QCD sum rules for hadron structure

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Quantum Chromodynamics

$$\mathcal{L} = \bar{\Psi}_a \left(i\gamma^\mu (D_\mu)_{ab} - m\delta_{ab} \right) \Psi_b - \frac{1}{4} G^A_{\mu\nu} G^{\mu\nu}_A$$

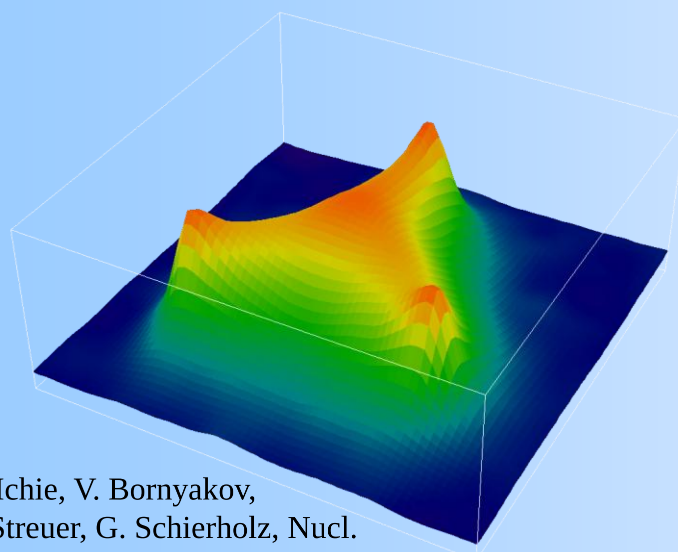
Very successful!

(but only really useful when dealing with hard processes)

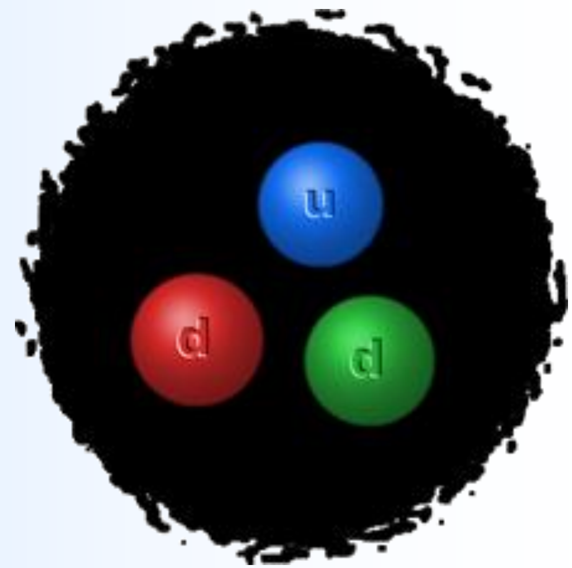
Hadrons and QCD

We would like to understand:

- The origin and dynamics of confinement.
- Understand the quark and gluon structure of hadrons based on QCD.



H. Ichie, V. Bornyakov,
T. Streuer, G. Schierholz, Nucl.
Phys. A**721**, 899 (2003)



Non-perturbative QCD

- **Potential models**
- **QCD Sum Rules**
- **Schwinger–Dyson Equation**
- **Lattice QCD**
- **AdS/CFT Correspondence**

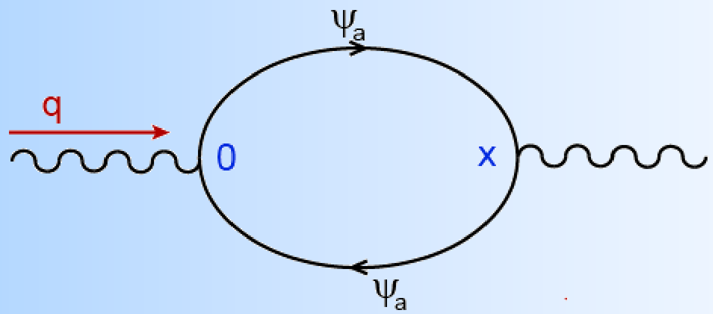
The QCD Sum Rules

- **1979, Shifman, Vainshtein, Zakharov**

⇨ Assumes confinement

⇨ relates hadronic parameters (masses, couplings, etc.) to a few characteristics of the “vacuum” of quantum chromodynamics (QCD): gluon and quark condensates.

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq.x}\langle 0|T\left[j_\mu(x)j^\dagger_\nu(0)\right]|0\rangle$$



The QCD Sum Rules

$\Pi(q)$

Quarks

Hadrons

The QCD Sum Rules

$$\Pi(q)$$

Quarks | **Hadrons**

$$\Pi(q) = i \int d^4x \; e^{iq.x} \langle 0 | T \left\{ j(x) j^\dagger(0) \right\} | 0 \rangle$$

eg: $j_D(x) = \bar{q}_a(x) i\gamma_5 c_a(x)$

The QCD Sum Rules

$\Pi(q)$

OPE Side

$$\Pi(q) = i \int d^4x \, e^{iq.x} \langle 0 | T \{ j(x) j^\dagger(0) \} | 0 \rangle$$

eg: $j_D(x) = \bar{q}_a(x) i\gamma_5 c_a(x)$

Wilson's OPE

$$i \int d^4x \, e^{iq.x} T \{ j(x) j^\dagger(0) \} = \sum_d C_d(q) \hat{O}_d$$

$$\Pi(q) = \sum_d C_d(q) \langle 0 | \hat{O}_d | 0 \rangle$$

Hadrons

The QCD Sum Rules

$$\Pi(q)$$

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$$\Pi(q) = \sum_d C_d(q) \langle 0 | \hat{O}_d | 0 \rangle$$

Hadrons

$j(x)$ ⇒ Hadronic field operator

$$\Pi(q) = \int_0^\infty ds \frac{\rho(s)}{s - q^2}$$

$$\rho(s) = \frac{1}{\pi} \mathrm{Im} \Pi(q)$$

The QCD Sum Rules

$\Pi(q)$

OPE Side

$$\Pi(q) = i\int d^4x\, e^{iq.x}\langle 0|T\left\{j(x)j^\dagger(0)\right\}|0\rangle$$

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$$\Pi(q) = \sum_d C_d(q)\langle 0|\hat{O}_d|0\rangle$$

Phenomenological Side

$j(x)$ ⟹ Hadronic field operator

$$\Pi(q) = \int_0^\infty ds \frac{\rho(s)}{s-q^2}$$

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(q)$$

Pole + Cont. approximation

$$\rho(s) \approx f_H\delta(s-m_H^2) + \rho^{\rm cont}(s)\theta(s-s_0)$$

$$\langle m_H|j|0\rangle = \underbrace{f_H m_H}_{\text{Hadronic parameters}}$$

$$\Pi(q) = \frac{m_H^2 f_H^2}{m_H^2 - q^2} + \int_{s_0}^\infty ds\frac{\rho^{\rm cont}(s)}{s-q^2}$$

The QCD Sum Rules

$\Pi(q)$

OPE Side

$$\Pi(q) = i\int d^4x\, e^{iq.x}\langle 0|T\left\{j(x)j^\dagger(0)\right\}|0\rangle$$

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Wilson's OPE

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$$\langle m_H|j|0\rangle = f_H m_H$$

Hadronic parameters

Quark - Hadron ⇔ **Duality**

$$\Pi(q) = \frac{m_H^2 f_H^2}{m_H^2 - q^2} + \int_{s_0}^\infty ds \frac{\rho^{\mathrm{cont}}(s)}{s-q^2}$$

The QCD Sum Rules

Condensates $\Longrightarrow$ $$\Pi^{OPE}(q^2) = \sum_d C_d(q^2)\langle 0|\hat{O}_d|0\rangle$$

$$d = 0 \begin{cases} \hat{O}_0 = \hat{1} \\ C_0(q^2) = \Pi^{pert}(q^2) \end{cases}$$

$$\nexists \hat{O}_d, \; d = 1, 2$$

$$d = 3 \Rightarrow \hat{O}_3 = \bar{\psi}\psi$$

$$\langle \bar{q}q\rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3$$

$q = u, d$

$$d = 4 \Rightarrow \hat{O}_4 = G^a_{\mu\nu}G^{a\mu\nu}$$

$$\langle g^2G^2\rangle = 0.88 \text{ GeV}^4$$

$$d = 5 \Rightarrow \hat{O}_5 = \bar{\psi}\sigma_{\mu\nu}\frac{\lambda}{2}G^{a\mu\nu}\psi$$

$$\langle \bar{q}g\sigma.Gq\rangle = (0.8)\langle\bar{q}q\rangle$$

$q = u, d$

The QCD Sum Rules

$$\Pi^{phen}(q^2) = \Pi^{OPE}(q^2)$$

Quark masses, values for condensates

Hadronic parameters

(masses, decay constants, form factors, magnetic moments, etc…)

$$\Pi^{phen}(q) = \frac{m_H^2 f_H^2}{m_H^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^{\text{cont}}(s)}{s - q^2} \quad = \quad \Pi^{OPE}(q^2) = \int_0^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}$$

$$\frac{m_H^2 f_H^2}{m_H^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^{\text{cont}}(s)}{s - q^2} = \int_0^{s_0} ds \frac{\rho^{OPE}(s)}{s - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}$$

Pole Contribution

Continuum Contribution

The QCD Sum Rules

Borel Transformation

$$\Pi(M^2) = \mathcal{B}\left[\Pi(q^2)\right] = \lim_{\substack{-q^2, n\to\infty \\ -q^2/n = M^2}} \frac{(-q^2)^{(n+1)}}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)$$

Improves OPE Convergence; Suppresses Continuum

$$\Pi^{phen}(M^2) = \Pi^{OPE}(M^2)$$

$$m_H^2 f_H^2 e^{-m_H^2/M^2} = \int_0^{s_0} ds\ \rho^{OPE}(s) e^{-s/M^2}$$

$$m_H^2 = \frac{\int_0^{s_0} ds\ s\ \rho^{OPE}(s) e^{-s/M^2}}{\int_0^{s_0} ds\ \rho^{OPE}(s) e^{-s/M^2}}$$

Validity Limits

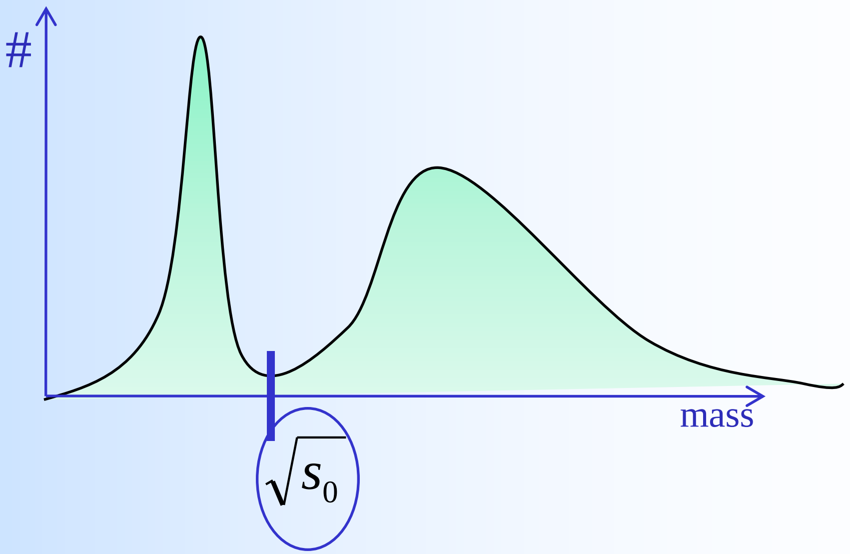
Some assumptions

OPE Side

Convergence

Spectral function

pole + continuun



Validity Limits

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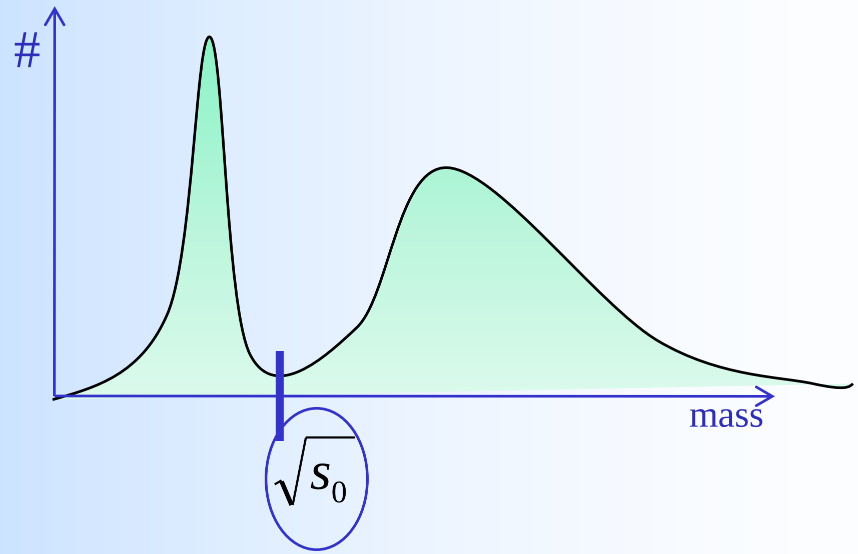

