Generating the electroweak scale from cosmological evolution

Ricardo D'Elia Matheus

IFT - Instituto de Física Teórica - UNESP

The Hierarchy Problem

Think about the Standard Model (SM) as an EFT with a cut-off at M_p:

$$V(H) = m_H^2(\alpha,\beta)H^2 + \lambda h^4 + \mathcal{O}(1/M_p^2)$$

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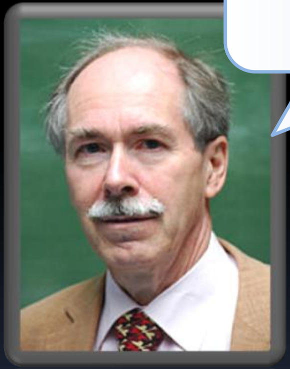

Technical Naturalness

- All dimensionless Wilson coefficients should be of order one.

The only mass scale is M_p!

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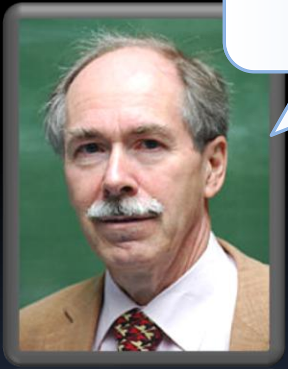


Technical Naturalness

$$m_H^2 \equiv \rho\, M_P^2$$

⇨ All dimensionless Wilson coefficients should be of order one.

$$Dim[\rho] = Dim[\lambda] = 0$$

$$\rho \approx \lambda \approx 1$$

Important exception: taking a coefficient to zero increases symmetry. In that case it can be arbitrarily small.

The Hierarchy Problem

Think about the Standard Mo … a cut-off at M_p:

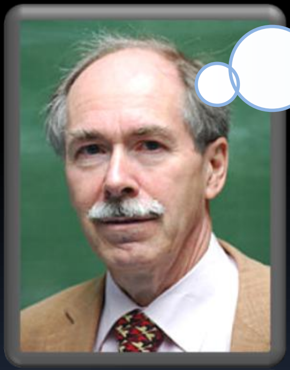
$V(H) = m_H^2$ … $H\rangle = v$

The only ma … fficients

Think about quark masses in QCD!

Each one can be as small as you want because you are always getting a new chiral symmetry

