THE NAMBU-JONA-LASINIO MODEL

AN EFFECTIVE THEORY OF QCD

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OUTLINE

- INTRODUCTION
- HISTORICAL OVERVIEW
- THE NAMBU-JONA-LASINIO MODEL
- NUMERICAL RESULTS
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INTRODUCTION

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$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\,(i\boldsymbol{\gamma}\cdot\boldsymbol{D} - m)\psi - \frac{1}{4}F^{a\ \mu\nu}F^{a}_{\mu\nu}$$

$$D_\mu = \partial_\mu - ig\tau^a\mathcal{A}^a_\mu,\ F^a_{\mu\nu} = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu + gf^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu.$$

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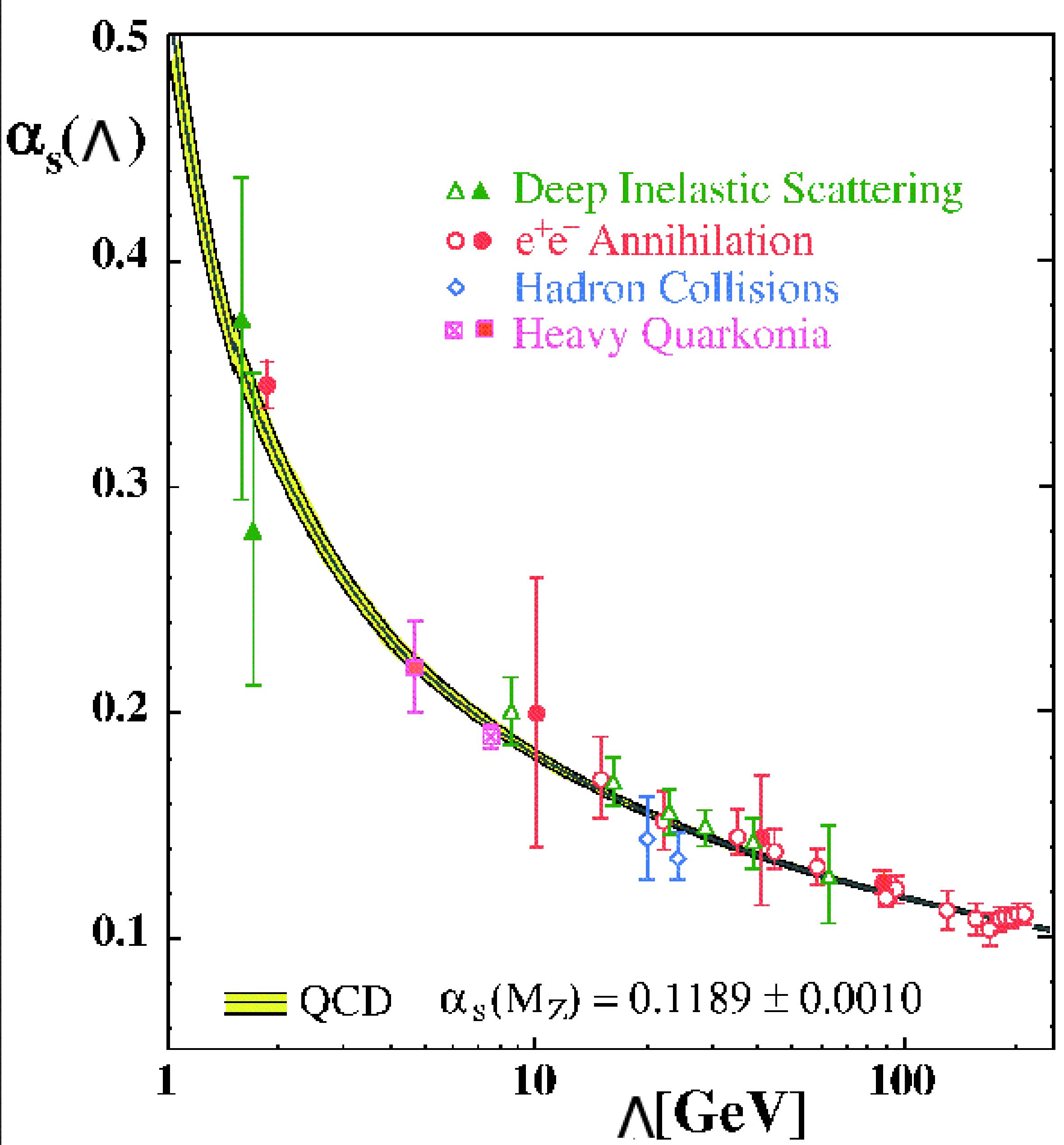

Figure 1. Avalible from: https://www.researchgate.net/figure/QCD-running-coupling-as-a-function-of-energy-scale-This-figure-is-adapted-from-Ref_fig5_263811655

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One way of circumventing this problem is Lattice QCD. However, even lattice methods have difficulties with some aspects of QCD, such as the sign problem when one tries to compute observables at finite real chemical potential.

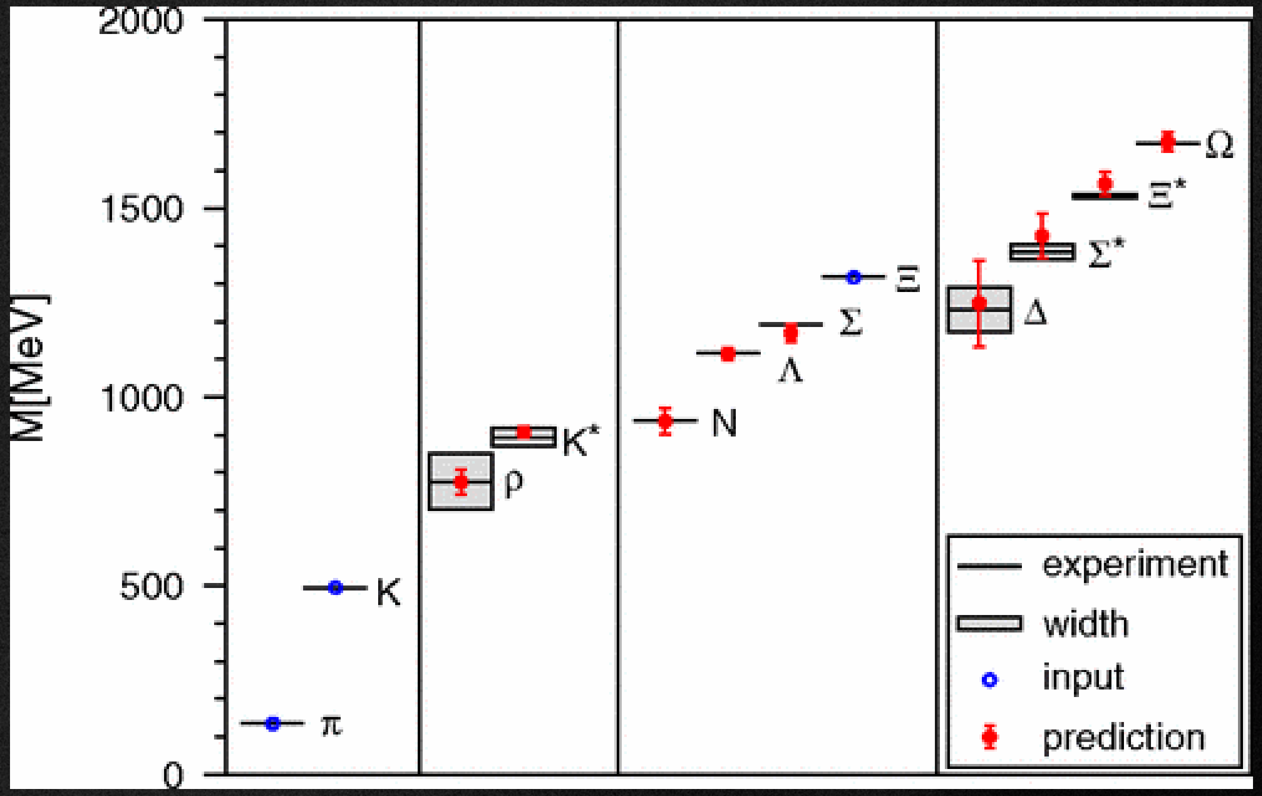

Figure 2. Avalible from: https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.84.449 [5].

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This can be done by starting from the full theory and deriving an effective theory in some limit, or by building models that incorporate these features. This is where the NJL model enters. Originally built to explain why nucleons are much more massive than pions, it is nowadays reinterpreted as an effective theory of QCD.

HISTORICAL OVERVIEW

- 1935 - Hideki Yukawa predicted the existence of a meson as the force carrier of the strong nuclear force.

