \beta(e_r)\frac{\partial}{\partial e_r} + 2\right] V(q; M, e_r) = 0$$

$$\boxed{\bar{e}(M; e_r) = e_r}$$

QUANTUM FIELD THEORY

The Running of the Electric Charge

$$\bar{e}^2(q) = \frac{e_r^2}{1-\left(\frac{e_r^2}{12\pi^2}\right)\ln\left(\frac{q^2}{M^2}\right)}$$

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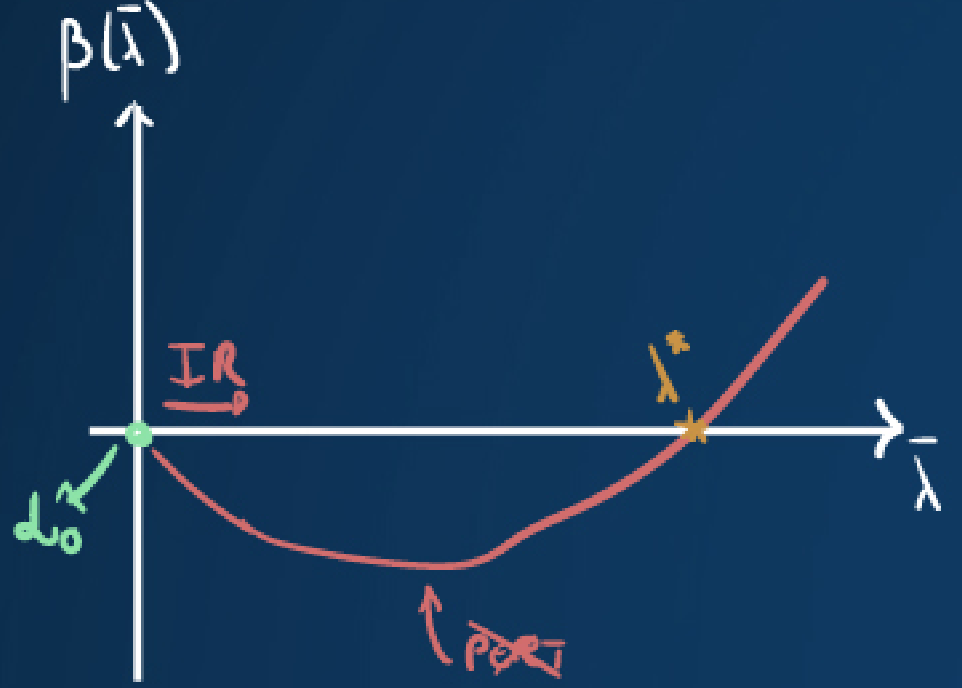

