Heat Kernel Expansion and the Gravitational Effective Field Theory

Lucas de Souza

UFABC, São Paulo

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One-Loop by Heat Kernel Approach

One-loop Effective Action

$$\Gamma^{(1)} = (-)^F \frac{i}{2} \mathrm{Tr}\ln\left[\left(-\Box + m^2 + X\right)_{ij}\right], \qquad \Box = g_{\mu\nu} D^\mu D^\nu$$

$\Downarrow$ Expanding in Heat Kernel Coefficients

$$\Gamma^{(1)} = (-)^F \frac{1}{2} \frac{1}{(4\pi)^{\frac{d}{2}}} \int_{\mathcal{M}} d^d x \sqrt{g} \sum_{r=0}^{\infty} \frac{\Gamma(r - \frac{d}{2})}{m^{2r-d}} \mathrm{tr}\, b_{2r}(x)$$

Heat Kernel Coefficients

- $\Omega_{\mu\nu} = [D_\mu, D_\nu]$ and I the identity matrix for the internal indices.

$$\begin{aligned}
b_0 &= I \\
b_2 &= \frac{1}{6} R I - X \\
b_4 &= \frac{1}{360}\left(12\Box R + 5R^2 - 2R_{\mu\nu}R^{\mu\nu} + 2R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) I \\
&\quad - \frac{1}{6}\Box X - \frac{1}{6} R X + \frac{1}{2} X^2 + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu}
\end{aligned}$$

$$
\begin{aligned}
b_6 = &\frac{1}{360}\Big(8D_\rho\Omega_{\mu\nu}D^\rho\Omega^{\mu\nu} + 2D^\mu\Omega_{\mu\nu}D_\rho\Omega^{\rho\nu} + 12\Omega_{\mu\nu}\Box\Omega^{\mu\nu} - 12\Omega_{\mu\nu}\Omega^{\nu\rho}\Omega_\rho{}^\mu \\
&+ 6R_{\mu\nu\rho\sigma}\Omega^{\mu\nu}\Omega^{\rho\sigma} - 4R_\mu{}^\nu\Omega^{\mu\rho}\Omega_{\nu\rho} + 5R\Omega_{\mu\nu}\Omega^{\mu\nu} \\
&- 6\Box^2 X + 60X\Box X + 30D_\mu X D^\mu X - 60X^3 \\
&- 30X\Omega_{\mu\nu}\Omega^{\mu\nu} - 10R\Box X - 4R_{\mu\nu}D^\nu D^\mu X - 12D_\mu R D^\mu X + 30XXR \\
&- 12X\Box R - 5XR^2 + 2XR_{\mu\nu}R^{\mu\nu} - 2XR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\Big) \\
&+ \frac{1}{7!}\Big(18\Box^2 R + 17D_\mu R D^\mu R - 2D_\rho R_{\mu\nu}D^\rho R^{\mu\nu} - 4D_\rho R_{\mu\nu}D^\mu R^{\rho\nu} \\
&+ 9D_\rho R_{\mu\nu\sigma\lambda}D^\rho R^{\mu\nu\sigma\lambda} + 28R\Box R - 8R_{\mu\nu}\Box R^{\mu\nu} \\
&+ 24R_{\mu\nu}D_\rho D^\nu R^{\mu\rho} + 12R_{\mu\nu\sigma\lambda}\Box R^{\mu\nu\sigma\lambda} + 35/9R^3 \\
&- 14/3RR_{\mu\nu}R^{\mu\nu} + 14/3RR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 208/9R_{\mu\nu}R^{\mu\rho}R^\nu{}_\rho \\
&+ 64/3R_{\mu\nu}R_{\rho\sigma}R^{\mu\rho\nu\sigma} - 16/3R^\mu{}_\nu R_{\mu\rho\sigma\lambda}R^{\nu\rho\sigma\lambda} \\
&+ 44/9R^{\mu\nu}{}_{\alpha\beta}R_{\mu\nu\rho\sigma}R^{\rho\sigma\alpha\beta} + 80/9R_\mu{}^\nu{}_\rho{}^\sigma R^{\mu\alpha\rho\beta}R_{\nu\alpha\sigma\beta}\Big)I
\end{aligned}
$$