Figure 2. Avalible from: https://pt.wikipedia.org/wiki/Ficheiro:Yukawa.jpg

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- 1950 to 1960 - A bunch of new hadrons were exprementaly found [3,4] .

HISTORICAL OVERVIEW

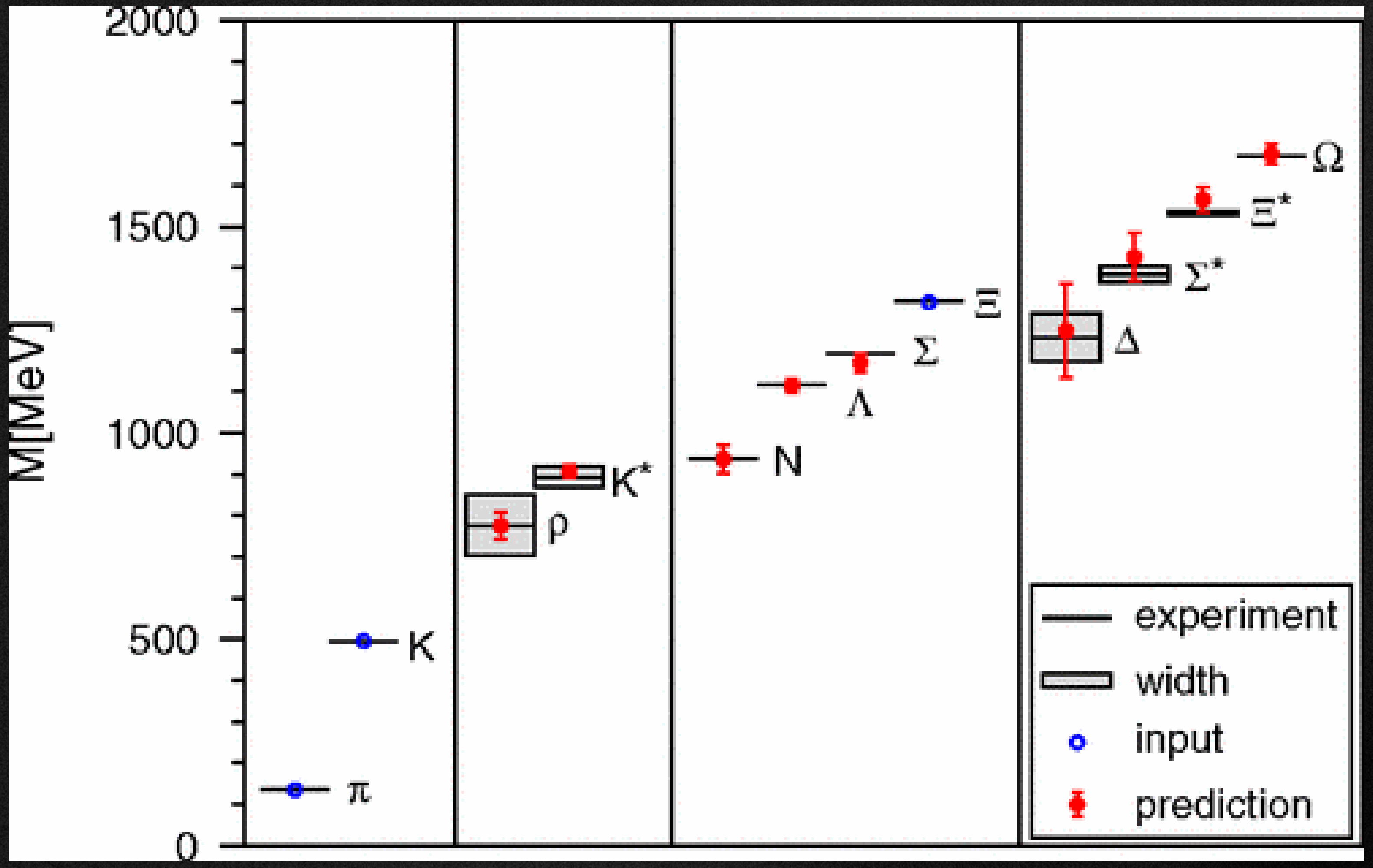
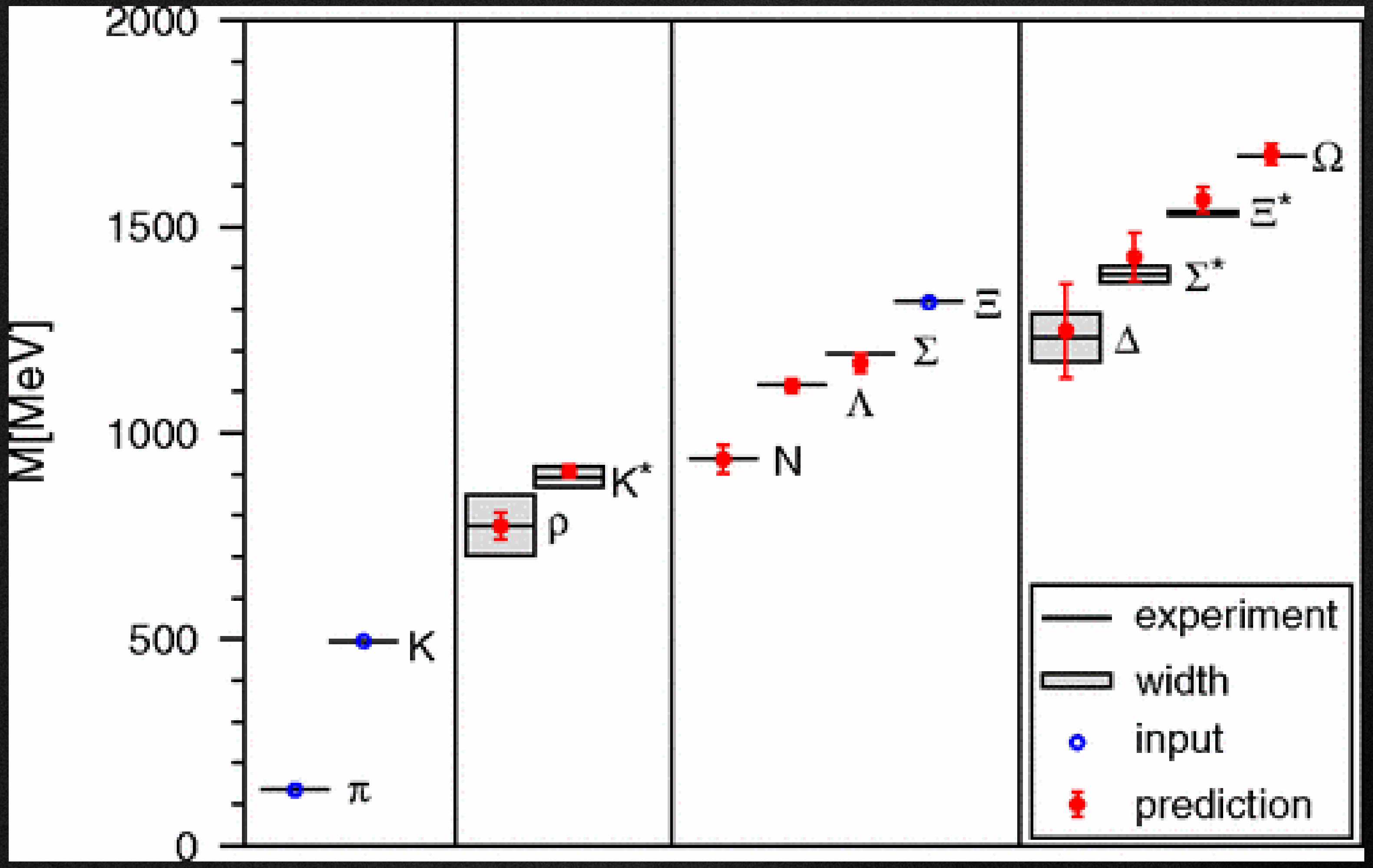


Figure 3. Avalible from: https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.84.449 [5].

HISTORICAL OVERVIEW

We have a problem!!!



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HISTORICAL OVERVIEW

- 1961 – Yoichiro Nambu and Giovanni Jona-Lasinio proposed a model to address this problem [6,7], marking the birth of the Nambu–Jona-Lasinio (NJL) model.