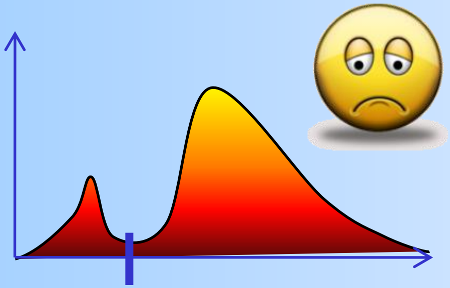

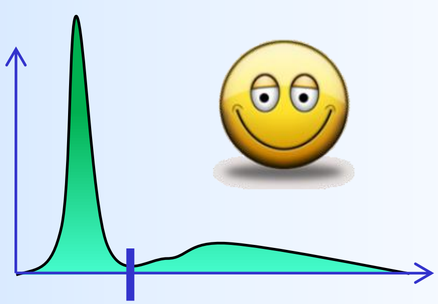

Validity Limits

Some assumptions

OPE Side

Convergence

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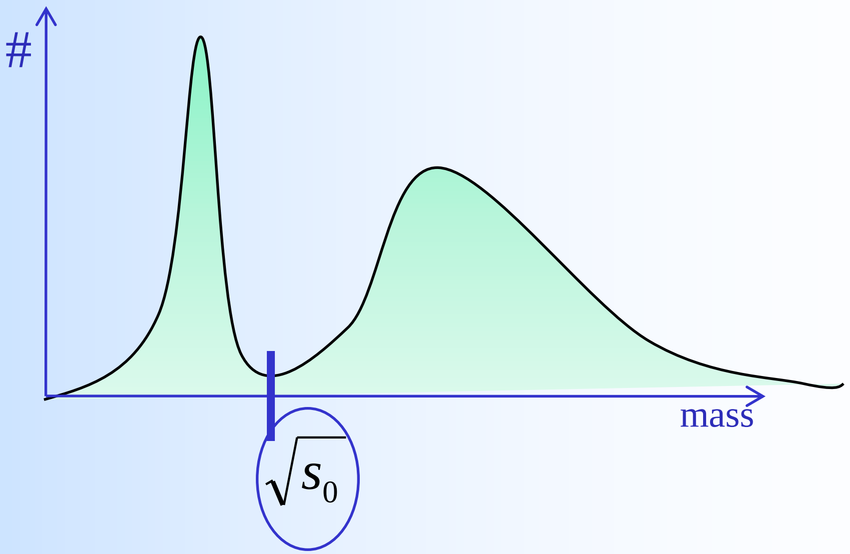


Apply Borel transform: $Q^2 \rightarrow M^2$

Validity Limits

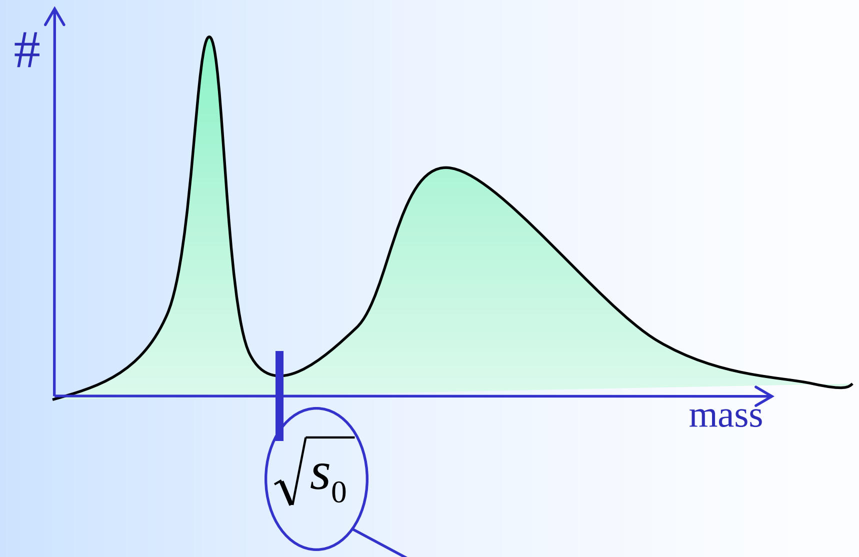
Some assumptions

OPE Side

Convergence

Spectral function

pole + continuun



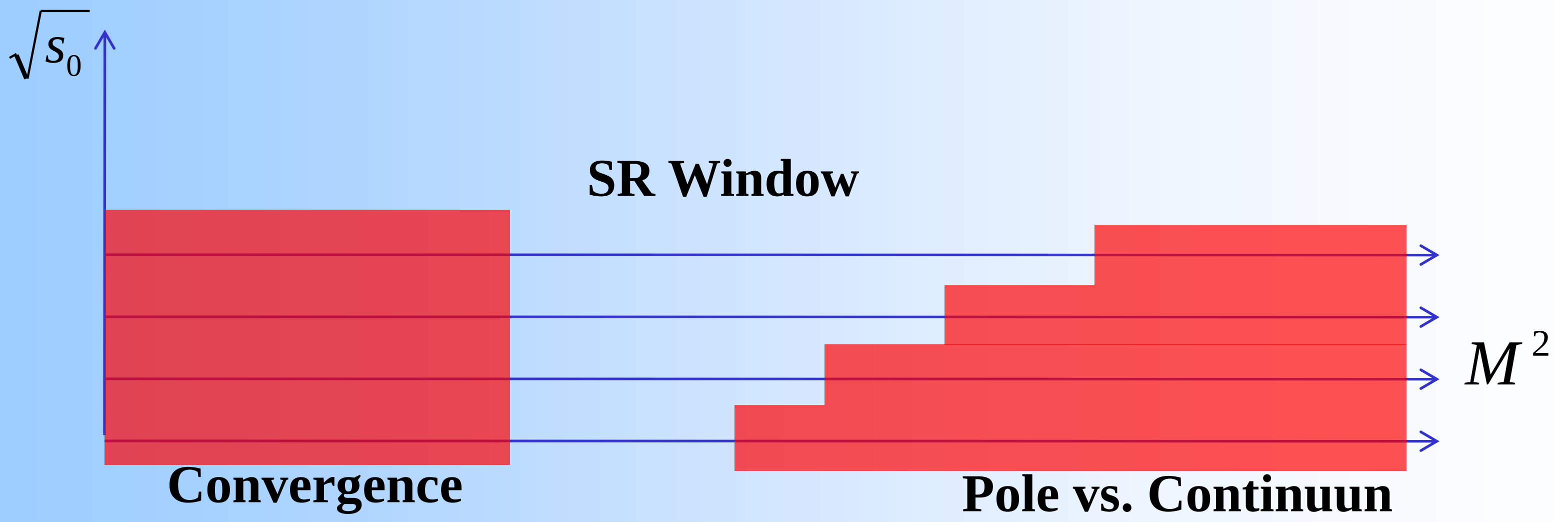
Apply Borel transform: $Q^2 \rightarrow M^2$

We want as much control as possible on these two parameters

Validity Limits

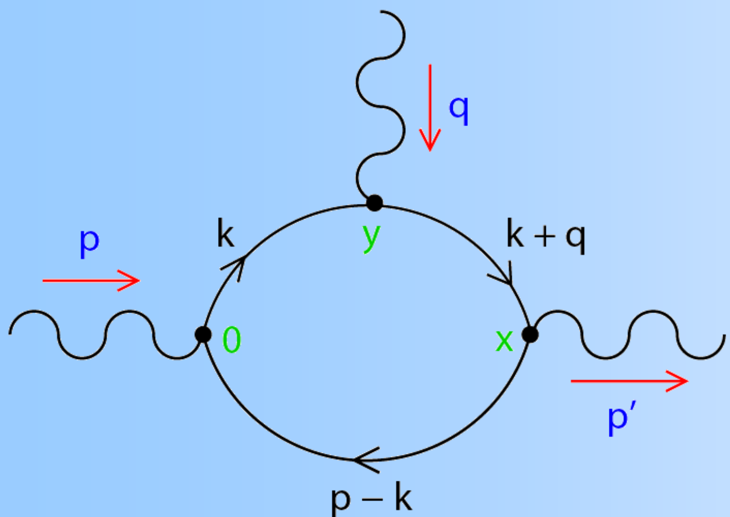
We demand:

- Stability on M^2
- OPE is converging
- Pole contribution is bigger than continuun contribution



3-Point Correlators

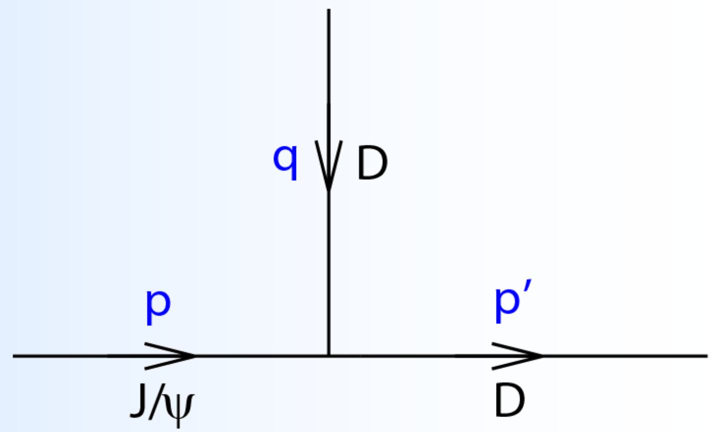
$$\Gamma(p,p',q) = \int d^4x \int d^4y \; e^{ip'x} \; e^{-iqy} < 0|T\{j_1(x) j_2^\dagger(y) j_3^\dagger(0)\}|0 >$$