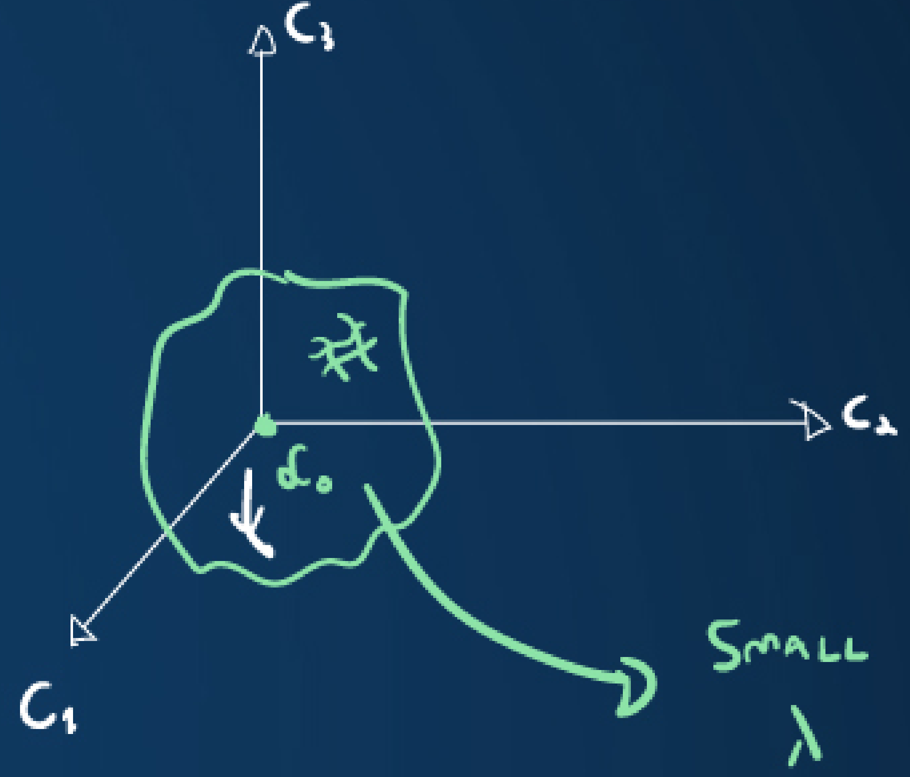

QUANTUM FIELD THEORY

Fixed Points and Anomalous Dimensions

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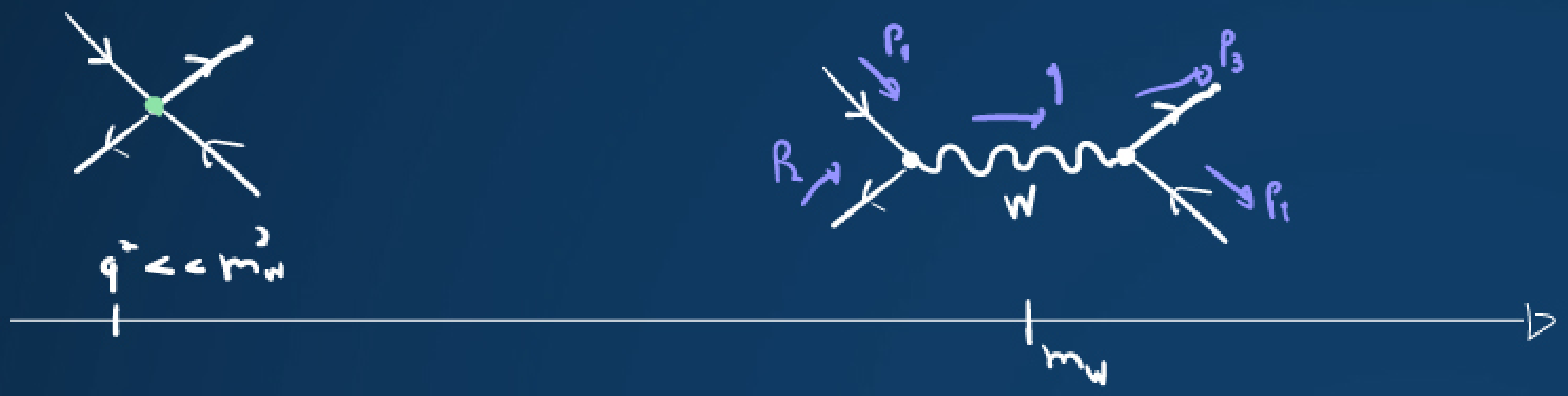
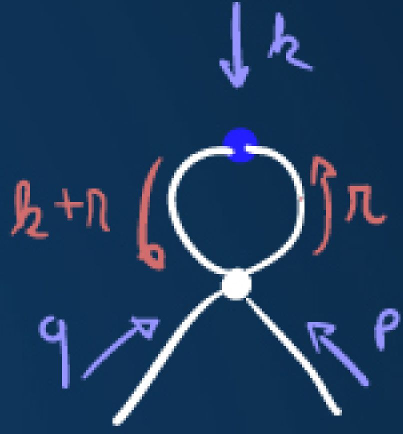
$$G^{(2)}(p, \lambda) = C\left(\frac{1}{p^2}\right)^{1-\gamma(\lambda^*)}$$