Figure 4. Avalible from: https://pt.wikipedia.org/wiki/Ficheiro:YoichiroNambu.jpg

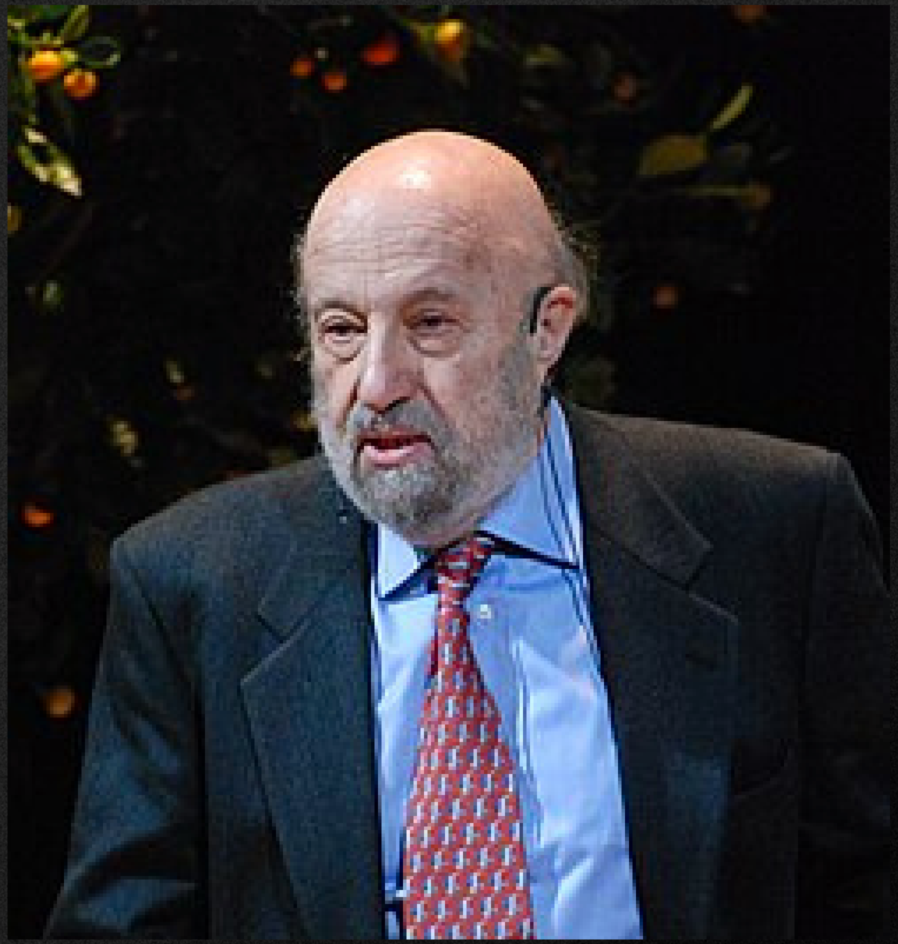

Figure 5. Avalible from: https://pt.wikipedia.org/wiki/Ficheiro:Giovanni_Jona-Lasinio-Nobel_Lecture-1.jpg

THE NJL MODEL

$$\mathcal{L}_{NJL} = \bar{\psi}(i\boldsymbol{\gamma} \cdot \boldsymbol{\partial} - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\boldsymbol{\tau}^a\psi)^2\right]$$

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The model generates nucleon masses through spontaneous chiral symmetry breaking.

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One can define projection operators that separate the left- and right-handed components of a spinor:

$$P_L = \frac{1}{2}(1 - \gamma^5),$$
$$P_R = \frac{1}{2}(1 + \gamma^5).$$

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These operators act as follows

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$$\mathcal{L} = \sum_{j=L,R} \bar{\psi}_j i\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\psi_j - \bar{\psi}_L m\psi_R - \bar{\psi}_R m\psi_L.$$

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Define the field transformation:

$$\psi_L \longrightarrow \psi'_L = L\psi_L = e^{-i\alpha_L^a \frac{\tau^a}{2}}\psi_L,$$
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$$\mathcal{L} = \sum_{j=L,R} \bar{\psi}_j i\boldsymbol{\gamma} \cdot \boldsymbol{\partial}\psi_j - \bar{\psi}_L m \psi_R - \bar{\psi}_R m \psi_L.$$

$SU_L(2) \times SU_R(2)$ chiral symmetry.

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$$\psi' = \left(1 - i\alpha_L^a \frac{\tau^a}{2} + \mathcal{O}(\alpha_L^2)\right)\psi_L + \left(1 - i\alpha_R^a \frac{\tau^a}{2} + \mathcal{O}(\alpha_R^2)\right)\psi_R$$

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$$\psi' = \left(1 - i\left(\frac{\alpha_R^a + \alpha_L^a}{2}\right)\frac{\tau^a}{2} - i\left(\frac{\alpha_R^a - \alpha_L^a}{2}\right)\gamma^5\frac{\tau^a}{2}\right)\psi$$

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$$\psi' = \left(1 - i\alpha_V^a \frac{\tau^a}{2} - i\alpha_A^a \gamma^5 \frac{\tau^a}{2}\right)\psi$$

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Thus, chiral symmetry can be expressed through the following transformations in the Dirac spinors

$$\psi \longrightarrow \psi' = V\psi = e^{-i\alpha_V^a \frac{\tau^a}{2}}\psi = \left[\mathbf{1}\cos\left(\frac{\alpha_V}{2}\right) - i\frac{\alpha_V^a \tau^a}{\alpha_V}\sin\left(\frac{\alpha_V}{2}\right)\right]\psi$$

$$\psi \longrightarrow \psi' = A\psi = e^{-i\gamma^5\alpha_A^a \frac{\tau^a}{2}}\psi = \left[\mathbf{1}\cos\left(\frac{\alpha_A}{2}\right) - i\gamma^5\frac{\alpha_A^a \tau^a}{\alpha_A}\sin\left(\frac{\alpha_A}{2}\right)\right]\psi$$

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where $\alpha_i = \sqrt{\alpha_i^a \alpha_i^a}$.

In this language chiral symmetry is broken if the Lagrangian is not invariant under the axial transformation.

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Now lets go back to the NJL Lagrangian

$$\mathcal{L}_{NJL} = \bar{\psi}(i\boldsymbol{\gamma} \cdot \boldsymbol{\partial} - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right].$$

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The two terms of the interaction part transform as

$$(\bar{\psi}\psi) \longrightarrow (\bar{\psi}\psi)\cos\alpha - (\bar{\psi}i\gamma^5\hat{\alpha}^a\tau^a\psi)\sin\alpha,$$

$$(\bar{\psi}i\gamma^5\tau^a\psi) \longrightarrow (\bar{\psi}i\gamma^5\tau^a\psi) + \hat{\alpha}^a(\bar{\psi}\psi)\sin\alpha - \hat{\alpha}^a(\bar{\psi}i\gamma^5\hat{\alpha}^b\tau^b\psi)(1-\cos\alpha),$$

where $\hat{\alpha}^a = \dfrac{\alpha^a}{\alpha}.$

THE NJL MODEL

$$(\bar{\psi}\psi)^2 \longrightarrow (\bar{\psi}\psi)^2 \cos^2\alpha - (\bar{\psi}\psi)(\bar{\psi}i\gamma^5\hat{\alpha}^a\tau^a\psi)\sin 2\alpha + (\bar{\psi}i\gamma^5\hat{\alpha}^a\tau^a\psi)^2\sin^2\alpha$$

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$$(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2 \longrightarrow (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2$$

In the case of massless fermions the NJL Lagrangian has chiral symmetry.

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$$Z = \mathcal{N} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{i\int d^4x \mathcal{L}_{NJL}},$$

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$$N \int \mathcal{D}\phi \exp\left[i \int d^4x(-A\phi^2 \pm B\phi)\right] = \exp\left[i \int d^4x \frac{B^2}{4A}\right]$$

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If one takes

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We get the following generating functional

$$Z = \mathcal{N}\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\varphi\mathcal{D}\Pi^a e^{i\int d^4x\mathcal{L}'_{NJL}},$$

where

$$\mathcal{L}'_{NJL} = \bar{\psi}\left(i\boldsymbol{\gamma}\cdot\boldsymbol{\partial} - m - \varphi - i\gamma^5\Pi^a\tau^a\right)\psi - \frac{\varphi^2}{4G} - \frac{(\Pi^a)^2}{4G}$$

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We made the Lagrangian linear in the fermionic fields, so we can integrate then out

$$Z = \mathcal{N} \int \mathcal{D}\varphi \mathcal{D}\Pi^a \det D^{-1}[\varphi, \Pi^a] e^{i \int d^4x \mathcal{L}'_{NJL}},$$

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with $D^{-1}[\varphi, \Pi^a] = i\boldsymbol{\gamma} \cdot \boldsymbol{\partial} - m - \varphi - i\gamma^5 \Pi^a \tau^a$. Now we exponenciate this determinant to obtain an effective action,

$$S_{\text{eff}} = -i \ln \det D^{-1} - \int d^4x \frac{\varphi^2}{4G} + \frac{(\Pi^a)^2}{4G}$$