‘t Hooft



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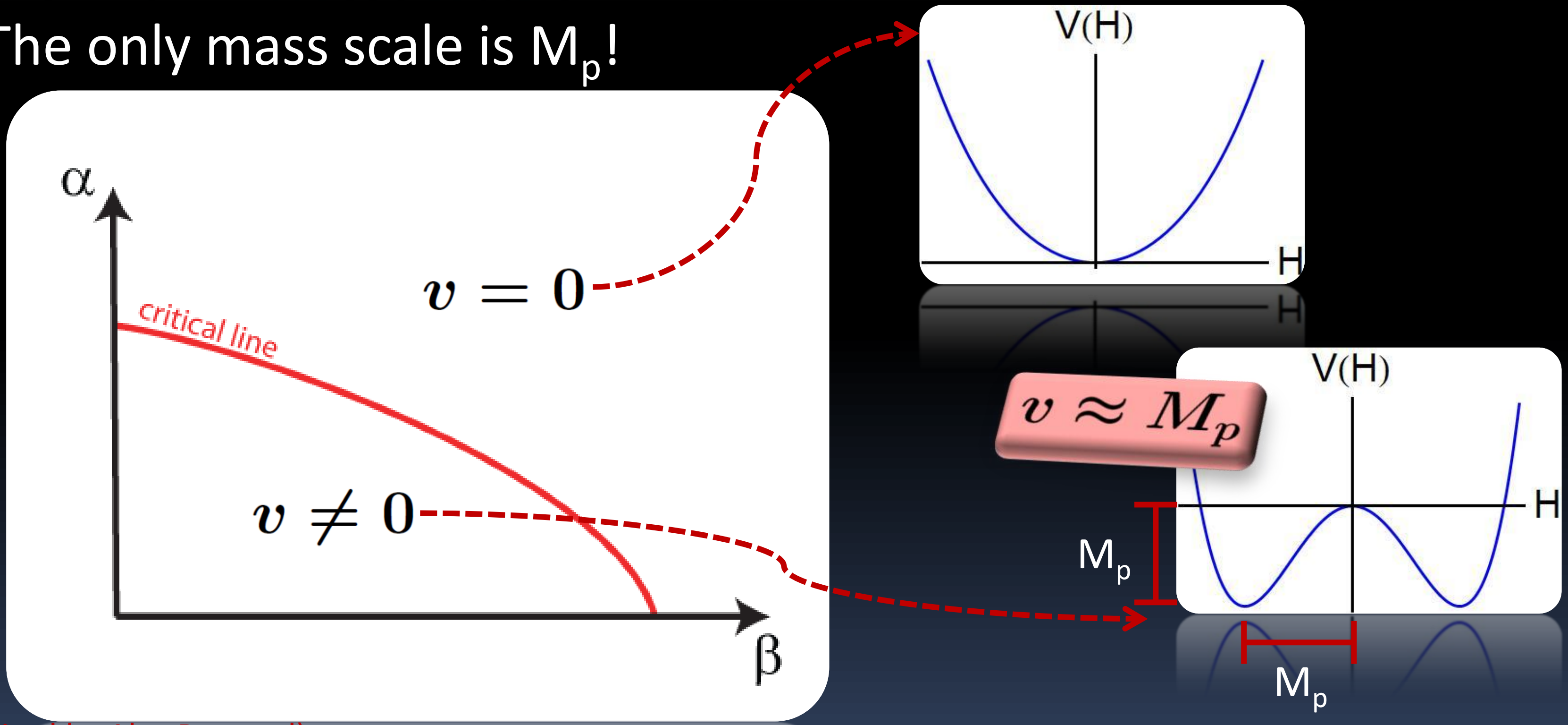

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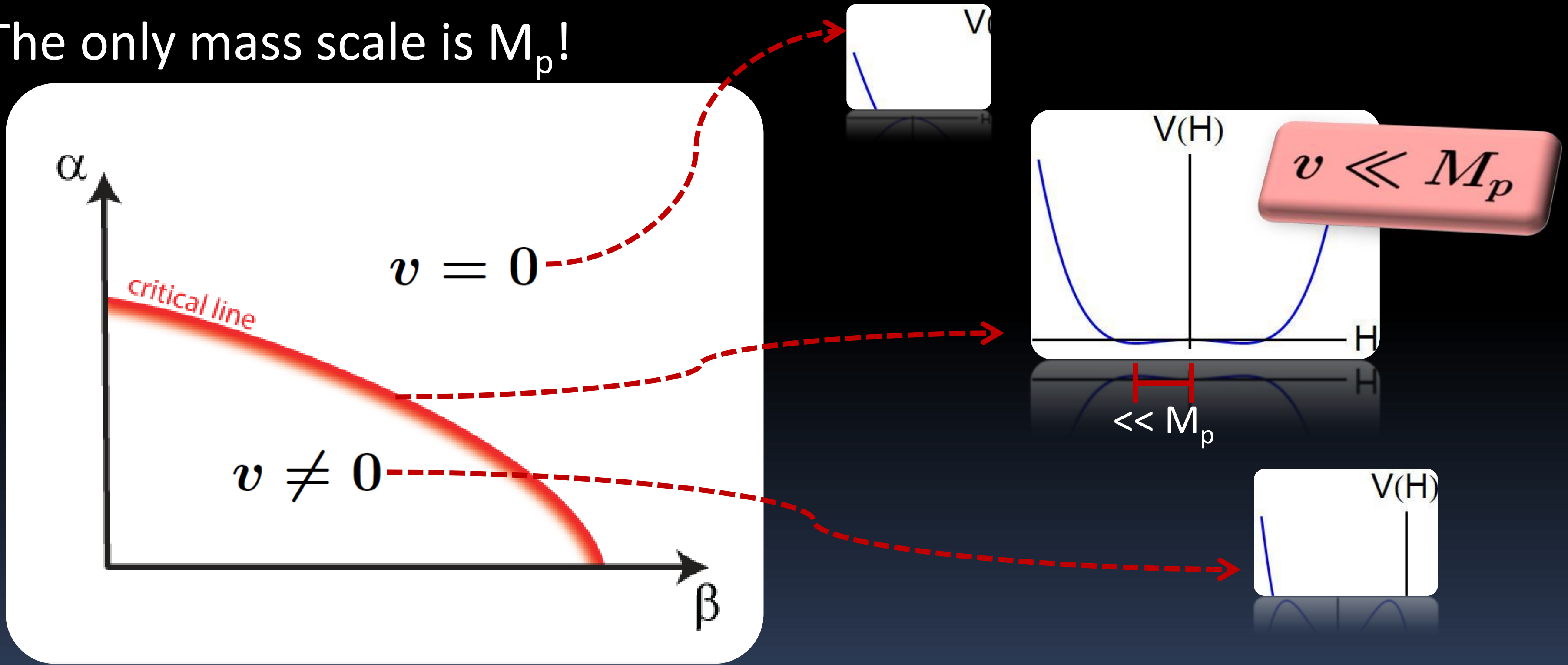