QUANTUM FIELD THEORY

Renormalization of Local Operators

$$\gamma_O(\lambda) = \frac{M}{Z}\frac{\partial Z_O}{\partial M} = M\frac{\partial}{\partial M}\ln\left[Z_O(M)\right]$$

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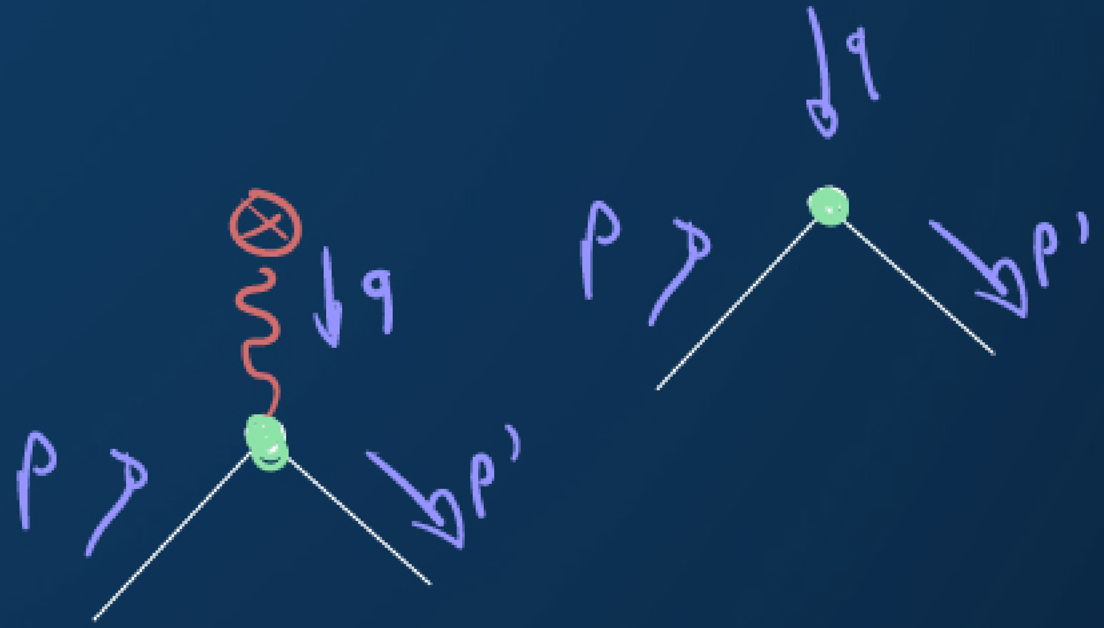

$$G^{(n)} = G^{(n;0)} + m^2 G^{(n;1)} + (m^2)^2 G^{(n;2)} + \ldots + (m^2)^\ell G^{(n,\ell)}$$

QUANTUM FIELD THEORY

Evolution of Mass Parameters

$$\bar{\rho}_m = \rho_m \left(\frac{P}{\mu}\right)^{-2}$$

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$$\left[\mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + n\gamma(\lambda) + \gamma_{\phi^2}(\lambda)\, m^2 \frac{\partial}{\partial m^2}\right] G^{(n)} = 0$$