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We will assume that the vacuum has the usual symmetries that we like, e.g. Poincaré symmetry, charge conjugation, parity, and time reversal. This imposes constraints on the vacuum expectation values of operators, for example:

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Now we expand the effective action on this classical configuration and discart the quatum fluctuantions. The classical configuration is the one which extremizes the action

$$\frac{\delta S_{eff}}{\delta \varphi(x)}\Big|_{\varphi=\bar{\phi}} = 0, \;\; \frac{\delta S_{eff}}{\delta \Pi^a(x)}\Big|_{\Pi^a=\bar{\pi}^a} = 0.$$

THE NJL MODEL

Using the following matrix identities

$$\ln \det A = \mathrm{Tr} \ln A, \quad \frac{d}{dx} \mathrm{Tr} \ln A = \mathrm{Tr}\left[A^{-1} \frac{dA}{dx}\right].$$

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And the inverse propagator

$$D^{-1}(x,y) = \delta^{(4)}(x-y) \left[i\boldsymbol{\gamma} \cdot \boldsymbol{\partial} - m - \varphi(x) - i\gamma^5 \Pi^a(x) \tau^a \right].$$

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We get

$$\frac{\delta S_{eff}}{\delta\varphi(x)}|_{\varphi=\bar\phi} = -i\mathrm{tr}D(x,x) - \frac{\bar\phi(x)}{2G} \implies \bar\phi(x) = -i2G\mathrm{tr}D(x,x)$$

$$\frac{\delta S_{eff}}{\delta\Pi^a(x)}|_{\Pi^a=\bar\pi^a} = i\mathrm{tr}\left[D(x,x)i\gamma^5\tau^a\right] - \frac{\bar\pi^a(x)}{2G} \implies \bar\pi^a(x) = -i2G\mathrm{tr}\left[D(x,x)i\gamma^5\tau^a\right]$$

THE NJL MODEL

Remembering that

$$D_{\alpha\beta}(x,y) = \langle\Omega|T\{\psi_\alpha(x)\bar{\psi}_\beta(y)\}|\Omega\rangle \implies \mathrm{tr}D(x,x) = -\langle\Omega|T\{\bar{\psi}_\alpha(x)\psi_\alpha(x)\}|\Omega\rangle$$

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$$\bar{\pi}^a = i2G\langle\bar{\psi}i\gamma^5\tau^a\psi\rangle = 0$$

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Which allows us to define constituent masses

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and gives the gap equation

$$M = m + 8iG\nu \int \frac{d^4k}{(2\pi)^4} \frac{M}{k^2 - M^2 + i\epsilon}$$

THE NJL MODEL

In the chiral limit

$$M = 8iG\nu \int \frac{d^4k}{(2\pi)^4} \frac{M}{k^2 - M^2 + i\epsilon},$$

which allows a non-zero solution.

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In the chiral limit

$$M = 8iG\nu \int \frac{d^4k}{(2\pi)^4} \frac{M}{k^2 - M^2 + i\epsilon},$$

which allows a non-zero solution. When this happens we get

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This is a manifestation of spontaneus chiral symmetry breaking.

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This is spontaneus symmetry breaking (SSB), when the Lagrangian has a symmetry but the ground state doesn't.

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To see this lets consider the first non-zero contribution of the pseudoscalar auxiliary field to the action

$$S_{eff}[\varphi, \Pi^a] = S_e ff[\bar{\phi}, \bar{\pi}^a] + \frac{1}{2}\int d^4x\, d^4y\, \pi^a(x) \frac{\delta^2 S_{eff}}{\delta \Pi^a(x) \delta \Pi^b(y)} \pi^b(y)$$

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When we do path integral we identify this as the inverse propagador of the field

$$\Delta^{-1}(x, y) = \frac{1}{2} \frac{\delta^2 S_{eff}}{\delta\Pi^a(x)\delta\Pi^b(y)}$$

THE NJL MODEL

$$\Delta^{ab^{-1}}(x,y) = \frac{1}{2}\left(\delta^{ab}\delta^{(4)}(x-y)\frac{1}{2G} - i\mathrm{tr}\left[D(y,x)i\gamma^5\tau^a D(x,y)i\gamma^5\tau^b\right]\right)$$

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$$\Delta^{-1}(q) = \frac{1-4G\Pi_{\mathrm{PS}}(q^2)}{4G}$$

THE NJL MODEL

$$4G\Delta^{-1}(q) = 1 + 16GN_c i \int \frac{d^4k}{(2\pi)^4} \frac{k \cdot q - k^2 + M^2}{(k^2 - M^2)((k-q)^2 - M^2)} 4G$$

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$$I(q^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - M^2][(k-q)^2 - M^2]}$$

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$$\Delta^{-1}(m_\pi^2) = 0 \implies m_\pi^2 = -\frac{m}{M} \frac{1}{i8GN_c I(m_\pi^2)}$$

NUMERICAL RESULTS

As I said before, the model is nonrenormalizable, but there is no need to worry, especially nowadays, when it is interpreted as an effective theory of QCD at low energies.

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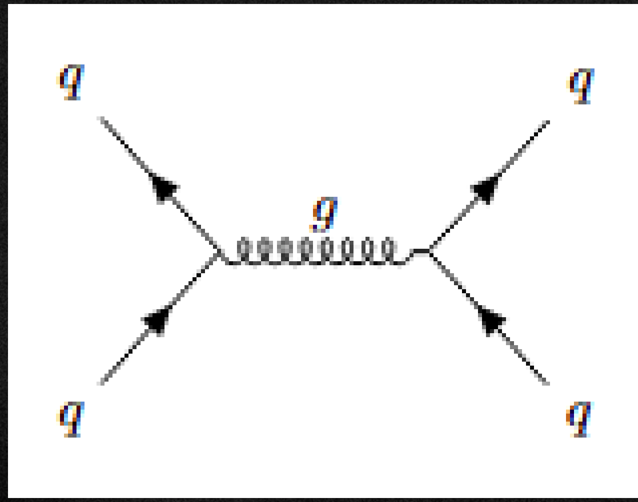

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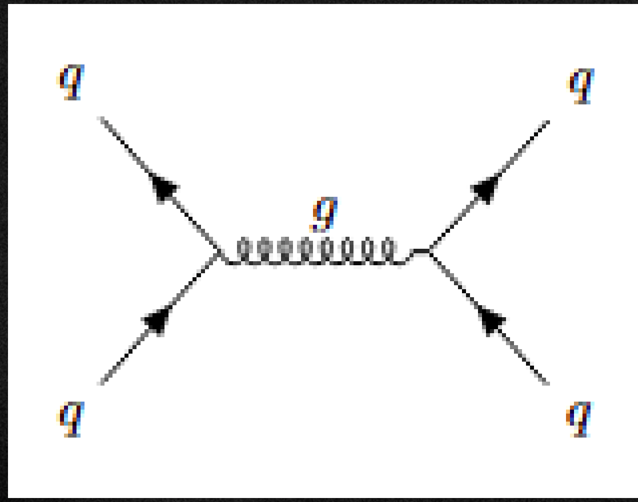

$$\xrightarrow{\text{I.R}} \quad G(q^2) \sim \frac{1}{q^2 - M_g^2}$$

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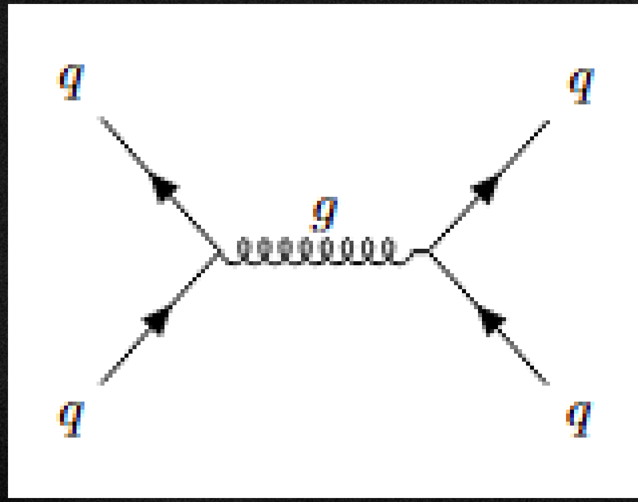
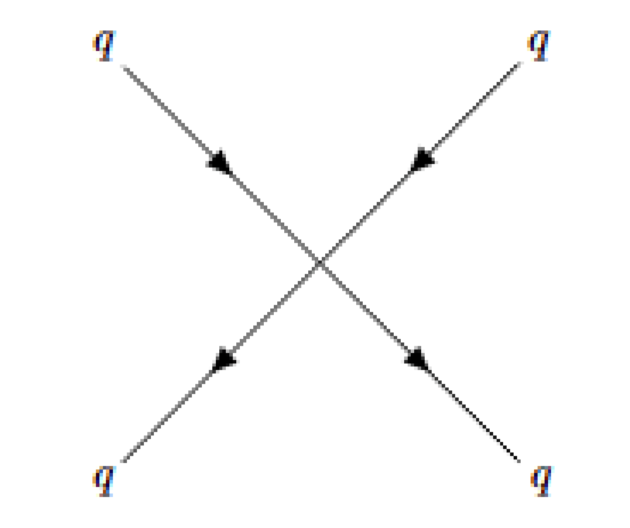


$$\xrightarrow{\text{I.R}} \quad G(q^2) \sim \frac{1}{q^2 - M_g^2} \quad \xrightarrow{q << M_g^2}$$