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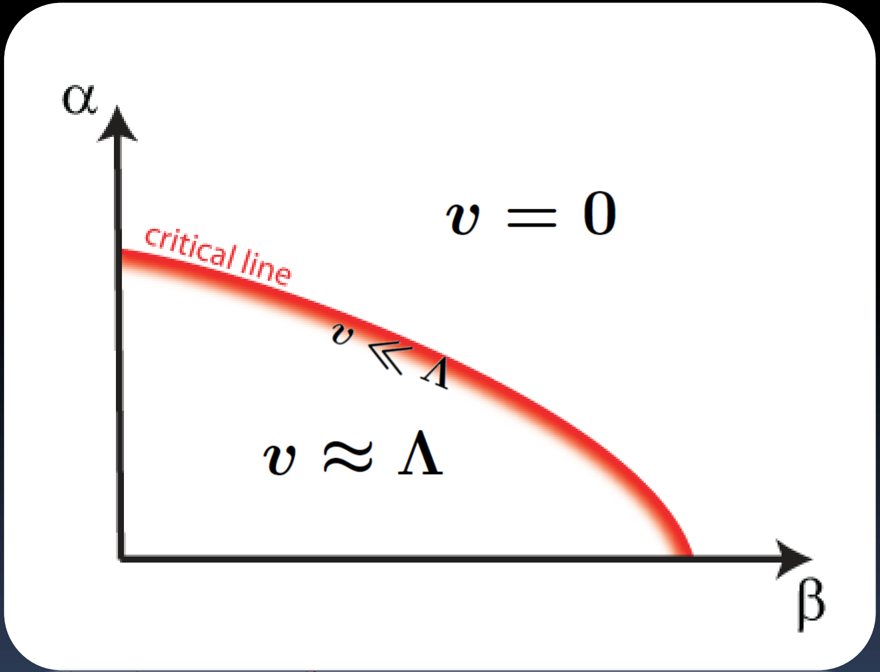
Solving the Hierarchy Problem

Question: how come we live so close to the line?