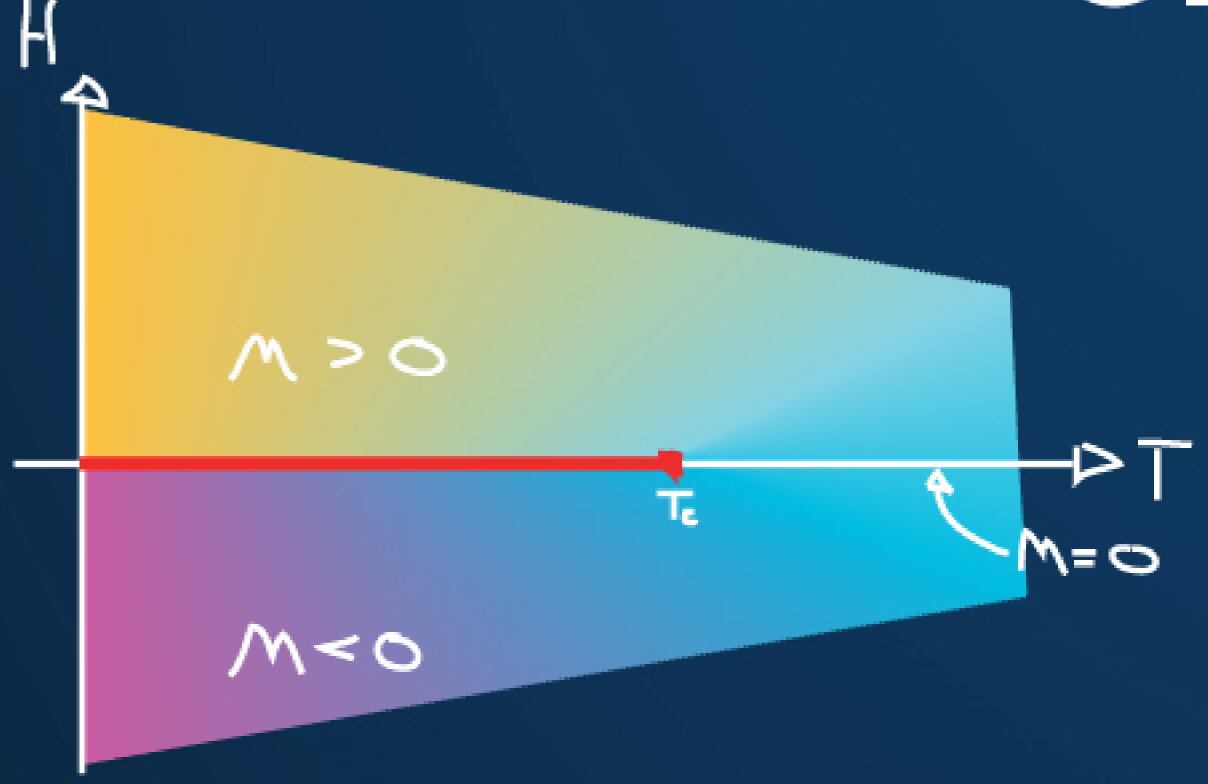
$$\rho_m \left(\frac{m}{\rho_0} \right)^{2-\gamma_{\phi^2}(\lambda^*)}$$

$$\xi \sim (T - T_c)^{-1/2}$$

$$\xi \sim (T - T_c)^{-\nu}$$

QUANTUM FIELD THEORY

Critical Exponents



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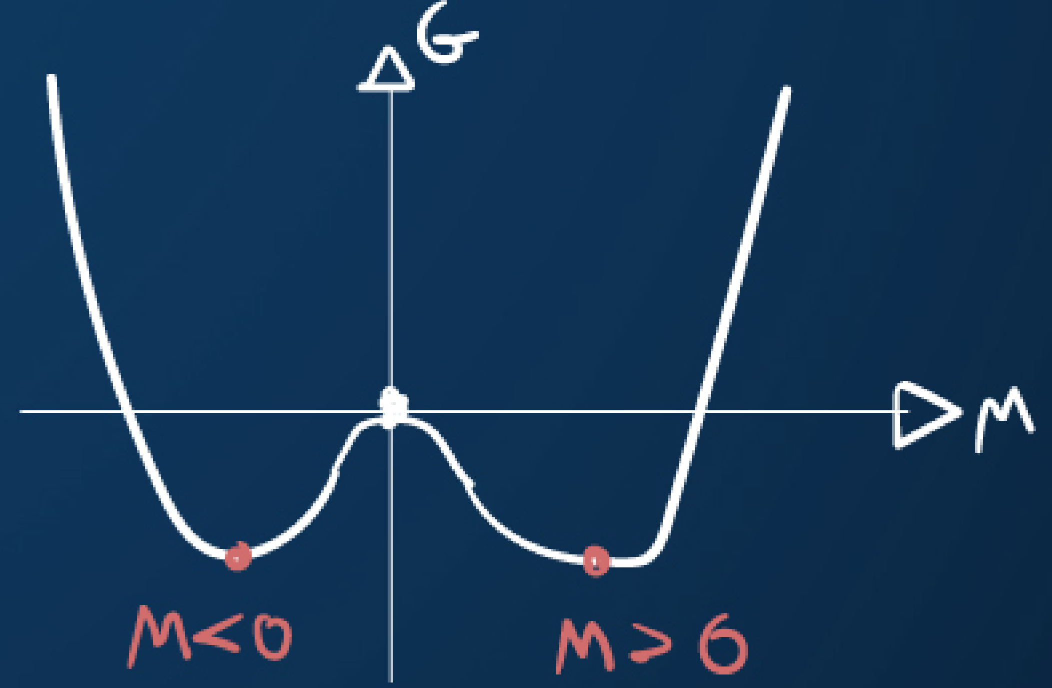