One-Loop by Heat Kernel Approach

Charged spin-0 particle

$$\mathcal{L}_0 = -|D_\mu \Phi|^2 - m^2 |\Phi|^2 - \xi |\Phi|^2 R$$

$$D_\mu \Phi = \partial_\mu \Phi + igq A_\mu \Phi$$

$$\Gamma_0^{(1)} = \frac{i}{2} \mathrm{Tr} \log \left[\left(-\Box + m^2 + \xi R \right) \right]$$

$$X = \xi R \,, \quad \Omega_{\mu\nu} = -igq F_{\mu\nu}$$

Charged spin-$\frac{1}{2}$ particle

$$\mathcal{L}_{1/2} = -\frac{1}{2} \bar{\Psi} (\not{D} - m) \Psi \,, \quad \not{D} = \gamma^\mu D_\mu \,,$$

where γ^μ are $n \times n$ Dirac matrices, with $n = 2^{\lfloor d/2 \rfloor}$.

$$\Gamma_{1/2}^{(1)} = -\frac{i}{4} \mathrm{Tr} \log \left[\left(-\Box + m^2 + \frac{1}{4} R + S^{\mu\nu} gq F_{\mu\nu} \right) \right] \,,$$

$$S^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu], \quad X = \frac{1}{4} R + \frac{i}{2} \gamma^\mu \gamma^\nu gq F_{\mu\nu},$$

$$\Omega_{\mu\nu} = -igq F_{\mu\nu} + \frac{1}{4} \gamma^\mu \gamma^\nu R_{\mu\nu\rho\sigma}$$

Effective Field Theories

Scattering amplitudes are obtained from

$$\frac{\delta Z}{\delta J_\ell}\,, \qquad Z[J_\ell] = \int \mathcal{D}\Phi_\ell \mathcal{D}\Phi_h e^{iS[\Phi_\ell,\Phi_h]\ +i\int d^dx \Phi_\ell J_\ell}$$

Integrating out Φ_h fields

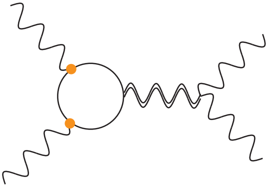
$$Z[J_\ell] = \int \mathcal{D}\Phi_\ell e^{i\Gamma_h[\Phi_\ell]+i\int dx^d \Phi_\ell J_\ell}$$

***Low energy* regime**

$$\Gamma_h[\Phi_\ell] \equiv \int dx^d \sqrt{-g}\,\mathcal{L}_{\text{eff}}[\Phi_\ell]\,, \qquad \mathcal{L}_{\text{eff}}[\Phi_\ell] \sim \sum_{a,b} \frac{\Phi_\ell^a(\partial\Phi_\ell)^{2b}}{m^{a+4b-4}}$$

Einstein-Maxwell Effective Field Theory

- Scattering amplitudes of photons in a gravitational theory



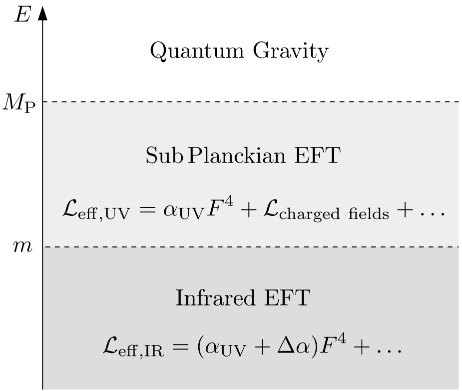
Einstein-Maxwell EFT

$$\begin{aligned}\mathcal{L}_{\text{EM}} = \mathcal{L}_{\text{kin}} & +\alpha_1 (F^{\mu\nu}F_{\mu\nu})^2 + \alpha_2 F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} \\ & +\alpha_3 \hat{R}^2 + \alpha_4 \hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + \alpha_5 \hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma} \\ & +\alpha_6 \hat{R}F^{\mu\nu}F_{\mu\nu} + \alpha_7 \hat{R}^{\nu}{}_{\mu}F^{\mu\rho}F_{\rho\nu} + \alpha_8 \hat{R}^{\rho\sigma}{}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma} \\ & +O(\hat{R}^3, \hat{R}^2F^2, \hat{R}F^4, F^6, \dots)\end{aligned}$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\hat{R}, \qquad \hat{R}_{\mu\nu\rho\sigma} \equiv M^{d-2}R_{\mu\nu\rho\sigma}$$