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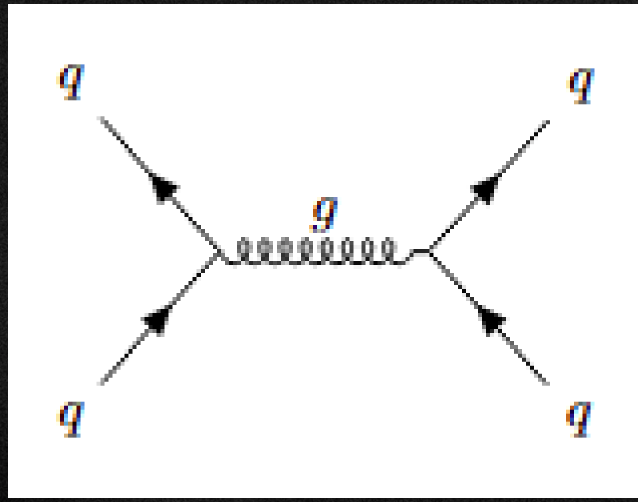
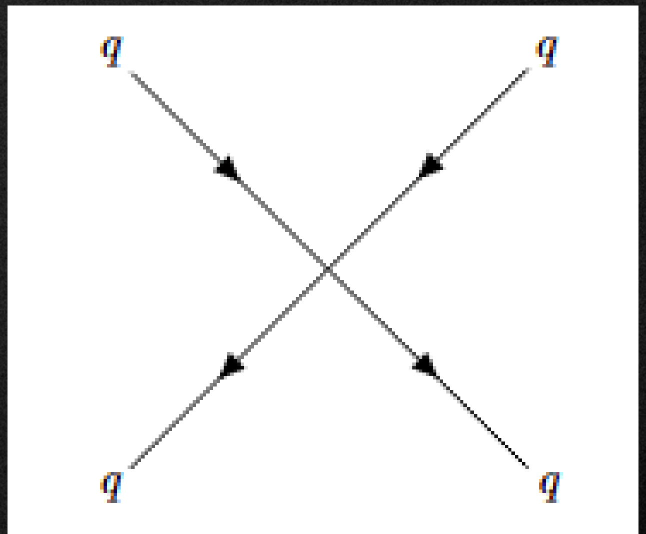


$$\xrightarrow{\text{I.R}} \quad G(q^2) \sim \frac{1}{q^2 - M_g^2} \quad \xrightarrow{q \ll M_g^2}$$



Since, in a nonrenormalizable model, all Green's functions are divergent and an infinite number of counterterms would be required to fix their values, we will instead avoid dealing with them explicitly and treat the regulator as a free parameter that must be fixed experimentally.

NUMERICAL RESULTS

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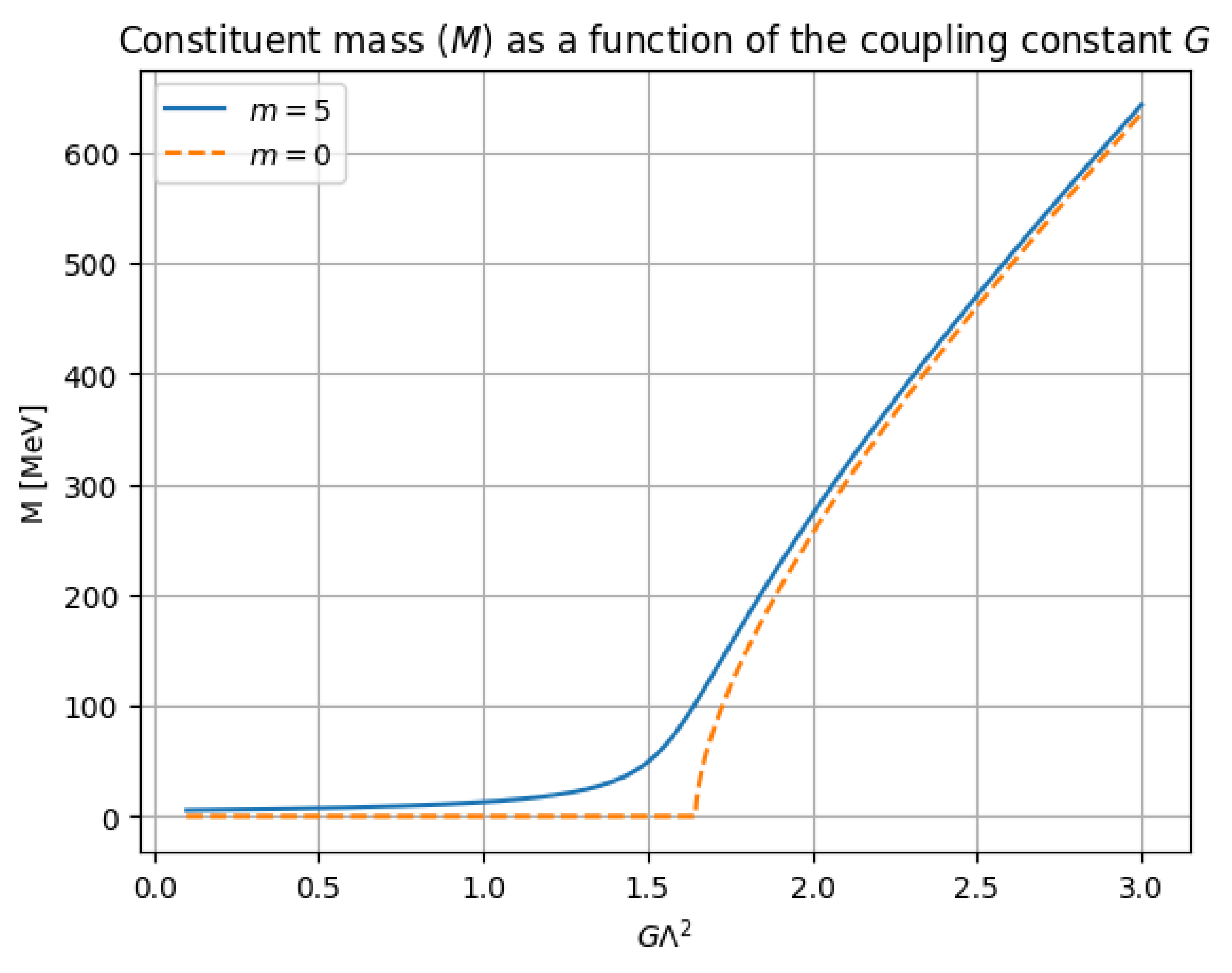
For $\Lambda = 664.3$