$$f^{ade} f^{bcd} + f^{bde} f^{cad} + f^{cde} f^{abd} = 0$$

$$SU(N)$$

$$u_R^A = \begin{pmatrix} u_R^R \\ u_R^G \\ u_R^B \end{pmatrix}$$

QUANTUM FIELD THEORY

Lie Algebras, the quick and dirty

Non-Abelian Theories, Part I

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$$(t_G^b)_{ac} \equiv i f^{abc}$$

$$[T^a, T^b] \equiv i f^{abc} T^c$$

$$\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{L}_{YM} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2$$

QUANTUM FIELD THEORY

Yang-Mills Theory

Non-Abelian Theories, Part II

$$A^m_\mu \to V(x)\left[A^m_\mu + \frac{i}{g}\partial_\mu\right]V^\dagger(x)$$

$$\frac{1}{2}\begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix}$$

$$G^a_\mu = (G^1_\mu, \dots, G^8_\mu)$$

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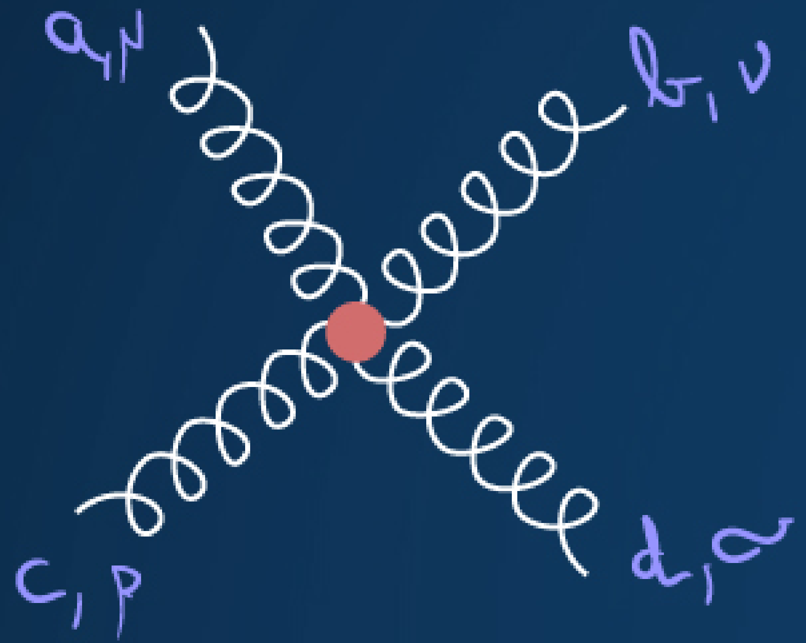
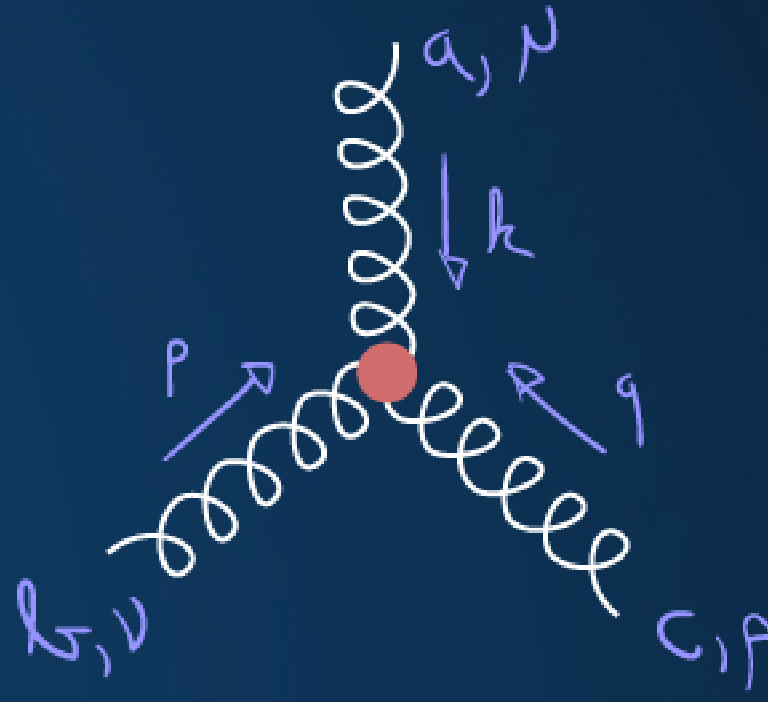
$$A^a_\mu = (A^1_\mu, A^2_\mu, A^3_\mu)$$