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Two answers: (1) Some symmetry forces it! (SUSY)

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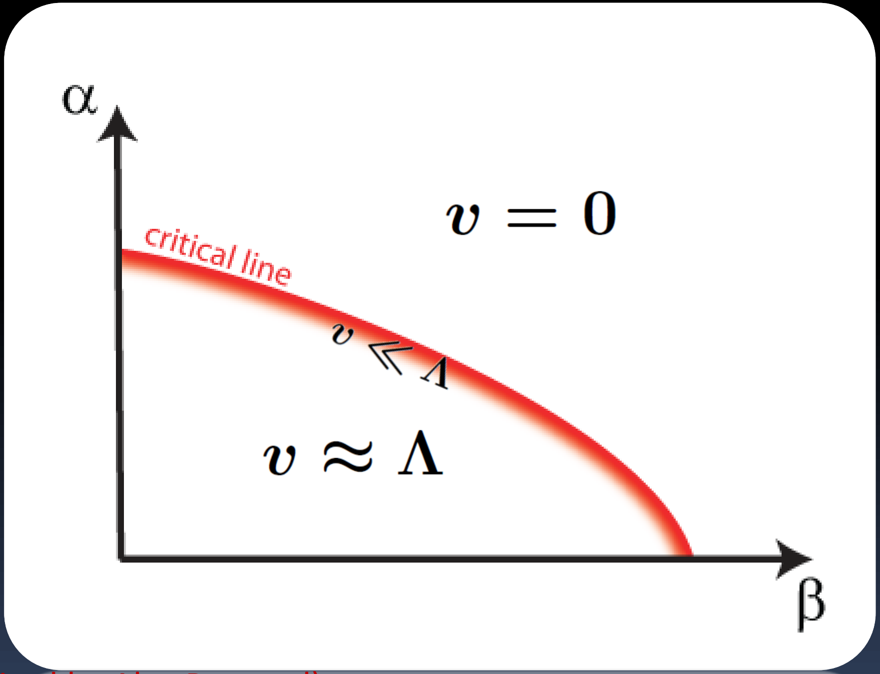
(Composite Models, Extra Dimensions et al.)



Both DEMAND new physics @ ~TeV

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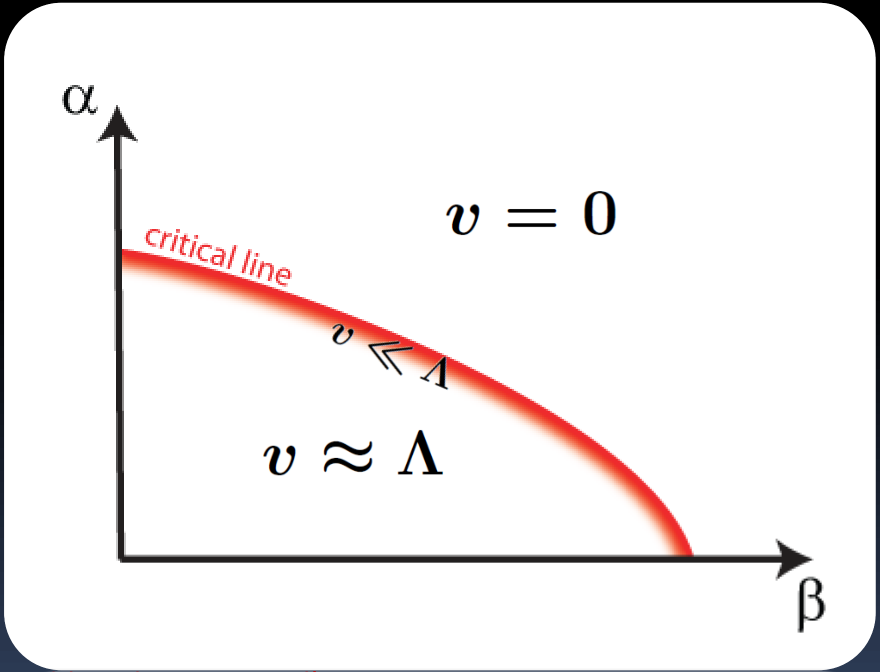