NUMERICAL RESULTS

Fitting the free parameters to the pion decay constant and mass we get

Λ [MeV]	$G\Lambda^2$	m [MeV]
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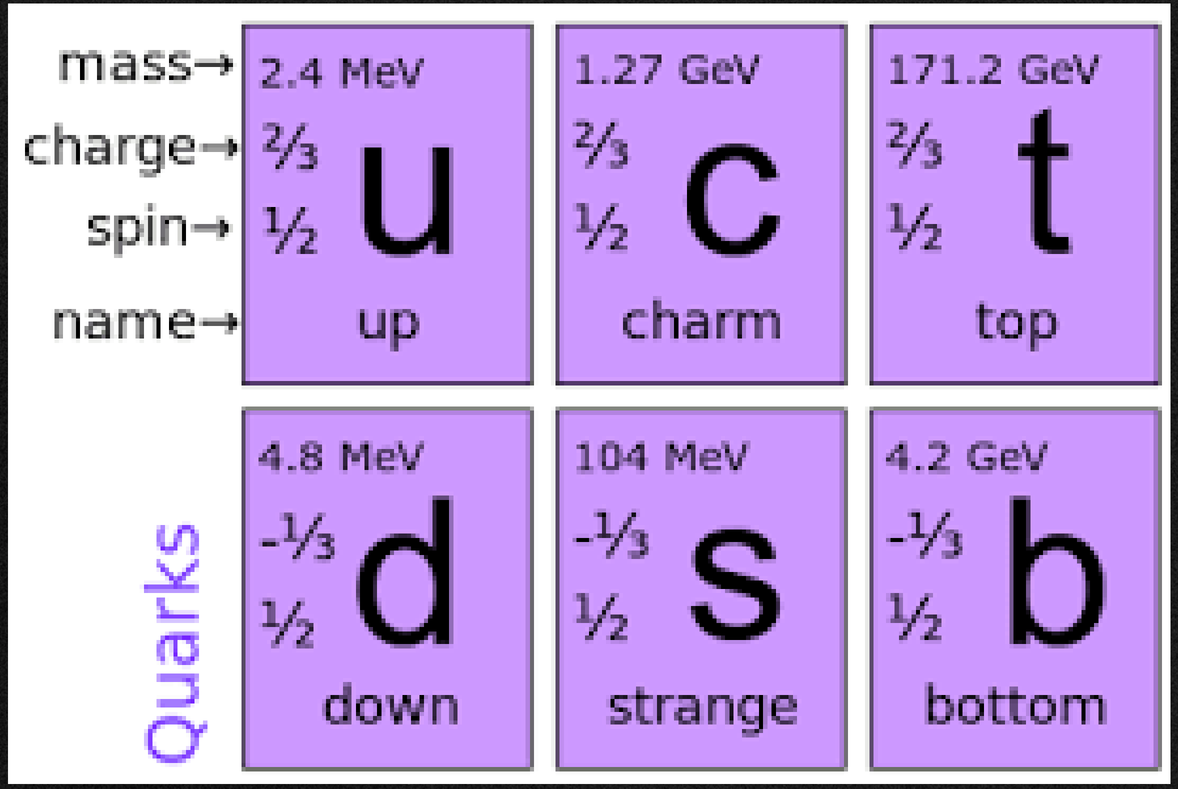

Figure 5. Avalible from: https://dragallur.wordpress.com/2015/05/03/3-particles-quarks/

Object	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Atomic mass unit (u)	1.660540×10^{-27}kg	1.000	931.5
Neutron	1.674929×10^{-27}kg	1.008664	939.57
Proton	1.672623×10^{-27}kg	1.007276	938.28
Electron	9.109390×10^{-31}kg	0.00054858	0.511

Figure 6. Avalible from: https://dragallur.wordpress.com/2015/05/03/3-particles-quarks/

NUMERICAL RESULTS

$$\int \frac{d^4k}{(2\pi)^4} \rightarrow T\sum_n \int \frac{d^3k}{(2\pi)^3},\ \ k_4 \longrightarrow \omega_n = \pi T(2n+1) - i\mu$$

NUMERICAL RESULTS

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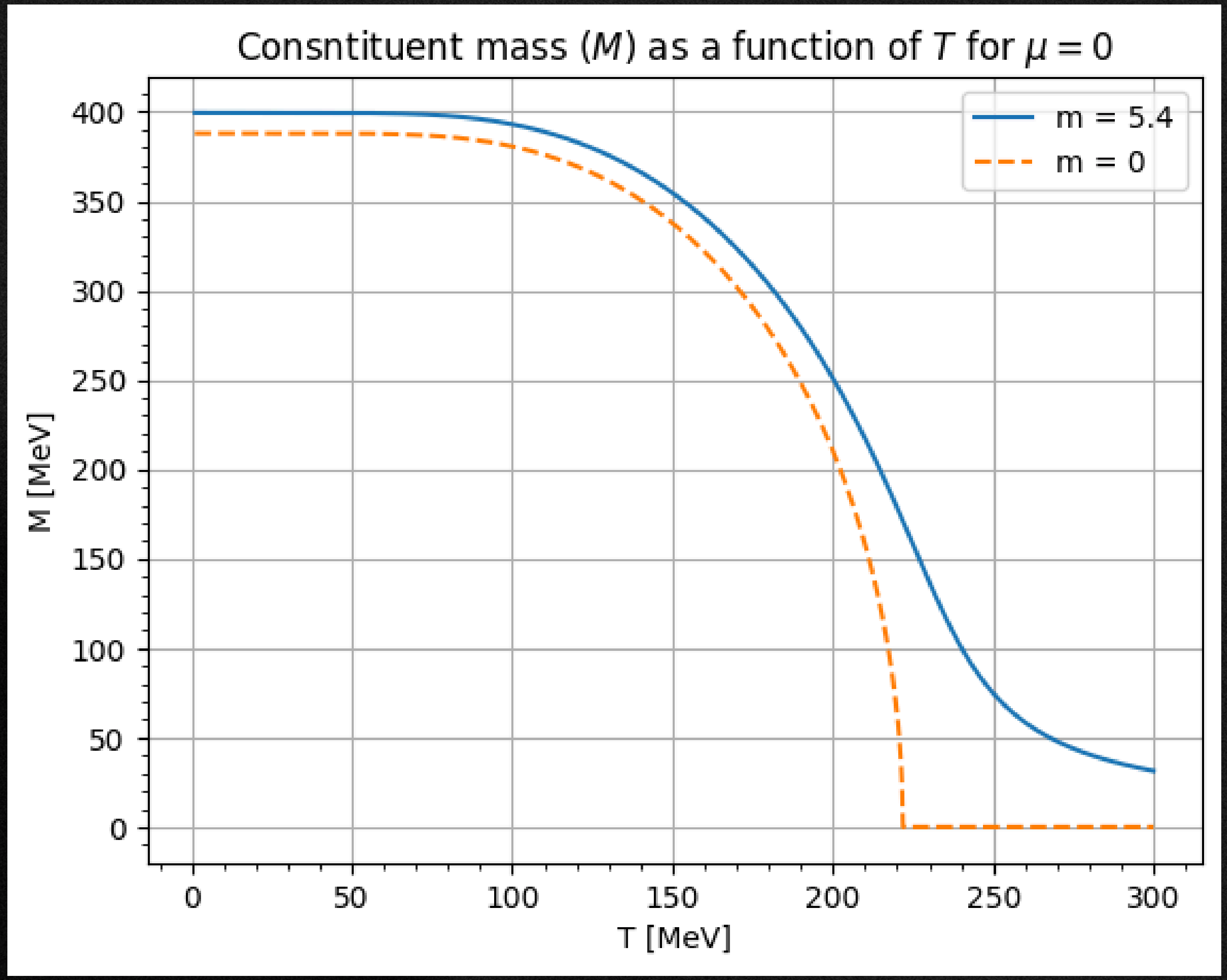
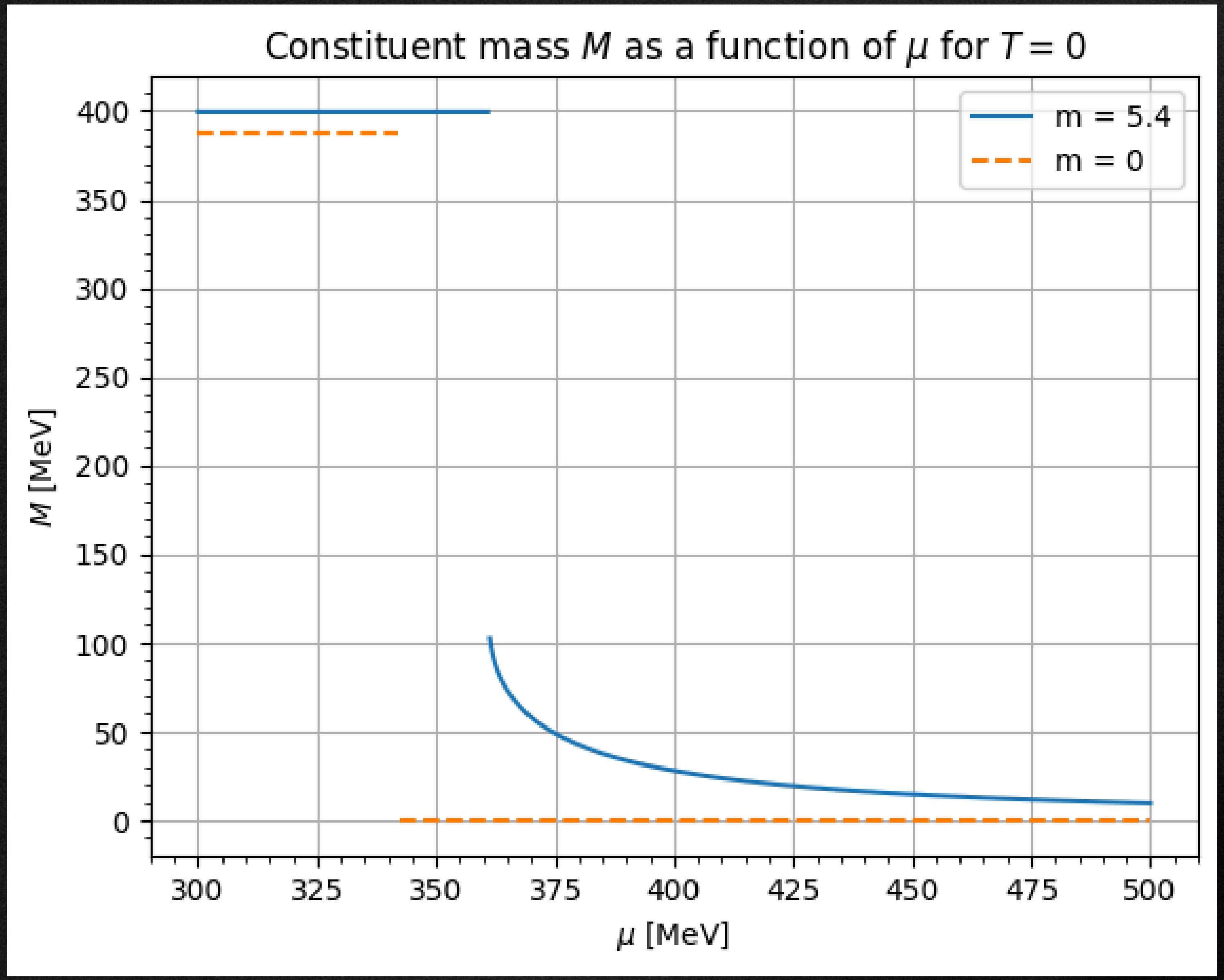
$$M = m + 4G\nu \int \frac{d^3p}{(2\pi)^3} \frac{M}{\sqrt{p^2+M^2}} [1 - n(T,\mu) - \bar{n}(T,\mu)]$$