Quark - Hadron

Duality

eg: J/Ψ-D-D Vertex

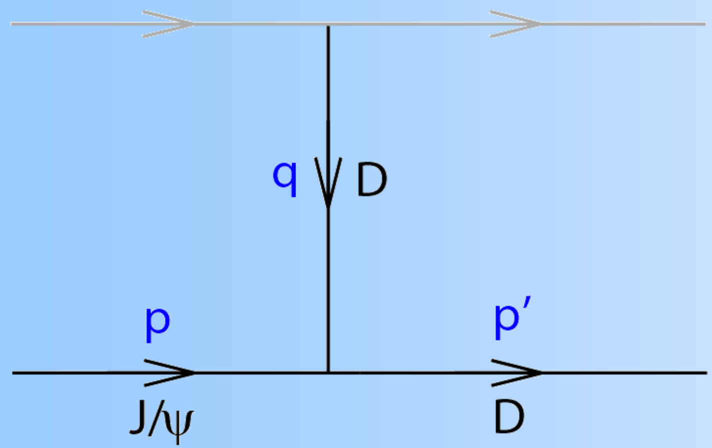


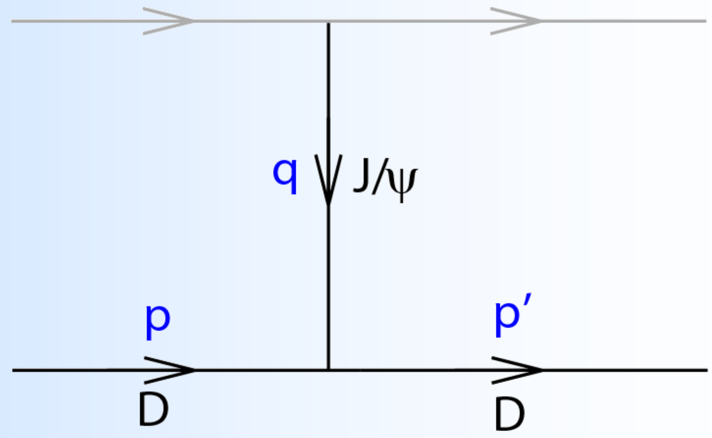
$$\mathcal{L}_{\psi DD} = i g_{\psi DD} \psi^{*\mu} (D \partial_\mu \bar{D} - \partial_\mu D \bar{D})$$

$$< m_{J/\psi}, \lambda, \vec{p}\,|(m_D, -\vec{p}\,'), (m_D, \vec{q}\,) >= -g_{\psi DD} \;\; (q + p')_\mu \;\; \epsilon^{(\lambda)\mu}(q)$$

$$\Gamma_\mu^{(D)}(p^2, p'^2, q^2) = -\frac{m_D^2 f_D}{m_c} m_{J/\psi} f_{J/\psi} \frac{m_D^2 f_D}{m_c} \frac{g_{J/\psi DD}^{(D)}(q^2)}{m_D^2 - q^2} \frac{1}{p^2 - m_{J/\psi}^2} \frac{1}{p'^2 - m_D^2} \left(-2p'_\mu + \frac{m_D^2 + m_{J/\psi}^2 - q^2}{m_{J/\psi}^2} p_\mu \right) + MS$$

3-Point Correlators



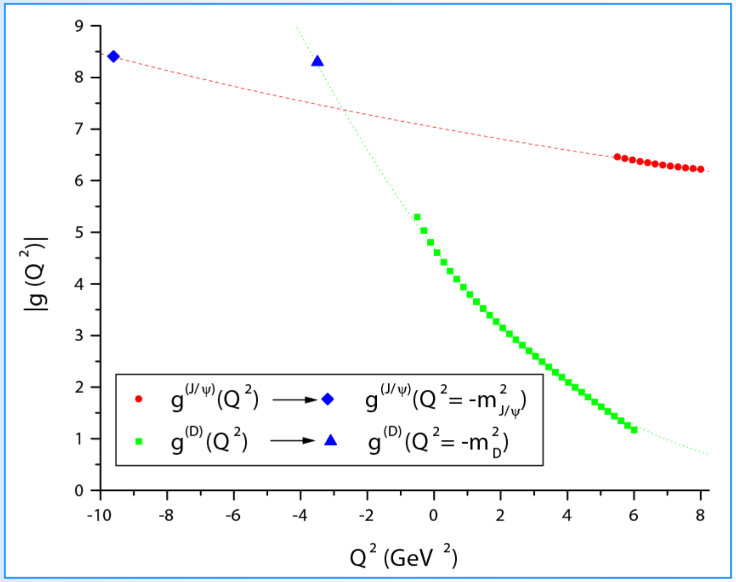


Borel transformation: p^2 & p'^2

Results depend on: q^2

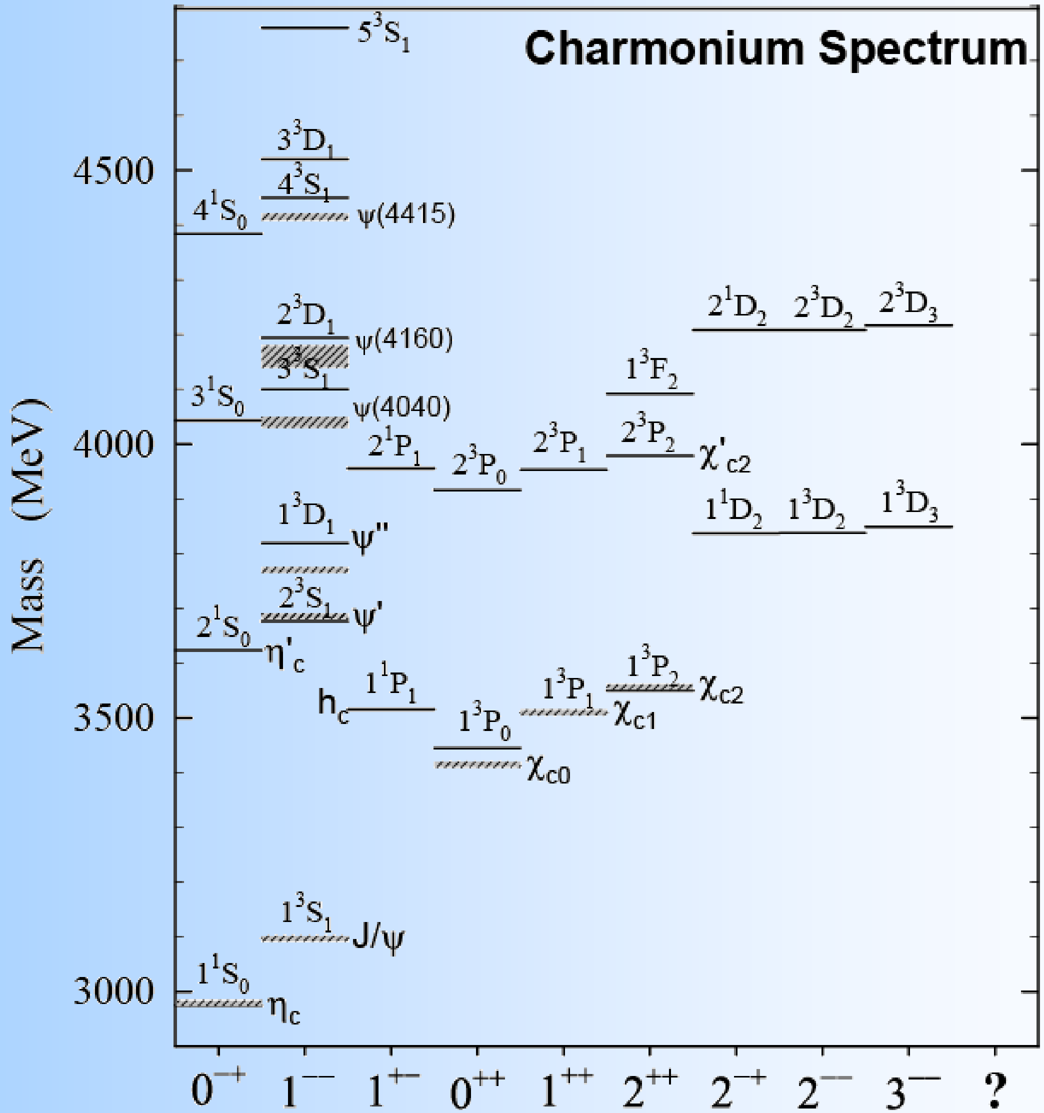
$$g_{J/\psi DD}^{(J/\psi_{off})}(q^2) \qquad g_{J/\psi DD}^{(D_{off})}(q^2)$$

$$g_{J/\psi DD} = g_{J/\psi DD}^{(D_{off})}(m_D^2) = g_{J/\psi DD}^{(J/\psi_{off})}(m_{J/\psi}^2)$$



Application: Hadron Spectroscopy

Quark Model



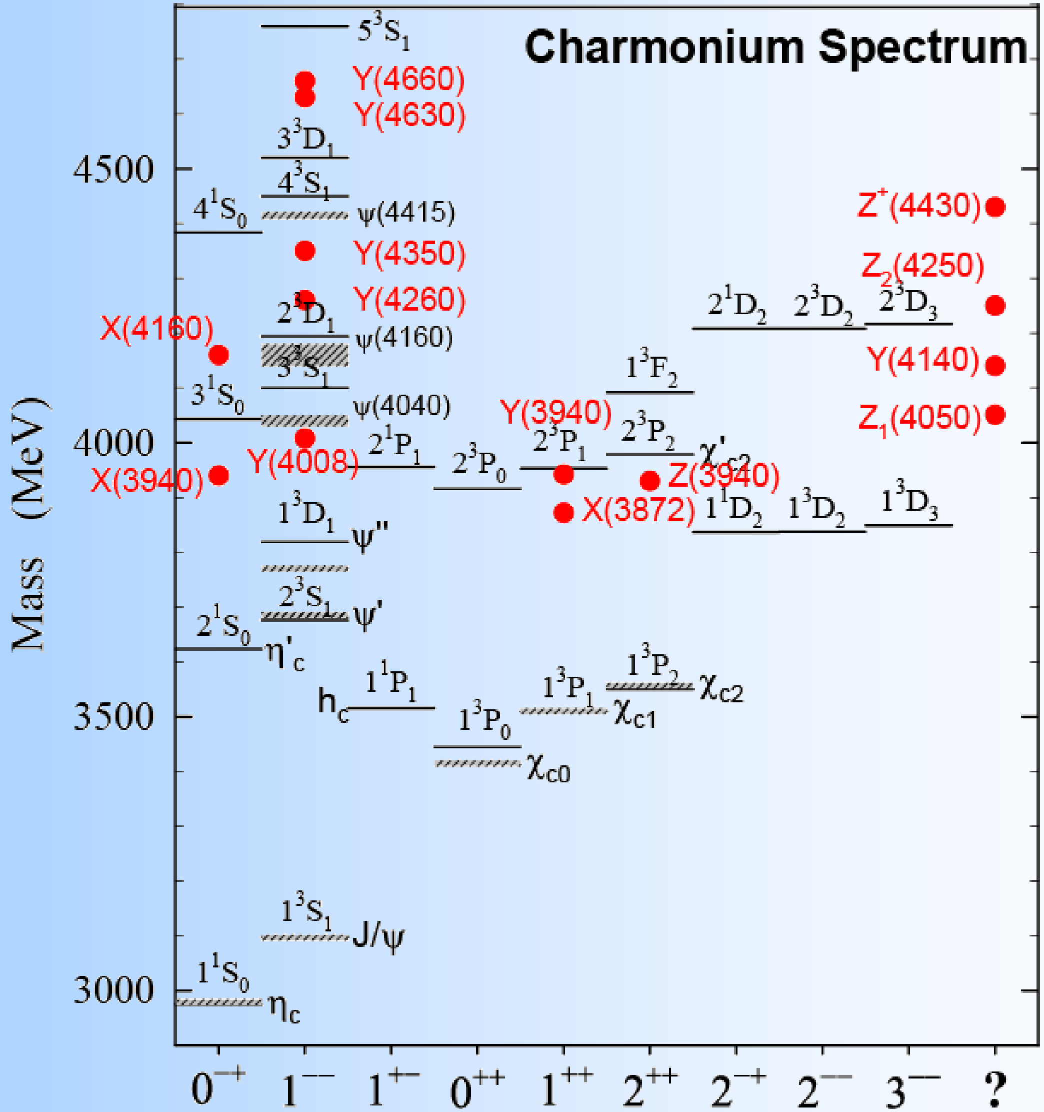
S. Godfrey, arXiv:0910.3409 [hep-ph]