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WHERE IS THE F**KING
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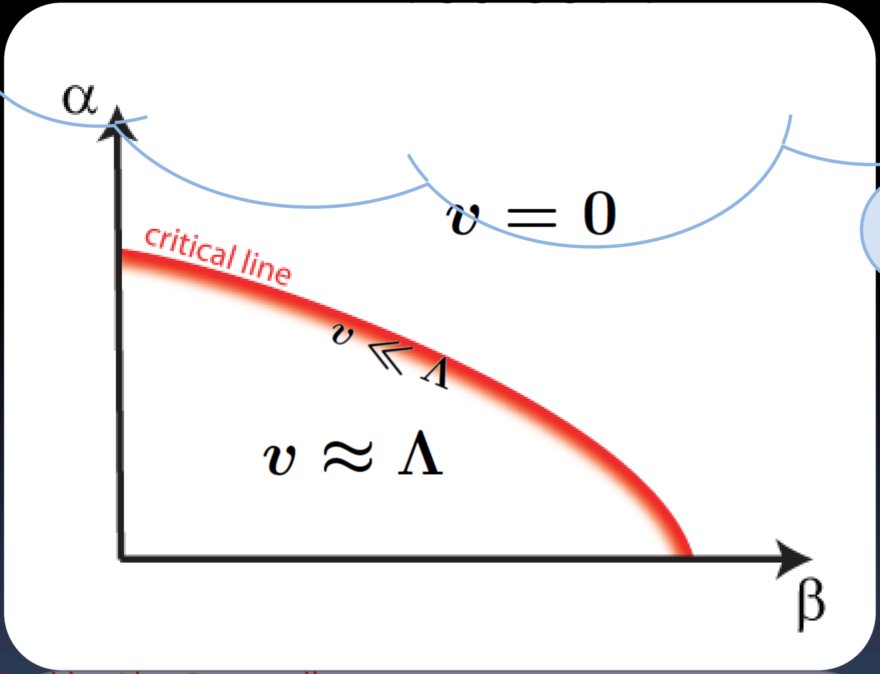

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A New Hope

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The Third Way: (3) History! Make α and β dynamical (fields in fact)

(stupid) Example: $m_H^2(\alpha,\beta)H^2 \rightarrow \alpha\beta H^2$ $m_H^2 = \langle\alpha\rangle\langle\beta\rangle$

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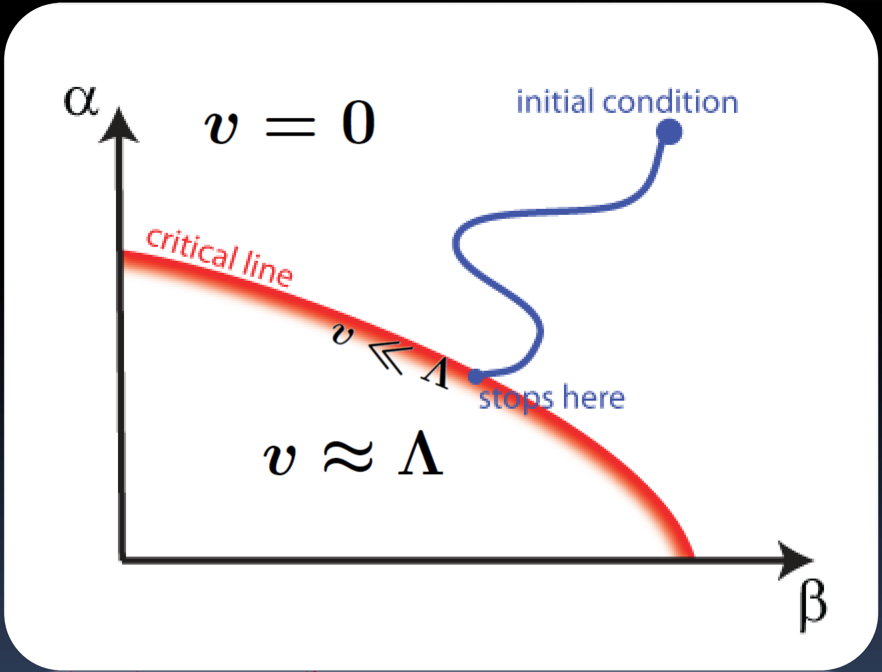