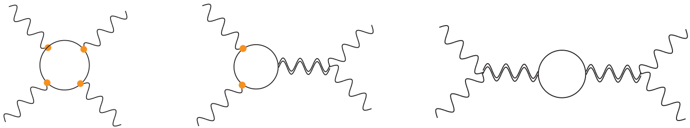
Einstein-Maxwell Effective Field Theory

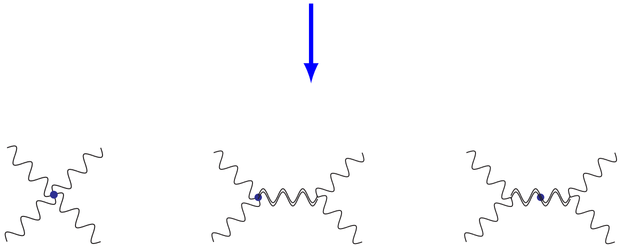
- The various scales and EFTs are summarized as follows:



Einstein-Maxwell Effective Field Theory

- In terms of scattering amplitudes:





- Reducing operator basis

Equations of Motion

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{M^{d-2}} T_{\mu\nu}\,, \qquad D^{\mu} F_{\mu\nu} = 0$$

Gauss Bonnet Combination

$$(R_{\mu\nu\rho\sigma})^2 - 4(R_{\mu\nu})^2 + R^2 = 0 + O(h^3)\,, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Downarrow$$

Reduced Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{EM,red}} = \;& \mathcal{L}_{\mathrm{kin}} + O(\hat{R}^3, \hat{R}^2 F^2, \hat{R} F^4, F^6) \\ & + \hat{\alpha}_1 (F^{\mu\nu} F_{\mu\nu})^2 + \hat{\alpha}_2 F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} F_{\sigma\mu} + \gamma \hat{C}^{\rho\sigma}{}_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} \end{aligned}$$

Einstein-Maxwell Effective Field Theory

Complete Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm EM} = \mathcal{L}_{\rm kin} &+\alpha_1 (F^{\mu\nu}F_{\mu\nu})^2 + \alpha_2 F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} \\
&+\alpha_3 \hat{R}^2 + \alpha_4 \hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + \alpha_5 \hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma} \\
&+\alpha_6 \hat{R}F^{\mu\nu}F_{\mu\nu} + \alpha_7 \hat{R}^{\nu}{}_{\mu}F^{\mu\rho}F_{\rho\nu} + \alpha_8 \hat{R}^{\rho\sigma}{}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma} + \cdots
\end{aligned}
$$

Reduced Coefficients

$$
\begin{aligned}
\hat{\alpha}_1 =\ & \alpha_1 + \frac{(d-4)^2}{4(d-2)^2}\alpha_3 + \frac{8-3d}{4(d-2)^2}\alpha_4 - \frac{d^2+4d-16}{4(d-2)^2}\alpha_5 \\
& + \frac{4-d}{4-2d}\alpha_6 + \frac{1}{2d-4}\alpha_7 - \frac{3}{(d-1)(d-2)}\alpha_8 \\
\hat{\alpha}_2 =\ & \alpha_2 + \alpha_4 + 4\alpha_5 - \alpha_7 + \frac{4}{d-2}\alpha_8\,, \qquad \gamma = \alpha_8\,.
\end{aligned}
$$

$$\Downarrow$$

Reduced Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm EM,red} =\ & \mathcal{L}_{\rm kin} + O(\hat{R}^3, \hat{R}^2F^2, \hat{R}F^4, F^6) \\
& +\hat{\alpha}_1 (F^{\mu\nu}F_{\mu\nu})^2 + \hat{\alpha}_2 F^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} + \gamma \hat{C}^{\rho\sigma}{}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}
\end{aligned}
$$

Einstein-Maxwell Effective Field Theory

A convenient change of basis

$$\mathcal{L}_{\mathrm{EM,red}} = \mathcal{L}_{\mathrm{kin}} + O(\hat{R}^3, \hat{R}^2 F^2, \hat{R} F^4, F^6) + \alpha \mathcal{O} + \beta \tilde{\mathcal{O}} + \gamma \hat{C}^{\rho\sigma}{}_{\mu\nu} F^{\mu\nu} F_{\rho\sigma}$$

$$\mathcal{O} = (F_{\mu\nu} F^{\mu\nu})^2 \,, \quad \tilde{\mathcal{O}} = 4 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} - 2 (F_{\mu\nu} F^{\mu\nu})^2$$

$$\hat{\alpha}_1 = \alpha - 2\beta \,, \quad \hat{\alpha}_2 = 4\beta$$

Positivity Conditions

- Positivity bounds derived using unitarity of forward amplitudes or causality

Infrared Consistency Conditions

$$\alpha_{\rm IR} \geq 0\,, \quad \beta_{\rm IR} \geq 0$$

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, Causality, analyticity and an IR obstruction to UV completion, JHEP 10 (2006) 014, [hep-th/0602178]