Application: Hadron Spectroscopy

BaBar

Belle

CDF & D0



S. Godfrey, arXiv:0910.3409 [hep-ph]

X (3872) – a case study

Facts:
(approximate)

Narrow resonant structure $\begin{cases} m_X = 3871.52 \pm 0.20 \text{ MeV} \\ \Gamma_X = 1.3 \pm 0.6 \text{ MeV} \end{cases}$

Probably: $J_{PC} = 1^{++}$ ($J_{PC} = 2^{-+}$ not excluded)

$$m[X(3872)] - [m(D^{*0}) + m(D^0)] = -0.42 \pm 0.39 \text{ MeV}.$$

$$X(3872) \rightarrow J/\psi\, \omega \rightarrow J/\psi\, \pi^+\pi^-\pi^0$$
$$X(3872) \rightarrow J/\psi\, \rho \rightarrow J/\psi\, \pi^+\pi^-$$
$$X(3872) \rightarrow J/\psi\, \gamma$$
$$X(3872) \rightarrow \psi(2S)\, \gamma$$
$$X(3872) \rightarrow D^{*0}\bar{D}^0 \rightarrow D^0\bar{D}^0\pi^0$$

$$\frac{\mathcal{B}(X \rightarrow J/\psi\ \pi^+\pi^-\pi^0)}{\mathcal{B}(X \rightarrow J/\psi\ \pi^+\pi^-)} = 1{,}0$$

X (3872) – a case study

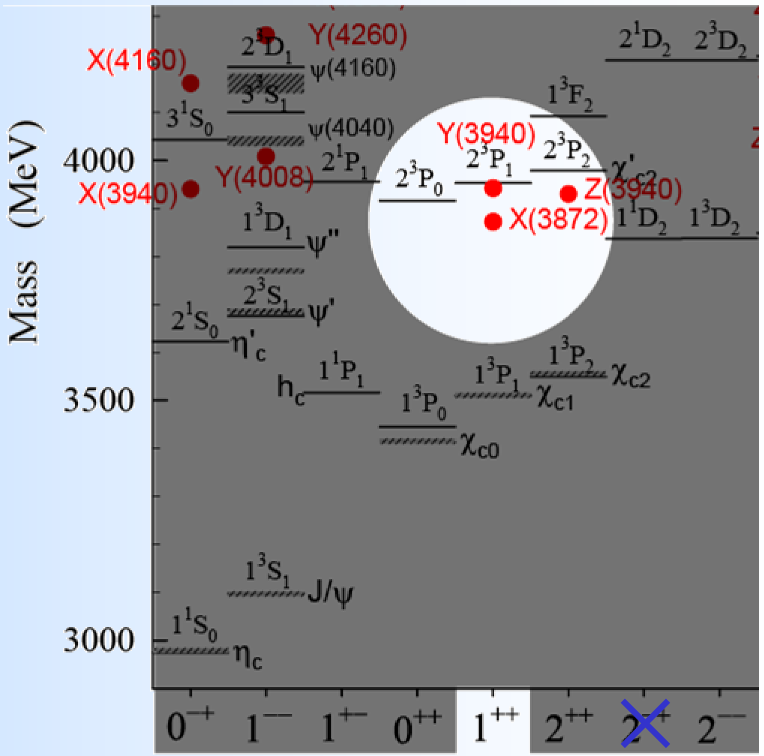
What is it?

Is it a Charmonium?

No quark model candidate

Barnes, Godfrey, Swanson, (2005)

Eichten, Lane, Quigg, (2006)



Kalashnikova, Nefediev (2010)

$$\frac{\mathcal{B}\left(X \to J/\psi\ \pi^+\pi^-\pi^0\right)}{\mathcal{B}\left(X \to J/\psi\ \pi^+\pi^-\right)} = 1{,}0 \pm 0{,}4 \pm 0{,}3$$

$c\bar{c}$ with isospin violating decay?

X (3872) – a case study

What is it?

Is it a D - D* molecule ? Swanson; Close, Page

Observed production rate is too large ! M. Suzuki, PRD (2005).

Observed radiative decay rate is too large !

$$\frac{\mathcal{B}\left(X \to \psi(2S)\gamma\right)}{\mathcal{B}\left(X \to \psi\,\gamma\right)} \sim 3.4 \pm 1.4 \quad \times \quad \frac{\Gamma\left(X \to \psi(2S)\gamma\right)}{\Gamma\left(X \to \psi\,\gamma\right)} \sim 4 \times 10^{-3}$$

BaBar, arXiv:0907.4575

Swanson, PLB (2004)

X (3872) – a case study

What is it?

Is it a tetraquark? Maiani *et al*

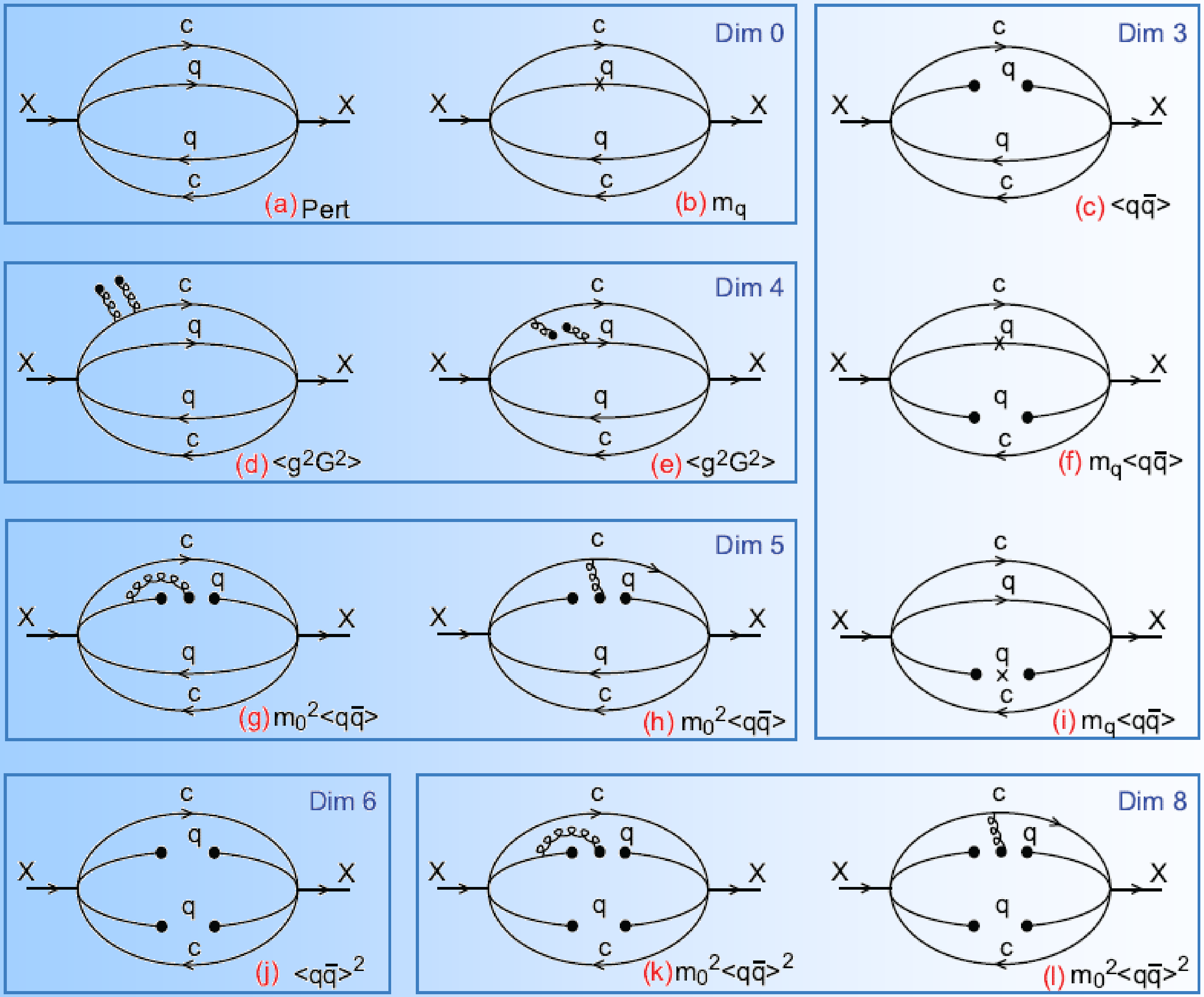
Proliferation of predicted states!

e.g. $cs\bar{c}\bar{s}$ ⇒

M(MeV)	Decay Channel $[\Gamma_{part}(\text{KeV})]$	Relative Wave
3834	-	
3927	Multihadron	–
4277(+15)	$J/\psi\ \phi$ [35], $D_s^{*+} D_s^{*-}$ [10]	P
4312(+30)	$J/\psi\ \phi$ [46], $D_s^{*+} D_s^{*-}$ [24]	P
4297(-5)	$\psi\ \eta(\eta')$ [245(110)], $D_s^{+} D_s^{*-}$ [500]	P
3890	Multihadron	–
3870	$J/\psi\ \eta$ [610]	S
3905	$J/\psi\ \eta$ [650]	S
4321(+15)	$J/\psi\ \phi$ [52]	P
4356 (+30)	$J/\psi\ \phi$ [64]	P
4330	$\psi\ \eta(\eta')$ [90(45)], $D_s^{(*)+} D_s^{(*)-}$ [27]; $J/\psi\ f_0(980)$	P; S
4341(-5)	$\psi\ \eta(\eta')$[92(48)], $D_s^{(*)+}\ D_s^{(*)-}$ [31]; $J/\psi\ f_0(980)$	P; S
4390(+40)	$\psi\ \eta(\eta')$ [100(58)], $D_s^{(*)+} D_s^{(*)-}$ [51]; $J/\psi\ f_0(980)$	P; S
4289(-41)	$\psi\ \eta(\eta')$ [83(36)], $D_s^{(*)+} D_s^{(*)-}$ [13]; $J/\psi\ f_0(980)$	P; S

NV Drenska, R Faccini, AD Polosa, PRD (2009).

QCDSR for the X (3872)



QCDSR for the X (3872)

Tetraquark

Matheus, Narison, Nielsen, Richard (2007)

$$j_\mu^{(q,\mathrm{di})} = \frac{i\,\epsilon_{abc}\,\epsilon_{dec}}{\sqrt{2}}\left[\,(\,q_a^T\,C\,\gamma_5\,c_b\,)\,(\,\bar{q}_d\,\gamma_\mu\,C\,\bar{c}_e^T\,) + (\,q_a^T\,C\,\gamma_\mu\,c_b\,)\,(\,\bar{q}_d\,\gamma_5 C\,\bar{c}_e^T\,)\,\right]$$

$$m_X = (3.92 \pm 0.13)\ \mathrm{GeV}$$

$$j_\mu^{X_l} = \cos\alpha\, j_\mu^{(u,\mathrm{di})} + \sin\alpha\, j_\mu^{(d,\mathrm{di})}$$

$$j_\mu^{X_h} = -\sin\alpha j_\mu^{(u,\mathrm{di})} + \cos\alpha j_\mu^{(d,\mathrm{di})}$$

$$\alpha = 45^\circ \rightarrow I_{X_l} = 0,\ I_{X_h} = 1$$

Maiani, Piccinini, Polosa, Riquer (2005)

$$\alpha = 20^\circ$$

$$|M(X_h) - M(X_l)| \backsimeq (2.6 - 3.9)\ \mathrm{MeV}$$

$\overline{\mathrm{D}}\mathrm{D}^*$ Molecule

Lee, Nielsen,Wiedner (2009)

$$j_\mu^{(q,\mathrm{mol})}(x) = \frac{1}{\sqrt{2}}\Big[\,(\,\bar{q}_a(x)\,\gamma_5\,c_a(x)\,\bar{c}_b(x)\,\gamma_\mu\,q_b(x)\,) - (\,\bar{q}_a(x)\,\gamma_\mu\,c_a(x)\,\bar{c}_b(x)\,\gamma_5\,q_b(x)\,)\,\Big]$$

$$m_X = (3.87 \pm 0.07)\ \mathrm{GeV}$$

$$(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$$

QCDSR for the X (3872)

Widths?