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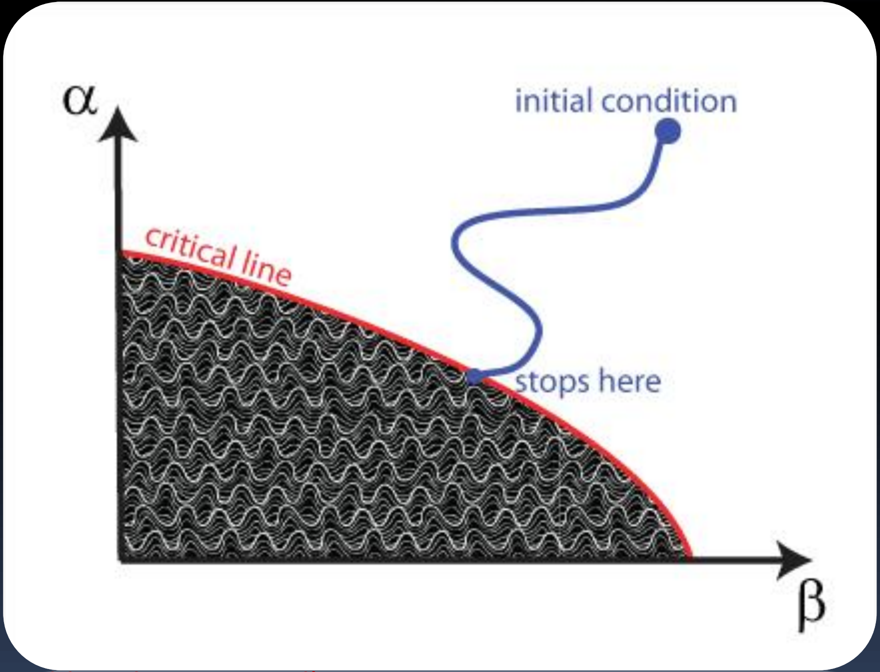


But how does the evolution stop?

Local Minima! A whole LOT OF local minima!

Can it be done in a (technically) natural way?

(spoiler: yes! But...)

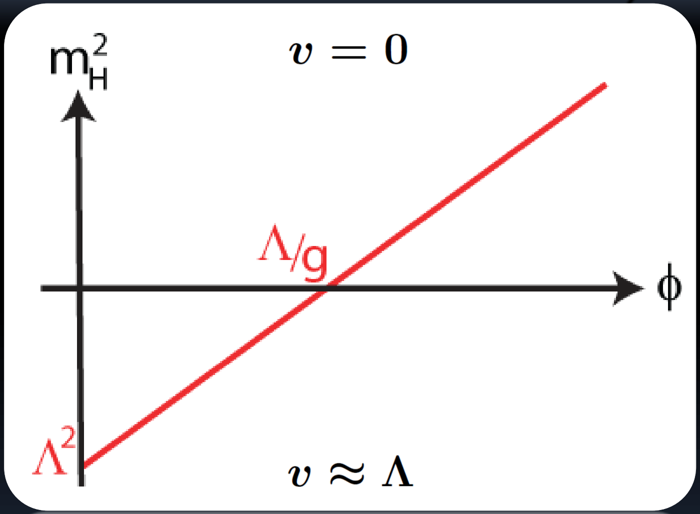
(inspired by Alex Pomarol)

Graham, Kaplan, Rajendran
arXiv:1504.07551

The Relaxion

Introduce one scalar field ϕ, and:

$$m_H^2 \to m_H^2(\phi) = -\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)$$