- Positivity bound based on the deviation of Extremality Bound $\frac{d-3}{d-2}\frac{M_\circ^2}{Q_\circ^2} \geq 1$

Extremal BH Decay condition — (BH) WGC

$$C_{\rm IR} = \alpha_1 + \frac{\alpha_2}{2} + \frac{(d-4)^2\alpha_3 + \left(2d^2 - 11d + 16\right)\alpha_4}{2(d-2)^2} + \frac{\left(2d^3 - 16d^2 + 45d - 44\right)\alpha_5}{(d-3)(d-2)^2} + \frac{(d-4)\alpha_6 + (d-3)(\alpha_7 + \alpha_8)}{2(d-2)} \geq 0$$

One-Loop by Heat Kernel Approach

Tree level and One-Loop EM Action

$$\Gamma^{(0)} = \int d^d x \left(\mathcal{L}_{\text{kin}} + \mathcal{L}_{F4,\text{UV}}\right)$$

$$\Gamma_s^{(1)} = \int d^d x \left(\Delta\hat{\alpha}_1^{(s)} (F_{\mu\nu}F^{\mu\nu})^2 + \Delta\hat{\alpha}_2^{(s)} F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} + \Delta\gamma^{(s)} \hat{C}^{\rho\sigma}{}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}\right)$$

Coefficients

$$\Delta\hat{\alpha}_{1,2}^{(s)} = \frac{1}{(4\pi)^{d/2}}\left[\frac{g^4q^4}{m^{8-d}}\Gamma\left(4-\frac{d}{2}\right)a_{1,2}^{(s)} + \frac{g^2q^2}{m^{6-d}M^{d-2}}\Gamma\left(3-\frac{d}{2}\right)b_{1,2}^{(s)} + \frac{1}{m^{4-d}M^{2d-4}}\Gamma\left(2-\frac{d}{2}\right)c_{1,2}^{(s)}\right]$$

$$\Delta\gamma^{(s)} = \frac{1}{(4\pi)^{d/2}}\frac{g^2q^2}{m^{6-d}M^{d-2}}\Gamma\left(3-\frac{d}{2}\right)d^{(s)}$$

- $a, b, c, d_i^{(s)}$ for each spin are obtained from the heat kernel coefficients

$$
\begin{aligned}
a_1^{(0)} &= \frac{1}{288}\,, & a_2^{(0)} &= \frac{1}{360}\,, \\
a_1^{(1/2)} &= -\frac{n}{144}\,, & a_2^{(1/2)} &= \frac{7n}{360}\,, \\
a_1^{(1)} &= \frac{d-49}{288}\,, & a_2^{(1)} &= \frac{d+239}{360}\,.
\end{aligned}
$$

$$
\begin{aligned}
b_1^{(0)} &= \frac{1}{720}\left[\left(30\xi - 5 + \frac{4}{(d-1)(d-2)}\right)(d-4) + \left(4 + \frac{8}{d-2}\right)\right]\frac{1}{(d-2)}\,, \\
b_1^{(1/2)} &= -\frac{n}{720}\left[\left(-5 + \frac{4}{(d-1)(d-2)}\right)\frac{d-4}{2(d-2)} - \frac{13(d-2)-4}{(d-2)^2}\right]\,, \\
b_1^{(1)} &= \frac{1}{720}\left[\left(\frac{4(d+59)}{(d-1)(d-2)} - 5\,(d-31)\right)\frac{d-4}{d-2} + \frac{4(d-1)(d+120)}{(d-2)^2}\right]\,, \\
b_2^{(0)} &= -\frac{1}{360}\left(4 + \frac{8}{d-2}\right)\,, \\
b_2^{(1/2)} &= -\frac{n}{360}\left(-\frac{4}{d-2} + 13\right)\,, \\
b_2^{(1)} &= -\frac{1}{360}\left(4(d+119) + \frac{8(d+59)}{d-2}\right)\,.
\end{aligned}
$$