Tetraquark

Navarra, Nielsen (2006)

$$\alpha \simeq 23.5^\circ \Longrightarrow \frac{\mathcal{B}\left(X \to J/\psi\ \pi^+\pi^-\pi^0\right)}{\mathcal{B}\left(X \to J/\psi\ \pi^+\pi^-\right)} = 1{,}0$$

$$g_{X\psi\omega} = 13.8 \pm 2.0$$
$$\Gamma(X \to J/\psi(n\pi)) = (50 \pm 15)\ \text{MeV}$$

$\not\Longrightarrow$

$$\alpha = 20^\circ$$
$$g_{X\psi\omega} \simeq 0.47$$

Maiani, Piccinini, Polosa, Riquer (2005)

Similar results for the $D\bar{D}^*$ Molecule!

Matheus, Navarra, Nielsen, Zanetti (2009)

The Decay Widths

$$L = g_{X\psi V}\ \varepsilon^{\mu\nu\rho\sigma}\ p_\mu\ X_\nu\ \psi_\rho\ V_\sigma$$

Maiani, Piccinini, Polosa, Riquer (2005)

$$\frac{d\Gamma}{ds}(X \to J/\psi f) = \frac{1}{8\pi m_X^2}|\mathcal{M}|^2 B_{V\to F}$$
$$\times \quad \frac{\Gamma_V m_V}{\pi} \frac{p(s)}{(s-m_V^2)^2 + (m_V \Gamma_V)^2},$$

$V = \rho, \omega$

$F = 2\pi, 3\pi$

$$\frac{\Gamma(X \to J/\psi\, \pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\, \pi^+\pi^-)} = 0.118 \left(\frac{g_{X\psi\omega}}{g_{X\psi\rho}}\right)^2$$

The Decay Widths

Assume that X is $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$

Three-point correlation function:

$$\Pi_{\mu\nu\alpha}(p,p',q) = \int d^4x d^4y \; e^{ip'.x} \; e^{iq.y} \; \langle 0|T[j_\mu^\psi(x) j_\nu^V(y) j_\alpha^{X\dagger}(0)]|0\rangle$$

Currents:

$$j_\mu^\psi = \bar{c}_a \gamma_\mu c_a$$

$$j_\nu^V = \frac{N_V}{2}(\bar{u}_a\gamma_\nu u_a + (-1)^{I_V}\bar{d}_a\gamma_\nu d_a)$$

OPE side:

$$\Pi_{\mu\nu\alpha}^V(p,p',q) = N_V \Pi_{\mu\nu\alpha}^{OPE}(p,p',q)$$

$$N_\rho = 1 \qquad I_\rho = 1$$

$$N_\omega = 1/3 \qquad I_\omega = 0$$

The Decay Widths

Assume that X is $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$

Phenomenological side:

$$\Pi^{(phen)}_{\mu\nu\alpha}(p,p',q) = \frac{i\lambda_X m_\psi f_\psi m_V f_V \; g_{X\psi V}}{(p^2 - m_X^2)(p'^2 - m_\psi^2)(q^2 - m_V^2)} \times \left(-\epsilon^{\alpha\mu\nu\sigma}(p'_\sigma + q_\sigma) - \epsilon^{\alpha\mu\sigma\gamma}\frac{p'_\sigma q_\gamma q_\nu}{m_V^2} - \epsilon^{\alpha\nu\sigma\gamma}\frac{p'_\sigma q_\gamma p'_\mu}{m_\psi^2}\right).$$

Sum rule:

$$\frac{i\lambda_X m_\psi f_\psi m_V f_V \; g_{X\psi V}}{(p^2 - m_X^2)(p'^2 - m_\psi^2)(q^2 - m_V^2)} = N_V \Pi^{OPE}(p,p',q)$$

Divide the two sum rules:

$$\frac{g_{X\psi\omega}}{g_{X\psi\rho}} = 1.14 \quad \Longrightarrow \quad \frac{\Gamma(X \to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\,\pi^+\pi^-)} \simeq 0.15$$

$\boldsymbol{\Gamma \simeq 50}$ MeV

The Decay Widths

Assume that X is $c\,\bar{c} + (D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ **(θ as mixing angle)**

$$\Pi_{\mu\nu\alpha} = -\frac{<\bar{u}\,u>}{(6\sqrt{2})}\cos(\theta)\;\Pi^{c\bar{c}}_{\mu\nu\alpha} + \sin(\theta)\;\Pi^{mol}_{\mu\nu\alpha}$$

$$\begin{cases} \Pi^{c\bar{c}}_{\mu\nu\alpha}(x,y) = \langle 0|T[j^{\psi}_{\mu}(x)j^{V}_{\nu}(y)j'^{(2)\dagger}_{\alpha}(0)]|0\rangle = 0 \\ \Pi^{mol}_{\mu\nu\alpha}(x,y) = \langle 0|T[j^{\psi}_{\mu}(x)j^{V}_{\nu}(y)j^{(4u)\dagger}_{\alpha}(0)]|0\rangle \end{cases}$$

$$\boldsymbol{\Pi_{\mu\nu\alpha}(x,y) = \sin(\theta)\,\Pi^{mol}_{\mu\nu\alpha}(x,y)} \;\Rightarrow\; \frac{\Gamma(X\to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X\to J/\psi\,\pi^+\pi^-)} \simeq 0.15$$

$$\boldsymbol{g_{X\psi V} \propto \sin\theta} \;\Rightarrow\; \boldsymbol{\Gamma \propto \sin^2\theta}$$

Smaller widths!

The Decay Widths

$$|X> = |c\,\bar{c}> + |D^{*0}\,\overline{D}^{0} + \overline{D}^{*0}\,D^{0}> + |D^{*+}\,\overline{D}^{-} + \overline{D}^{*-}\,D^{+}>$$

$$J^q_\mu(x) = \sin\theta\, j^{(4q)}_\mu(x) + \cos\theta\, j^{(2q)}_\mu(x)$$

$$j^X_\mu(x) = \cos\alpha\, J^u_\mu(x) + \sin\alpha\, J^d_\mu(x)$$

$$\Pi_{\mu\nu\alpha}(p,p',q) = \sin(\theta)\frac{N_V}{2\sqrt{2}}\big(\cos\alpha + (-1)^{I_V}\sin\alpha\big)\Pi^{OPE}_{\mu\nu\alpha}(p,p',q)$$

$$\Pi^{(phen)}_{\mu\nu\alpha}(p,p',q) = \frac{i(\cos\alpha+\sin\alpha)\lambda^q m_\psi f_\psi m_V f_V\ g_{X\psi V}}{(p^2-m_X^2)(p'^2-m_\psi^2)(q^2-m_V^2)} \times \left(-\epsilon^{\alpha\mu\nu\sigma}(p'_\sigma+q_\sigma) - \epsilon^{\alpha\mu\sigma\gamma}\frac{p'_\sigma q_\gamma q_\nu}{m_V^2} - \epsilon^{\alpha\nu\sigma\gamma}\frac{p'_\sigma q_\gamma p'_\mu}{m_\psi^2}\right).$$

The Decay Widths

$$|X> = |c\,\bar{c}> + |D^{*0}\,\bar{D}^{0} + \bar{D}^{*0}\,D^{0}> + |D^{*+}\,\bar{D}^{-} + \bar{D}^{*-}\,D^{+}>$$

$$\frac{g_{X\psi\omega} f_\omega}{g_{X\psi\rho} f_\rho} = \frac{N_\omega\left(\cos\alpha + \sin\alpha\right)}{N_\rho\left(\cos\alpha - \sin\alpha\right)}.$$

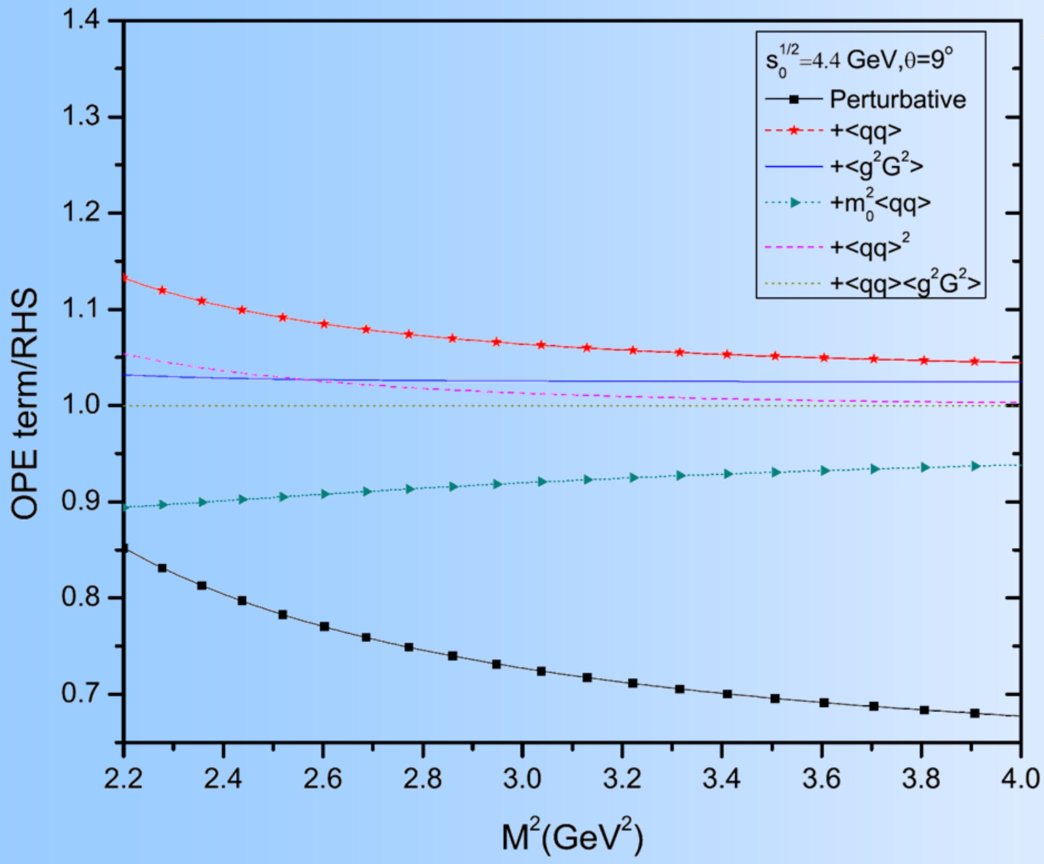
$$\frac{\Gamma(X \to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\,\pi^+\pi^-)} \simeq 0.15\left(\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}\right)^2$$

$$\frac{X \to J/\psi\,\pi^+\pi^-\pi^0}{X \to J/\psi\pi^+\pi^-} = 1.0 \quad \Rightarrow \quad \alpha \sim 20^0$$

Mixed X(3872)