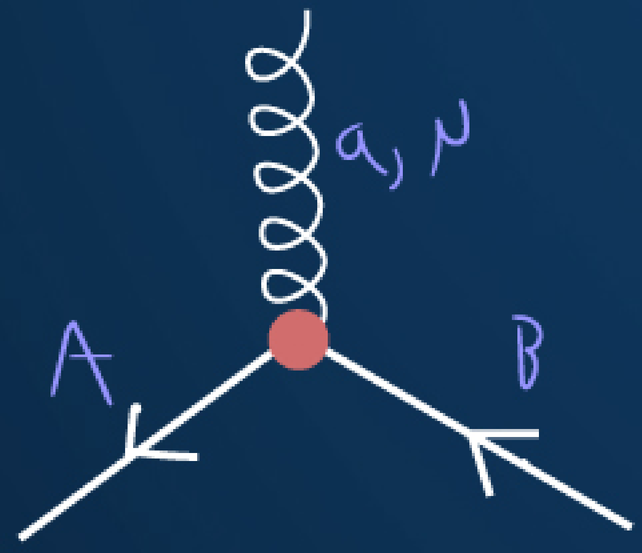




QUANTUM FIELD THEORY

Quantization of a Non-Abelian Gauge Field

Non-Abelian Theories, Part III



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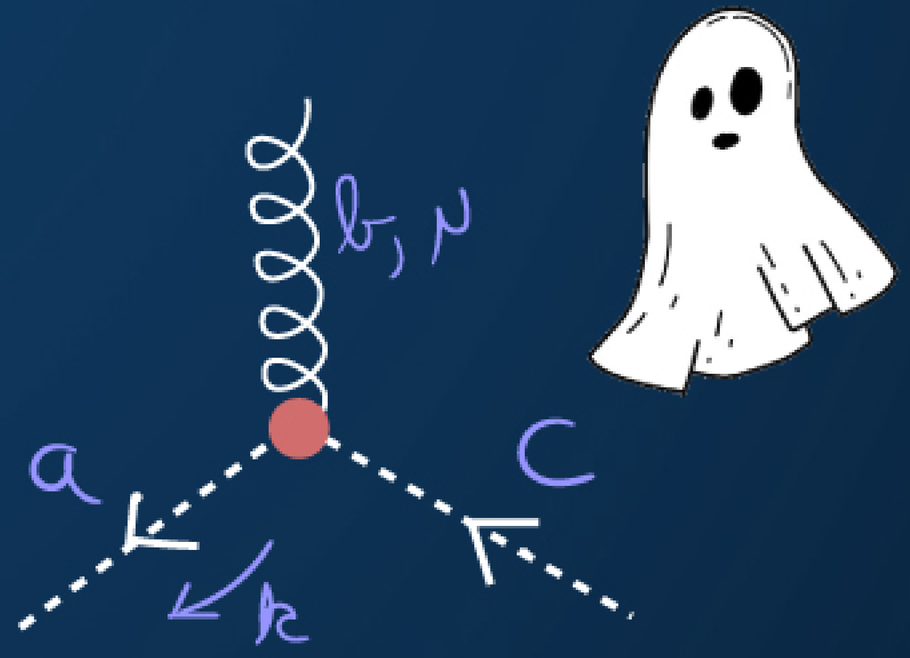