- $a, b, c, d_i^{(s)}$ for each spin are obtained from the heat kernel coefficients

$$
\begin{aligned}
c_1^{(0)} &= \frac{1}{720}\left[\frac{6+\xi\left(\xi-\frac{1}{3}\right)}{4}\frac{(d-4)^2}{(d-2)^2}-\frac{3(3d-8)}{(d-2)^2}\right], & c_2^{(0)} &= \frac{1}{60}, \\
c_1^{(1/2)} &= -\frac{n}{960}\left(\frac{3(3d-8)}{(d-2)^2}+\frac{(d-4)^2}{(d-2)^2}\right), & c_2^{(1/2)} &= \frac{n}{80}, \\
c_1^{(1)} &= \frac{1}{240}\left[\frac{(d-11)(d-4)^2}{2(d-2)^2}-\frac{(d+9)(3d-8)}{(d-2)^2}\right], & c_2^{(1)} &= \frac{d+9}{60}.
\end{aligned}
$$

$$
\begin{aligned}
d^{(0)} &= -\frac{1}{180}, \\
d^{(1/2)} &= \frac{n}{360}, \\
d^{(1)} &= -\frac{d+59}{180}.
\end{aligned}
$$

One-Loop by Heat Kernel Approach

- We introduce the charge-to-mass ratio $z = \frac{g|q|}{m} M^{\frac{d-2}{2}}$ such that

$$\Delta\alpha = \frac{1}{m^{4-d} M^{2d-4}} \left(a z^4 + b z^2 + c \right)$$

and we work with the dimensionless z-polynomial

$$\Delta\bar{\alpha} = a z^4 + b z^2 + c \equiv m^{4-d} M^{2d-4} \Delta\alpha$$

Infrared Consistency Conditions

$$\bar{\alpha}_{\rm IR} = \bar{\alpha}_{\rm UV} + \Delta\bar{\alpha} \geq 0 \, , \quad \bar{\beta}_{\rm IR} = \bar{\beta}_{\rm UV} + \Delta\bar{\beta} \geq 0$$

Extremal BH Decay condition — (BH) WGC

$$\bar{C}_{\rm IR} = \bar{C}_{\rm UV} + \Delta\bar{C} \geq 0$$

Results

Let's look at the results of imposing positivity conditions in some dimensions

- The simplest case:

Spin	$\Delta\bar{\alpha}$
0	$\frac{7z^4}{1920} + \frac{(1-10\xi)z^2}{480} + \frac{60\xi^2-20\xi+3}{480}$
$\frac{1}{2}$	$\frac{z^4}{240} - \frac{z^2}{480} + \frac{1}{240}$
1	$\frac{127z^4}{960} - \frac{11z^2}{60} + \frac{1}{30}$

Table: Reduced coefficients $\Delta\bar{\alpha}$ in $d = 3$.

Results

Spin	Condition for z bounded	Bound if $\bar{\alpha}_{\rm UV} = 0$
0	$\bar{\alpha}_{\rm UV} < \begin{cases} -\frac{60\xi^2-20\xi+3}{480} & \text{if } \xi \leq \frac{1}{10} \\ -\frac{16\xi^2-6\xi+1}{168} & \text{if } \xi > \frac{1}{10} \end{cases}$	Unbounded
$\frac{1}{2}$	$\bar{\alpha}_{\rm UV} < -\frac{1}{256}$	Unbounded
1	$\bar{\alpha}_{\rm UV} < \frac{23}{762}$	$z \leq 0.464$ or $z \geq 1.08$

Table: Condition for the existence of IR consistency bounds on the charge-to-mass ratio z and IR consistency bounds on z if $\bar{\alpha}_{\rm UV} = 0$ in $d = 3$.

- C. Cheung and G. N. Remmen, Infrared Consistency and the Weak Gravity Conjecture, JHEP 12 (2014) 087, [arXiv:1407.7865].

Results

Spin	Bound if $\bar{\alpha}_{\mathrm{UV}} = 0$	Bound if $\bar{\beta}_{\mathrm{UV}} = 0$	Bound if $\bar{C}_{\mathrm{UV}} = 0$
0	ξ-dependent	$z \geq 3.87$	ξ-dependent
$\frac{1}{2}$	$z \geq 2.77$	$z \geq 1.99$	$z \geq 2.78$
1	$z \geq 2.48$	$z \geq 2.39$	$z \geq 2.43$

Table: Positivity Consistency Bounds on z if UV coefficients are negligible in $d = 5$.