Matheus, Navarra, Nielsen, Zanetti (2009)

OPE Convergence



$$j_\mu^X(x) = \cos\alpha\, J_\mu^u(x) + \sin\alpha\, J_\mu^d(x)$$

$$J_\mu^q(x) = \sin\theta\, j_\mu^{(4q)}(x) + \cos\theta\, j_\mu^{(2q)}(x)$$

$$5^\circ \le \theta \le 13^\circ\ ;\ \alpha \sim 20^\circ$$

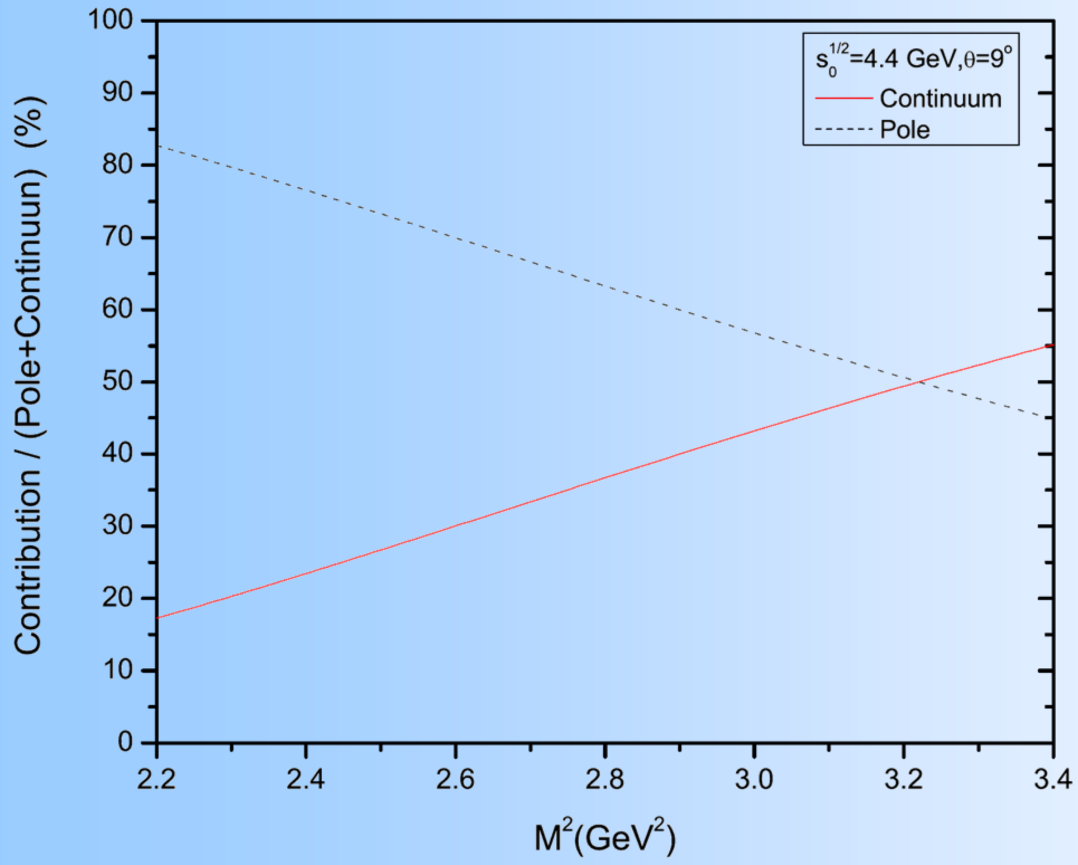
$$m_c(m_c) = (1.23 \pm 0.05)\ \text{GeV},$$
$$\langle \bar{u}u \rangle - (0.23 \pm 0.03)^3\ \text{GeV}^3,$$
$$\langle \bar{u} g\sigma.G u \rangle = m_0^2 \langle \bar{u}u \rangle,$$
$$m_0^2 = 0.8\ \text{GeV}^2,$$
$$\langle g^2 G^2 \rangle = 0.88\ \text{GeV}^4.$$

$$M^2 \ge 2.6\ \text{GeV}^2$$

Mixed X(3872)

Matheus, Navarra, Nielsen, Zanetti (2009)

Pole contribution vs. Continuun contribution

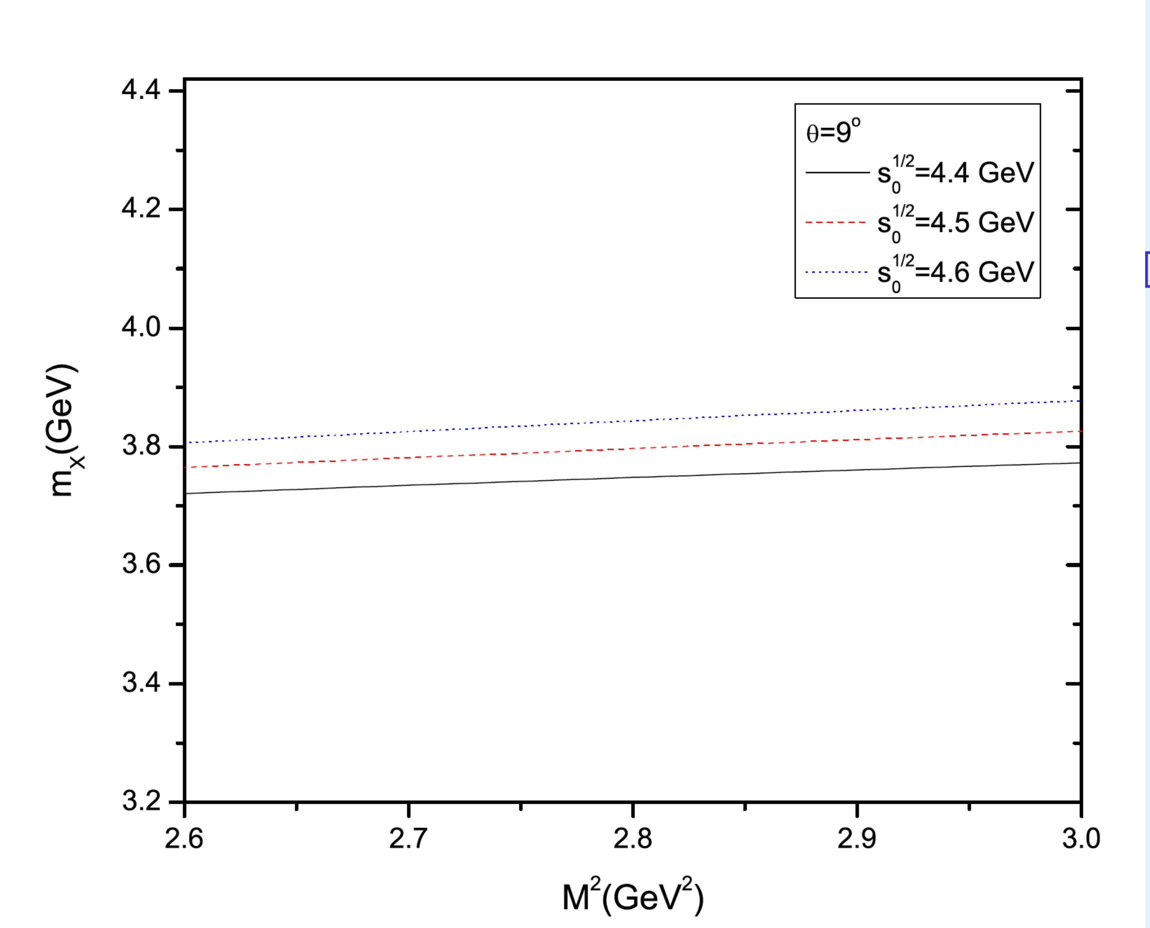


$$M^2 < 3.3\ GeV^2$$

Mixed X(3872)

Matheus, Navarra, Nielsen, Zanetti (2009)

Sum Rule Window ⇨ $2.6\ \mathrm{GeV}^2 \leq M^2 \leq 3.0\ \mathrm{GeV}^2$



$5^\circ \leq \theta \leq 13^\circ$

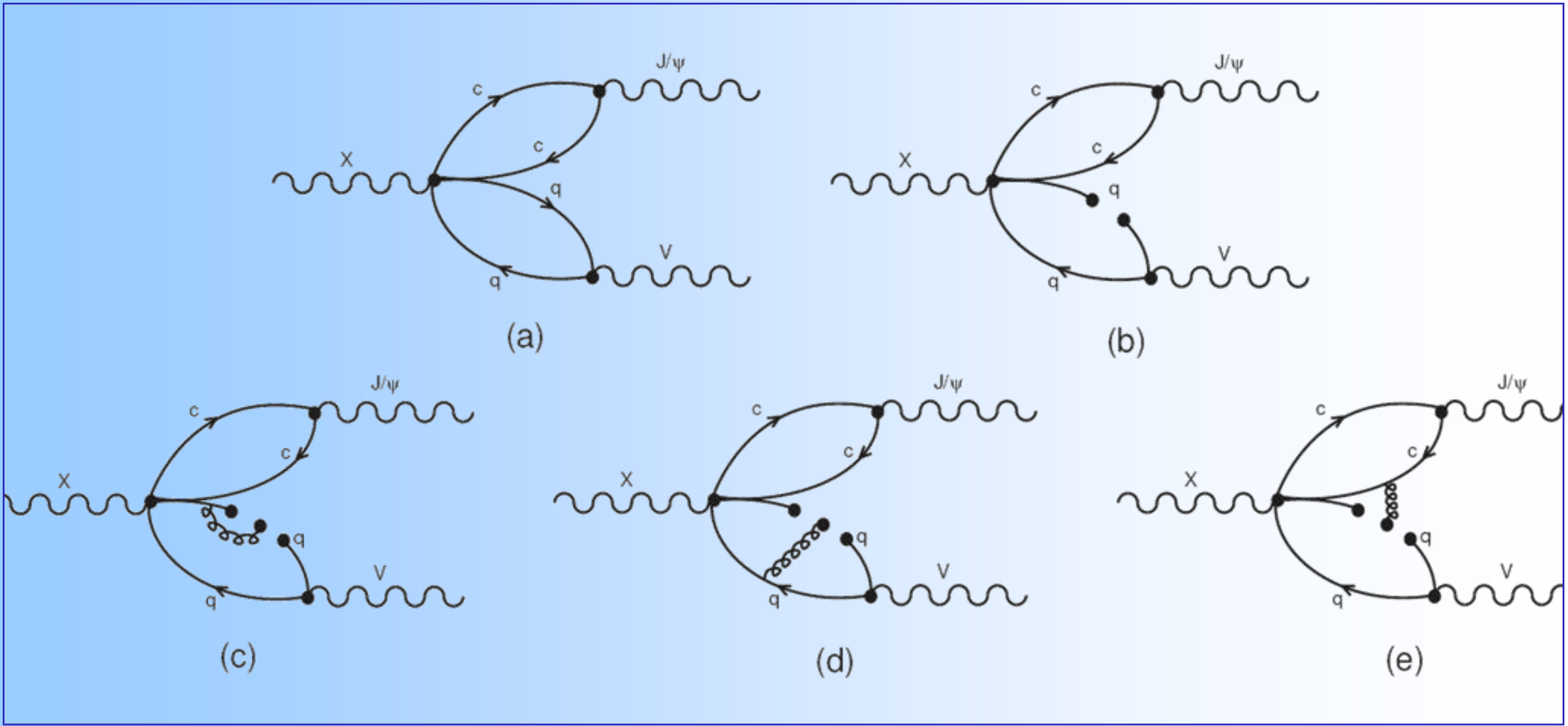
$$m_X = (3.77 \pm 0.18)\ \mathrm{GeV}.$$