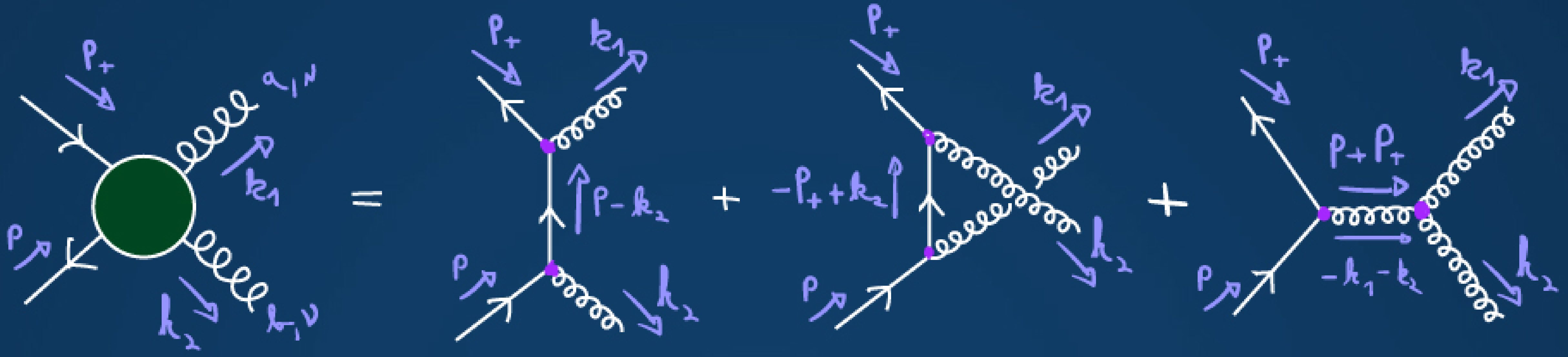

QUANTUM FIELD THEORY

Faddeev-Popov Ghosts and Unitarity

Non-Abelian Theories, Part IV

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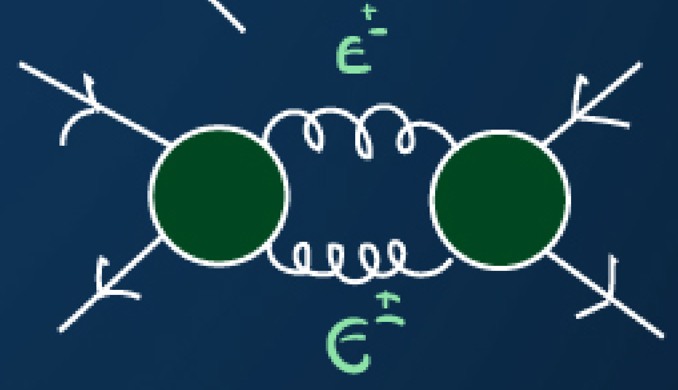