$$n(T,\mu) = \left(1 + e^{\frac{E_p-\mu}{T}}\right)^{-1},\ \ \bar{n}(T,\mu) = \left(1 + e^{\frac{E_p+\mu}{T}}\right)^{-1}.$$

NUMERICAL RESULTS

For $\Lambda = 587.9,\ G\Lambda^2 = 2.44$





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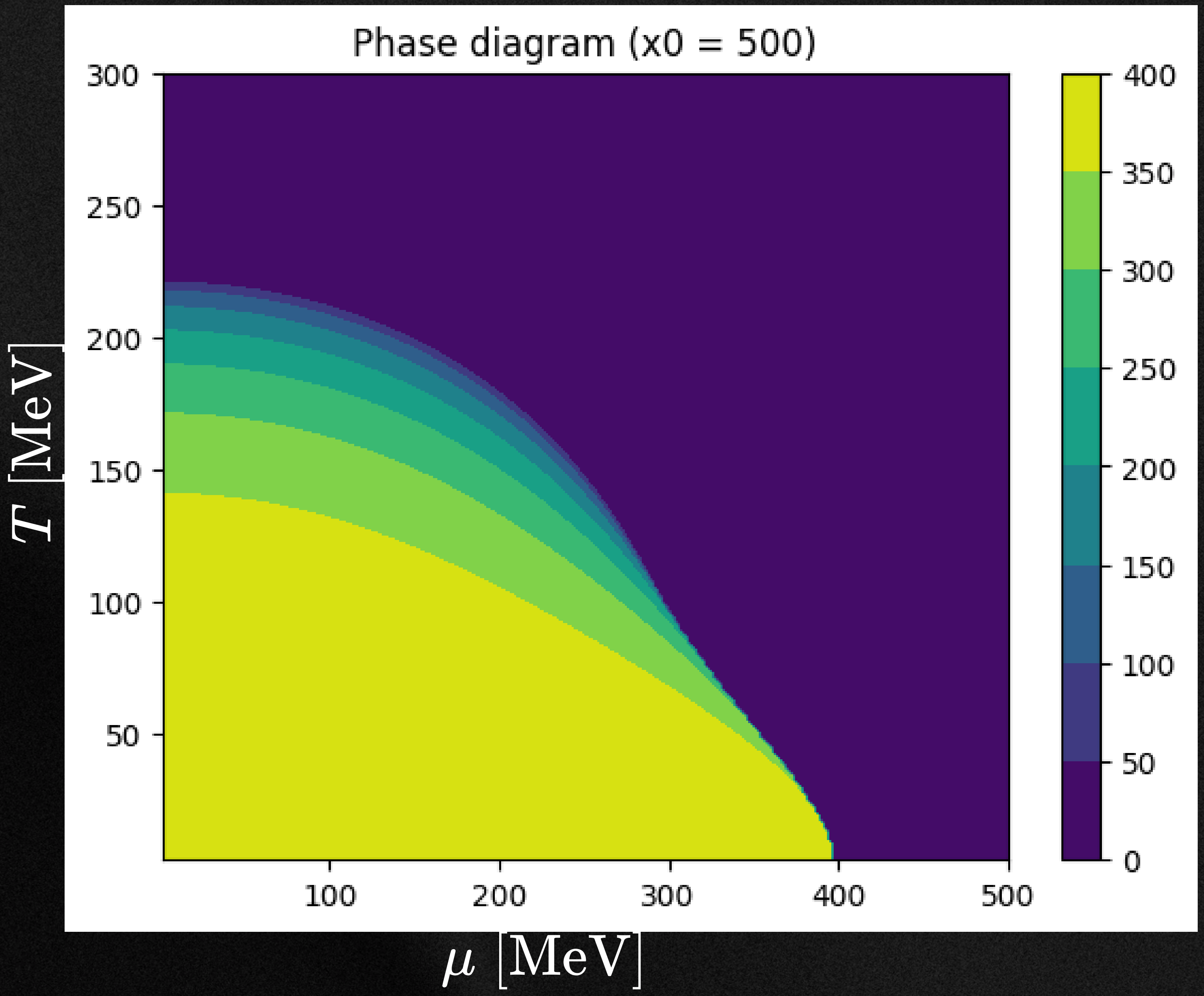

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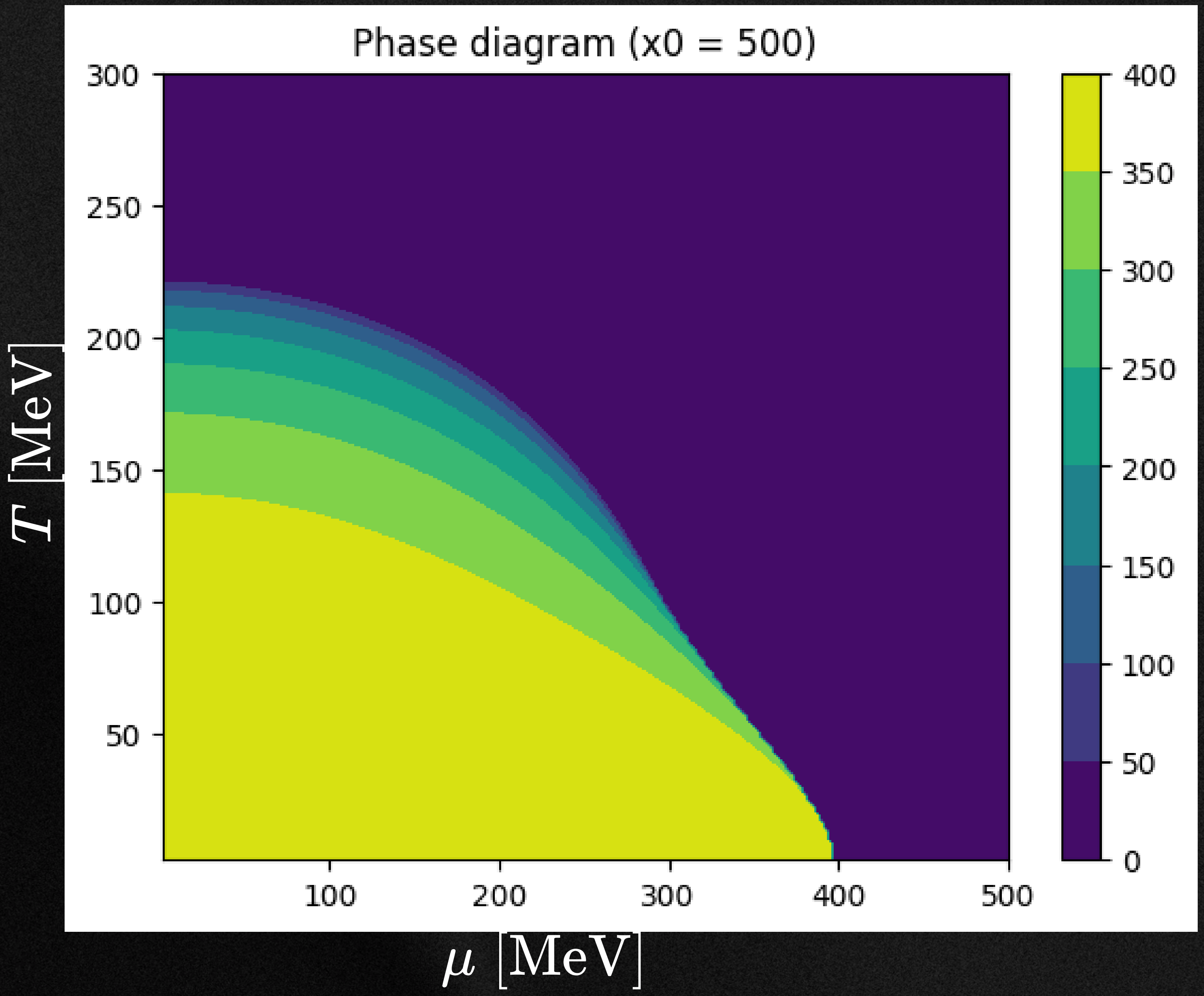