Mixed X(3872)

Matheus, Navarra, Nielsen, Zanetti (2009)

Decay Width

$X \to J/\psi\omega$



$5^\circ \leq \theta \leq 13^\circ$

$\Rightarrow g_{X\psi\omega} = 5.4 \pm 2.4 \Rightarrow \Gamma\left(X \to J/\psi\pi^+\pi^-\pi^0\right) = (9.3 \pm 6.9)\ \text{MeV}$

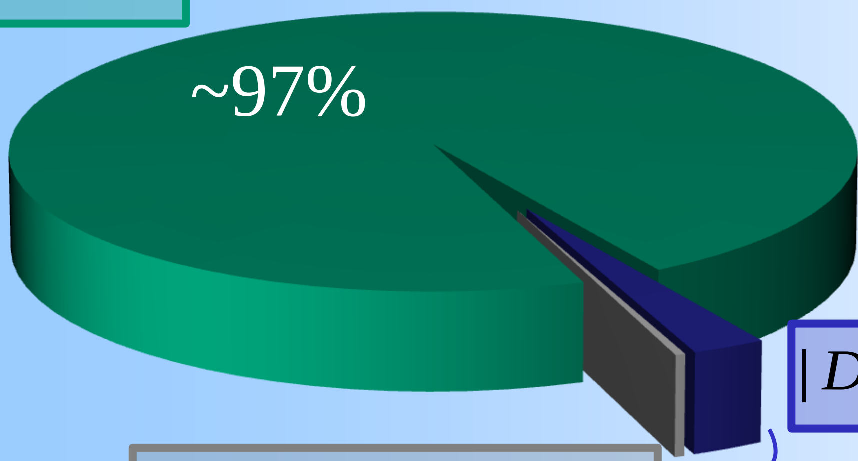
Mixed X(3872)

$$j_\mu^X(x) = \cos\alpha\, J_\mu^u(x) + \sin\alpha\, J_\mu^d(x)$$

$$J_\mu^q(x) = \sin\theta\, j_\mu^{(4q)}(x) + \cos\theta\, j_\mu^{(2q)}(x)$$

$$5^\circ \leq \theta \leq 13^\circ\,;\ \alpha \sim 20^\circ$$

$|c\,\bar{c}>$



$$m_X = (3{,}77 \pm 0{,}18)\ \mathrm{GeV}$$

$$\Gamma(X \to J/\psi\pi^+\pi^-\pi^0) = (9{,}3 \pm 6{,}9)\ \mathrm{MeV}$$

$|D^{*0}\,\bar{D}^0 + \bar{D}^{*0}\,D^0>$

$|D^{*+}\,\bar{D}^- + \bar{D}^{*-}\,D^+>$

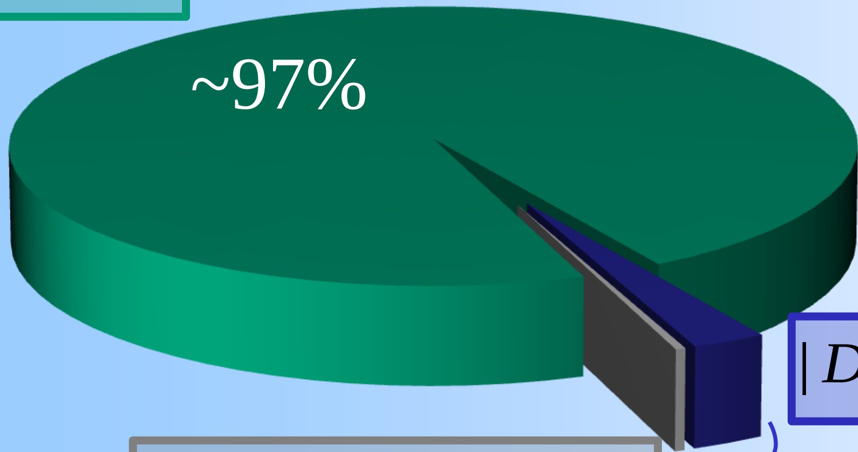
$(\sim 88\%\ |\underline{D^{*0}\,\bar{D}^0} + \ldots > +\ 12\%\,|\underline{D^{*+}\,\bar{D}^-} + \ldots >)$

Matheus, Navarra, Nielsen, Zanetti (2009)

Mixed X(3872)

$| c\, \bar{c} >$

$5^\circ \leq \theta \leq 13^\circ ;\ \alpha \sim 20^\circ$



$| D^{*0}\, \bar{D}^0 + \bar{D}^{*0}\, D^0 >$

$| D^{*+}\, \bar{D}^- + \bar{D}^{*-}\, D^+ >$

$(\sim 88\%\ | D^{*0}\, \bar{D}^0 + \ldots > + 12\% | D^{*+}\, \bar{D}^- + \ldots >)$

Nielsen, Zanetti (2010)

$$\frac{\Gamma(X \to J/\psi\ \gamma)}{\Gamma(X \to J/\psi\ \pi^+\pi^-)} = 0.19 \pm 0.13$$

Belle (2005)

$$\frac{\Gamma(X \to J/\psi\gamma)}{\Gamma(X \to J/\psi\,\pi^+\pi^-)} = 0.14 \pm 0.05$$

Mixed X(3872)

What about the production?

$$J_\mu^q(x) = \sin\theta\, j_\mu^{(4q)}(x) + \cos\theta\, j_\mu^{(2q)}(x)$$
$$j_\mu^X(x) = \cos\alpha\, J_\mu^u(x) + \sin\alpha\, J_\mu^d(x)$$
$$5^\circ \leq \theta \leq 13^\circ\, ;\, \alpha \sim 20^\circ$$

Zanetti, Nielsen, Matheus (2011)

$$\mathcal{B}(B \to X(3872)K) = (1.00 \pm 0.68) \times 10^{-5}$$

$$\theta = 0^\circ \Rightarrow \mathcal{B}(B \to X_{\bar{c}c}K) = (2.68 \pm 0.50) \times 10^{-5}$$

$$\theta = 90^\circ \Rightarrow \mathcal{B}(B \to X_{\mathrm{mol}}K) = (0.38 \pm 0.06) \times 10^{-6}$$

Babar

$$\mathcal{B}(B^\pm \to K^\pm X(3872)) < 3.2 \times 10^{-4}$$

Conclusion

• There is strong evidence of hadronic states which are neither usual mesons nor barions. Understanding what these are and why some structures are present in the spectrum and some are not might reveal new information about QCD

•With enough work, QCD Sum Rules can be quite discriminating

• What we should look for are interpolating current operators that CAN NOT describe the state. This is much more revealing than positive results.

• Widths are much more sensitive to changes in the operator

Scales in the correlator

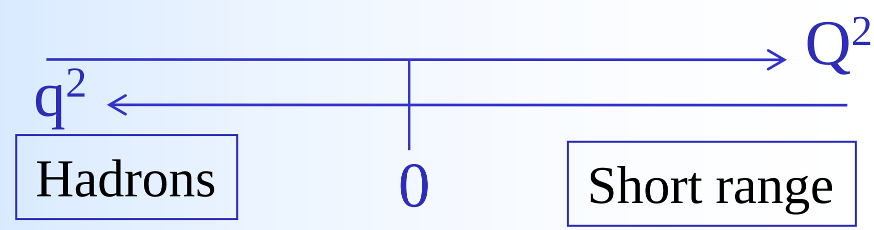
$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|\, T\{j_\mu(x)j_\nu(0)\}\,|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2)$$

$$\langle 0|T\{j_\mu(x)j^\mu(0)\}\,|\rangle = f(x^2) = \int d\tau\, e^{i\tau x^2} f(\tau)$$

$$3q^2\Pi(q^2) = -i\int d\tau \int d^4x\, e^{i\tau x^2} e^{iQ^2/4\tau} f(\tau)$$

$$\left.\begin{array}{l}\tau \sim 1/x^2 \\ \tau \sim Q^2\end{array}\right\} \Longrightarrow x^2 \sim 1/Q^2$$

$$\left.\begin{array}{l}q_0 = 0 \\ \vec{q}^{\,2} = Q^2\end{array}\right\} \Longrightarrow |\vec{x}| \sim 1/\sqrt{Q^2}$$



Quark condensate from OPE

$$i\int d^4x\, e^{iq\cdot x}\, T\{\bar\psi(x)\gamma_\mu\psi(x),\bar\psi(0)\gamma_\nu\psi(0)\} = (q_\mu q_\nu - q^2 g_{\mu\nu})\sum_d C_d(q^2) O_d$$

$$\Pi_{\mu\nu}^{(\bar\psi\psi)}(q) = i\int d^4x e^{iq\cdot x}\langle 0 \mid \{\bar\psi^i(x)\gamma_\mu S^{ij}(x,0)\gamma_\nu\psi^j(0) + \bar\psi^j(0)\gamma_\nu S^{ji}(0,x)\gamma_\mu\psi^i(x)\} \mid 0\rangle$$

$$\psi(x) = \psi(0) + x^\rho \overrightarrow{D}_\rho \psi(0) + ..$$

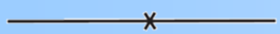
$$\bar\psi(x) = \bar\psi(0) + \bar\psi(0) \overleftarrow{D}_\rho x^\rho + ..$$

$$\langle 0 \mid \bar\psi^i_\alpha \psi^j_\beta \mid 0\rangle = A\delta^{ij}\delta_{\alpha\beta} \Longrightarrow A = \frac{1}{12}\langle 0 \mid \bar\psi\psi \mid 0\rangle$$

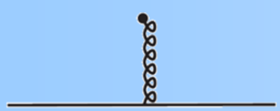
$$C_3(q^2) = \frac{2m}{q^4} \Longrightarrow \Pi_{\mu\nu}^{(\bar\psi\psi)}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu})\frac{2m}{q^4}\langle 0 \mid \bar\psi\psi \mid 0\rangle$$

OPE corrections to the propagator

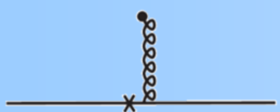
$\delta_{ab}\frac{i}{2\pi^2x^4}\not{x}$



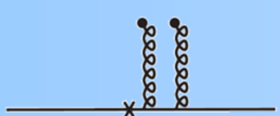
$-\delta_{ab}\frac{m_q}{4\pi^2x^2}$



$-\tau^A_{ab}\frac{i}{32\pi^2x^2}g_sG^A_{\mu\nu}(0)\,(\not{x}\,\sigma^{\mu\nu}+\sigma^{\mu\nu}\not{x}\,)$



$-\tau^A_{ab}\frac{m_q}{32\pi^2}g_sG^A_{\mu\nu}(0)\sigma^{\mu\nu}ln(-x^2)$

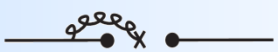


$-\delta_{ab}\frac{m_q\langle g^2G^2\rangle}{2^93\pi^2}x^2ln(-x^2)$

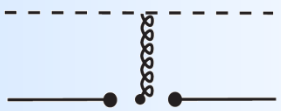
$-\delta_{ab}\frac{\langle\bar{q}q\rangle}{12}$

$\delta_{ab}\frac{im_q\langle\bar{q}q\rangle}{48}\not{x}$

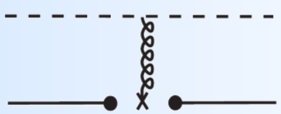
$-\delta_{ab}\frac{x^2}{2^63}\langle\bar{q}g\sigma.Gq\rangle$



$\delta_{ab}\frac{ix^2m_q}{2^73^2}\langle\bar{q}g\sigma.Gq\rangle\not{x}$



$-\tau^A_{ab}\frac{1}{2^63}\langle\bar{q}g\sigma.Gq\rangle\sigma_{\mu\nu}$



$\tau^A_{ab}\frac{i}{2^83}m_q\langle\bar{q}g\sigma.Gq\rangle(\not{x}\,\sigma_{\mu\nu}+\sigma_{\mu\nu}\not{x}\,)$