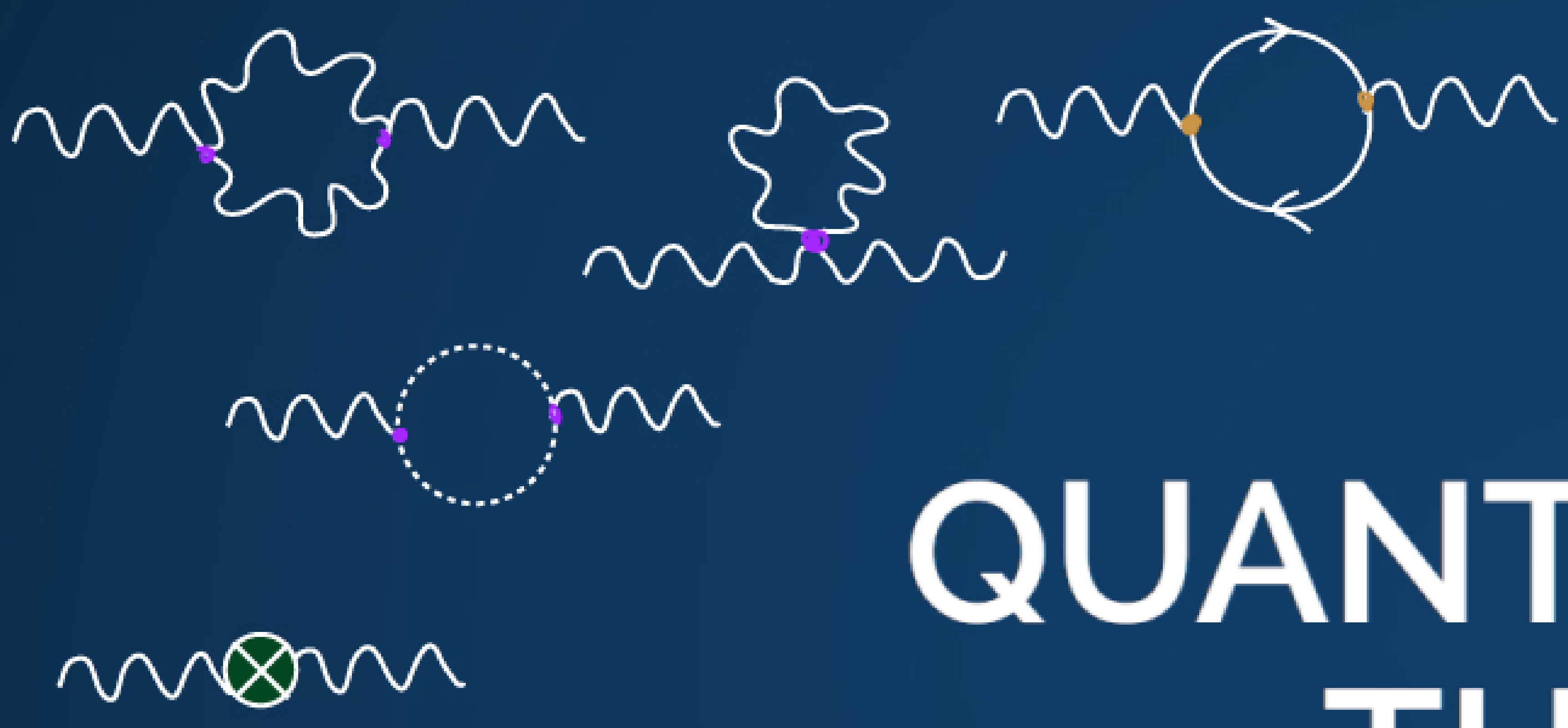

QUANTUM FIELD THEORY

Asymptotic Freedom (in Gauge Theories)

Non-Abelian Theories, Part V

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$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3} C_2(G) - \frac{4}{3} n_f C(r)\right]$$