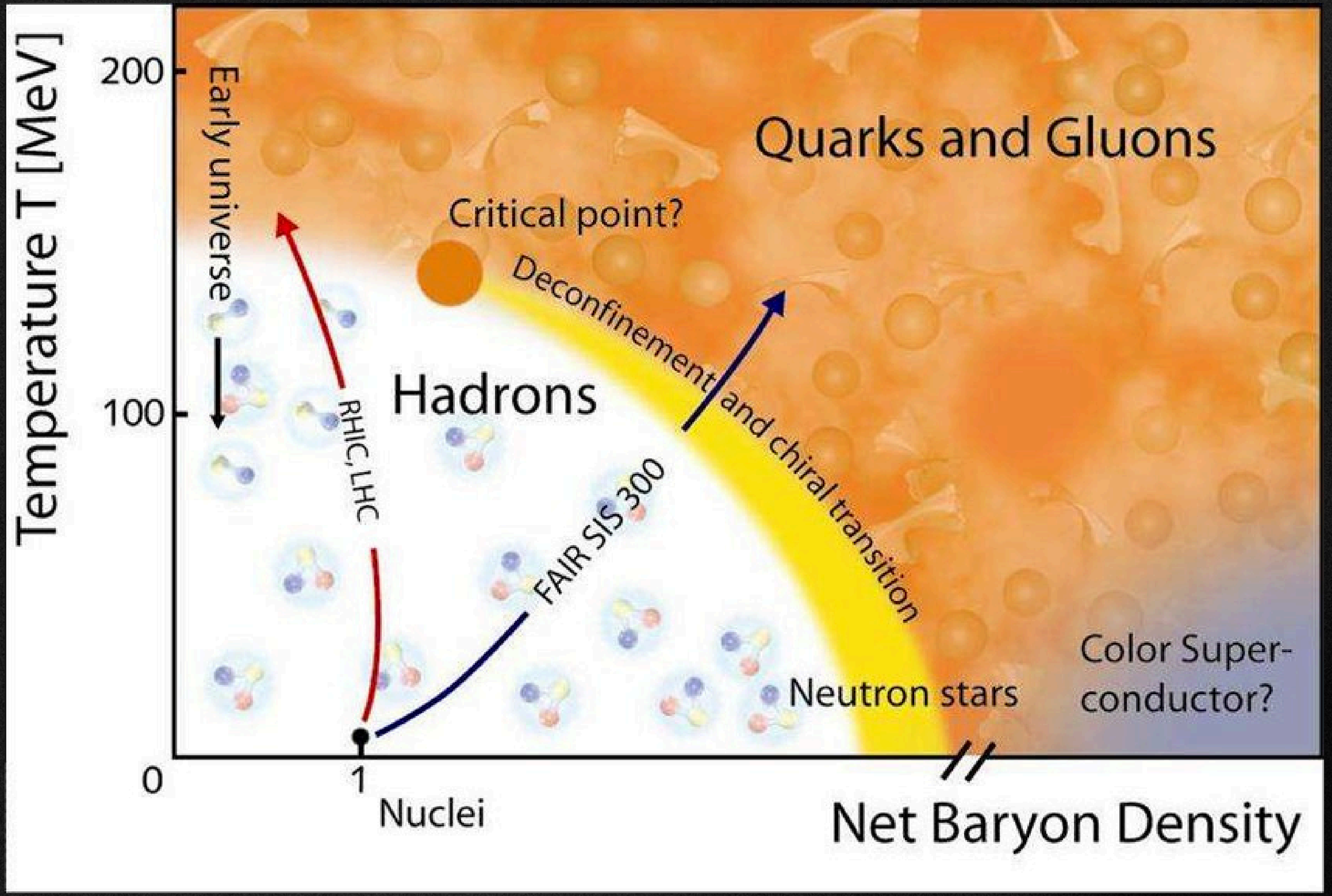

Figure 7. Avalible from: https://www.researchgate.net/figure/The-phase-diagram-of-QCD_fig1_282844145

CONCLUSION

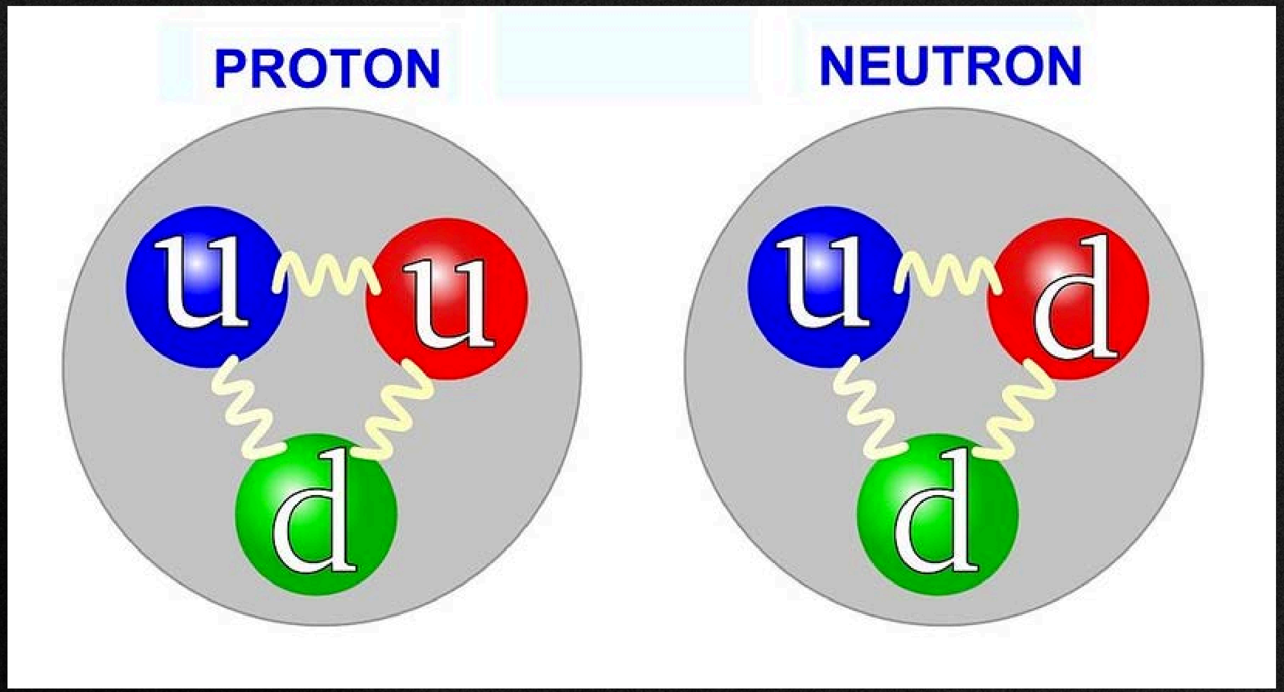

Figure 8. Avalible from: https://sites.ifi.unicamp.br/hadrex/qgp/

- The NJL model isolates a specific feature of QCD, chiral symmetry, and allows us to study its consequences.
- The model provides a mechanism—spontaneous chiral symmetry breaking—to explain why protons and neutrons are much more massive than the sum of the masses of the quarks that compose them.
- We can use it as an effective theory of QCD to explore regions of the phase diagram that are not accessible to the lattice community and to obtain qualitative insights into its behavior.

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