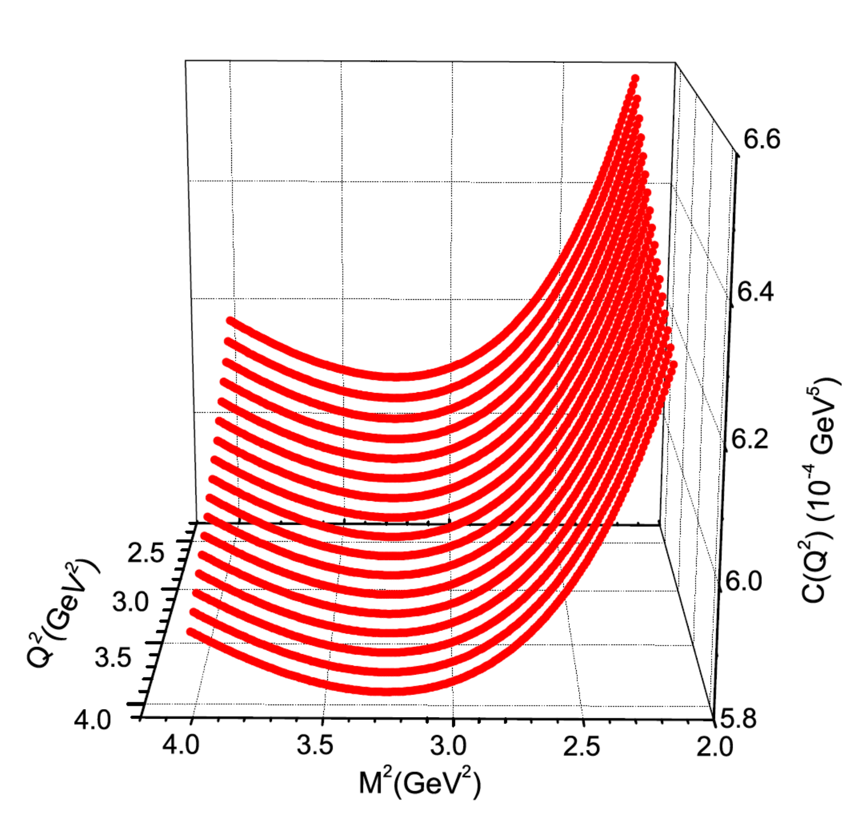
Borel Transformation

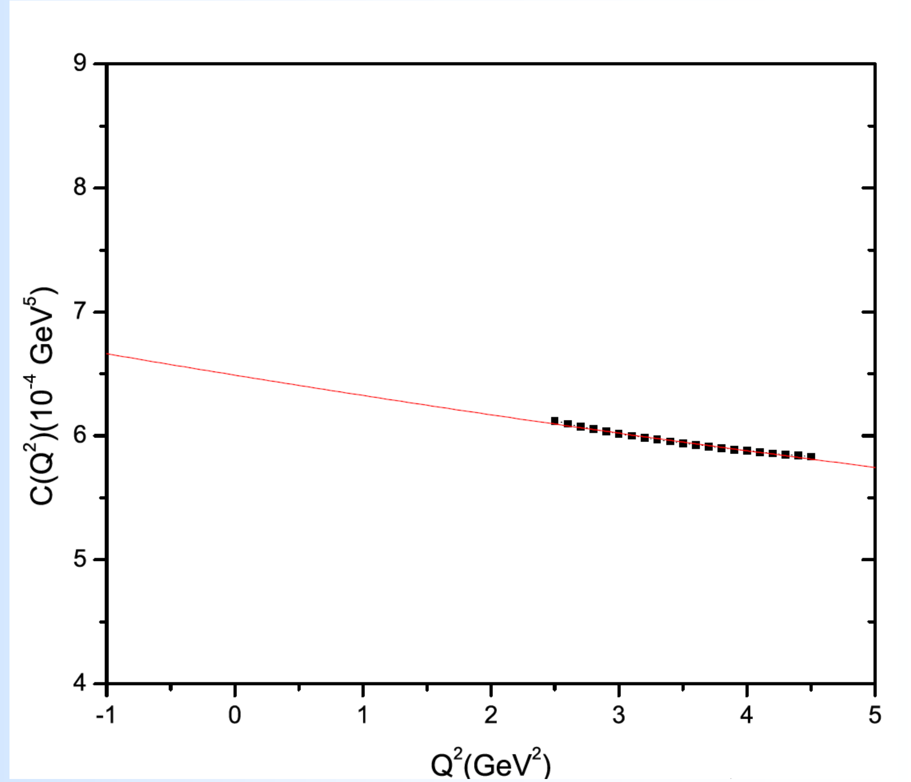
$$\beta[f(Q^2)] = \hat{f}(M^2) = lim_{Q^2,n\to\infty}^{Q^2/n\to M} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{\partial}{\partial Q^2}\right)^n f(Q^2)$$

$$\begin{cases} \beta\left[\frac{1}{(Q^2)^k}\right] = \frac{1}{(k-1)!}\left(\frac{1}{M^2}\right)^{k-1} \\ \beta\left[\frac{1}{s+Q^2}\right] = e^{-s/M^2} \end{cases}$$

OPE: k[th] term goes like: $1/(q^2)^k$

Phen.: $\Pi(Q^2) = \sum_{j=0}^{\infty} K_j \frac{1}{Q^2+m_j^2} \Longrightarrow \beta\left[\Pi(Q^2)\right] = \sum_j K_j e^{-m_j/M^2}$





$$C(Q^2)\left(e^{-m_\psi^2/M^2} - e^{-m_X^2/M^2}\right) + B\, e^{-s_0/M^2} = (Q^2 + m_\omega^2)\Pi^{(OPE)}(M^2, Q^2)$$

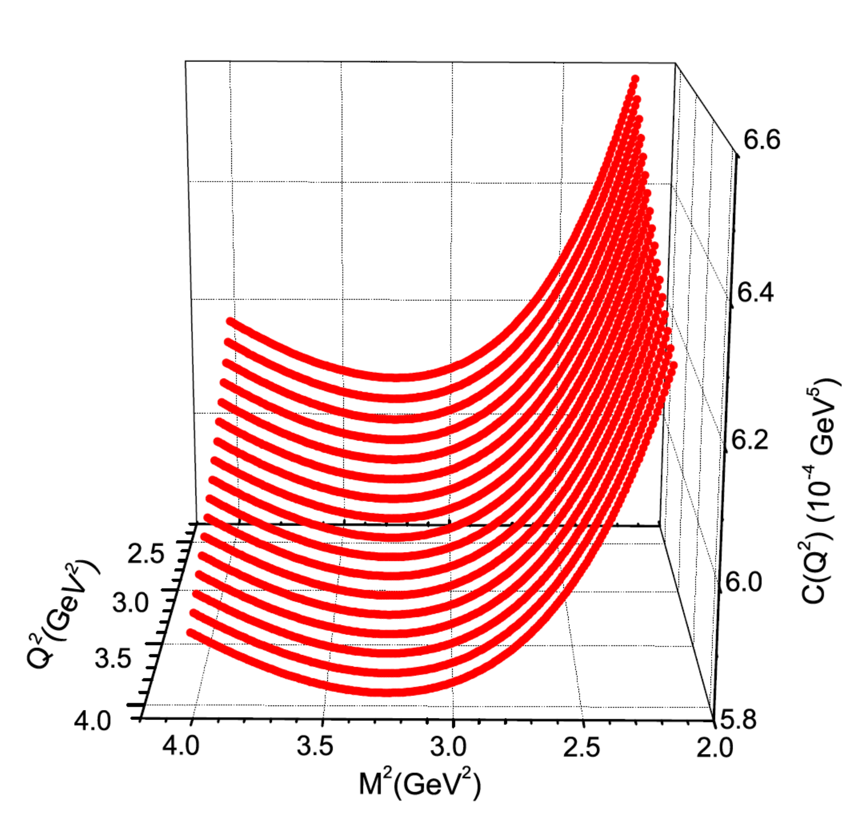
$$\epsilon^{\alpha\nu\sigma\gamma} p'_\sigma q_\gamma p'_\mu \qquad p^2 = p'^2 = -P^2$$

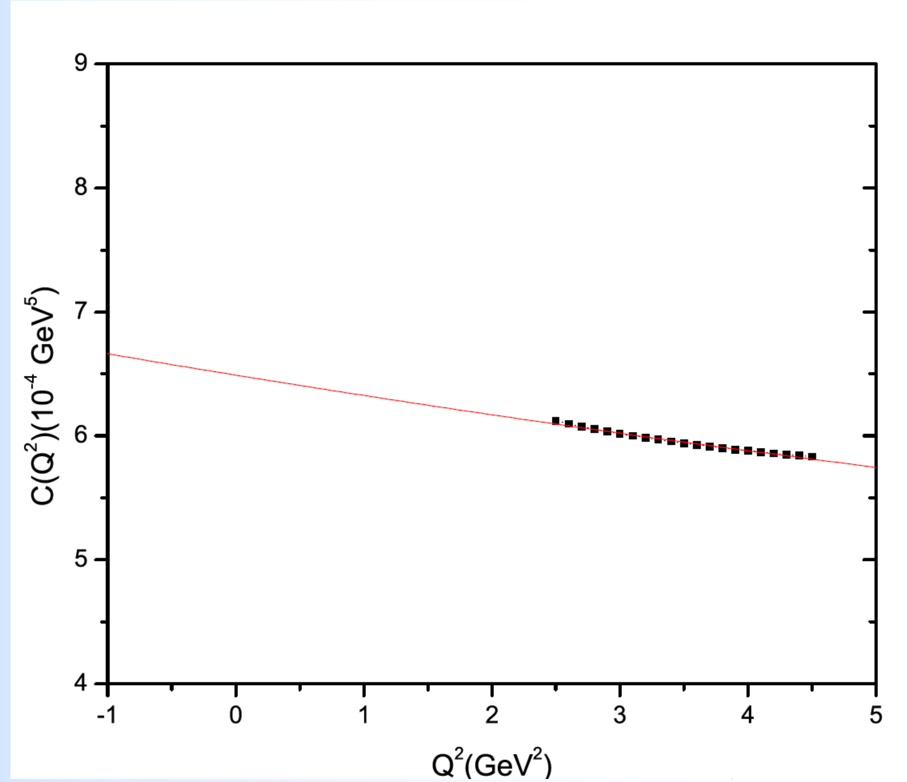
$$P^2 \to M^2,$$

$$C(Q^2) = \frac{6}{\sin(\theta)} m_\omega f_\omega \frac{f_\psi \lambda^q}{m_\psi(m_X^2 - m_\psi^2)} g_{X\psi\omega}(Q^2)$$

$$C(Q^2) = \frac{c_1}{Q^2 + c_2}$$

$$c_1 = 2.5 \times 10^{-2}\ \mathrm{GeV}^7,$$
$$c_2 = 38\ \mathrm{GeV}^2,$$





$$g_{X\psi\omega} = g_{X\psi\omega}(-m_\omega^2) = 5.4 \pm 2.4$$

$$\Gamma\left(X \to J/\psi\pi^+\pi^-\pi^0\right) = g_{X\psi\omega}^2 \frac{m_\omega \Gamma_\omega}{8\pi^2 m_X^2} B_{\omega\to\pi\pi\pi} I_\omega$$

$$\Gamma\left(X \to J/\psi\pi^+\pi^-\pi^0\right) = (9.3 \pm 6.9) \quad \text{MeV}$$