(QFT) WGC $\longleftarrow$ **IR Consistency** $\overset{\sim}{\longleftrightarrow}$ (BH) WGC

Results

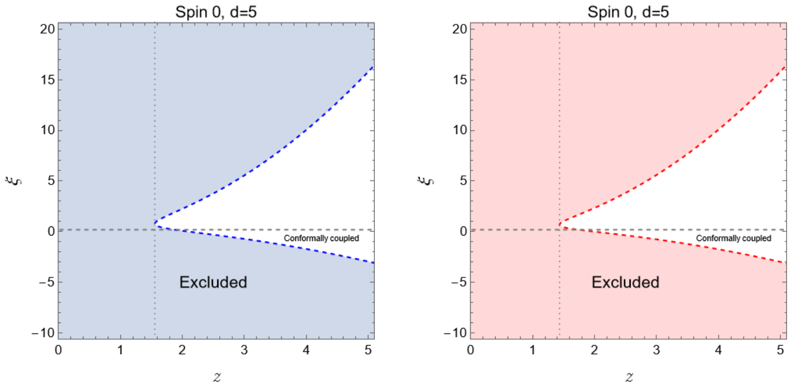


Figure: Infrared Consistency (left) and Extremal BH Decay (right) Bounds on the charged spin 0 particle in $d = 5$.

Results

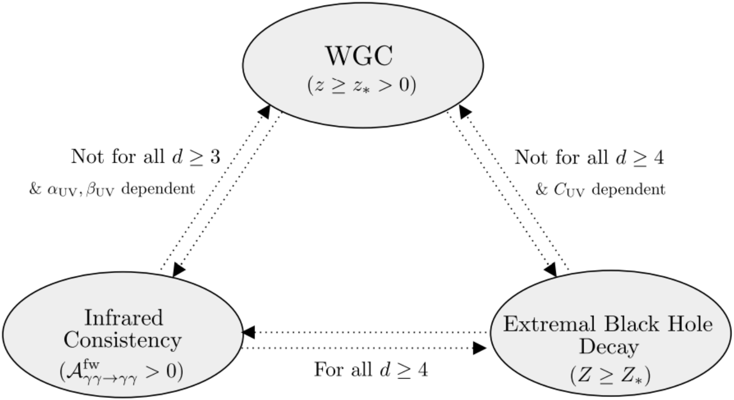
Spin	$\Delta\bar{\alpha}$	$\Delta\bar{\beta}$
0	$-\frac{7z^4}{720}-\frac{(37-100\xi)z^2}{2520}-\frac{1500\xi^2-500\xi+183}{44100}$	$-\frac{z^4}{720}-\frac{z^2}{210}-\frac{1}{450}$
$\frac{1}{2}$	$-\frac{4z^4}{45}-\frac{83z^2}{315}-\frac{424}{11025}$	$-\frac{7z^4}{45}-\frac{58z^2}{315}-\frac{2}{75}$
1	$-\frac{37z^4}{90}-\frac{247z^2}{315}-\frac{1397}{22050}$	$-\frac{31z^4}{90}-\frac{172z^2}{315}-\frac{1}{25}$

Spin	$\Delta\bar{C}$
0	$-\frac{7z^4}{720}-\frac{(61-150\xi)z^2}{3780}-\frac{13500\xi^2-4500\xi+947}{396900}$
$\frac{1}{2}$	$-\frac{4z^4}{45}-\frac{34z^2}{135}-\frac{2591}{99225}$
1	$-\frac{37z^4}{90}-\frac{1669z^2}{1890}-\frac{15023}{198450}$

Table: Reduced coefficients $\Delta\bar{\alpha}$, $\Delta\bar{\beta}$ and $\Delta\bar{C}$ in $d=9$.

(QFT) WGC $\not\longleftrightarrow$ **IR Consistency** $\overset{\sim}{\longleftrightarrow}$ (BH) WGC

Results


WGC
$(z \geq z_* > 0)$
Not for all $d \geq 3$
& $\alpha_{\mathrm{UV}}, \beta_{\mathrm{UV}}$ dependent
Not for all $d \geq 4$
& C_{UV} dependent
Infrared Consistency
$(\mathcal{A}^{\mathrm{fw}}_{\gamma\gamma\to\gamma\gamma} > 0)$
Extremal Black Hole Decay
$(Z \geq Z_*)$
For all $d \geq 4$

References

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