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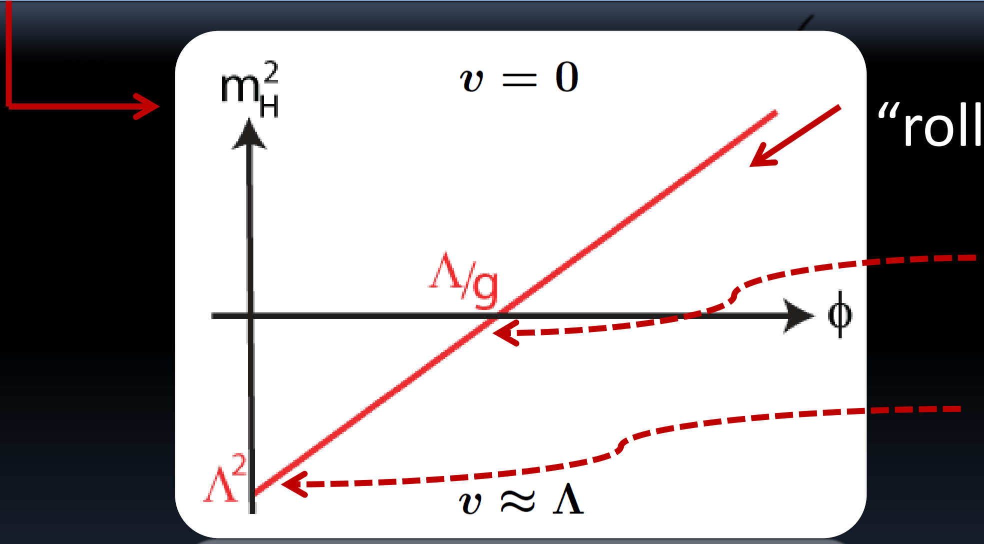

“rolls” down

Must stop here…

… not here

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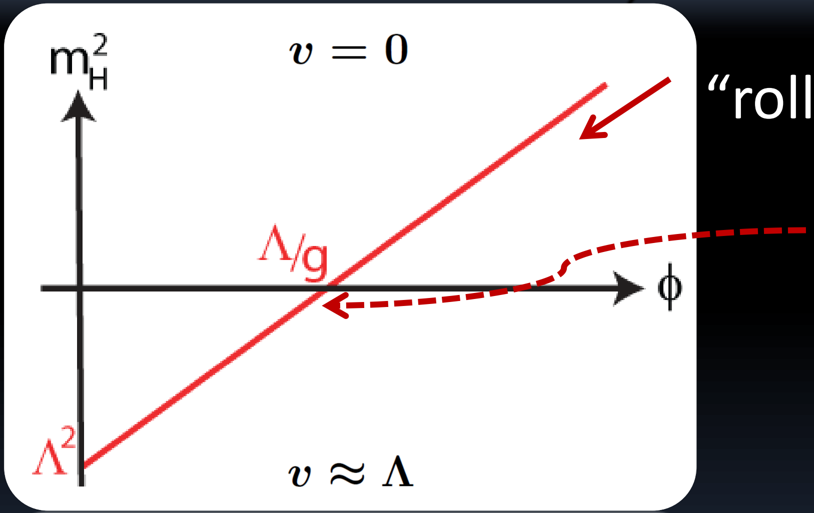

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Must stop here...

Large Field Excursions!

$\phi_c \equiv \Lambda/g$ If $g \ll 1$ → $\phi \approx \Lambda/g \gg \Lambda$

Espinosa et al.
arXiv: 1506.09217

The Relaxion

The minimal model:

$$V(\phi, H) = \underbrace{\Lambda^3 g\phi}_{\text{Linear slope for } \phi} \underbrace{- \frac{1}{2}\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)}_{\frac{1}{2} m_H^2} H^2 + \underbrace{\epsilon\Lambda_c^2 H^2 \cos(\phi/f)}_{\text{Local minima in } \phi}$$

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Both *g* and ε break shift symmetries (more about that later) and can be naturally small !

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Λ is the cut-off for the SM

Λ_c is the scale at which the periodic potential is generated

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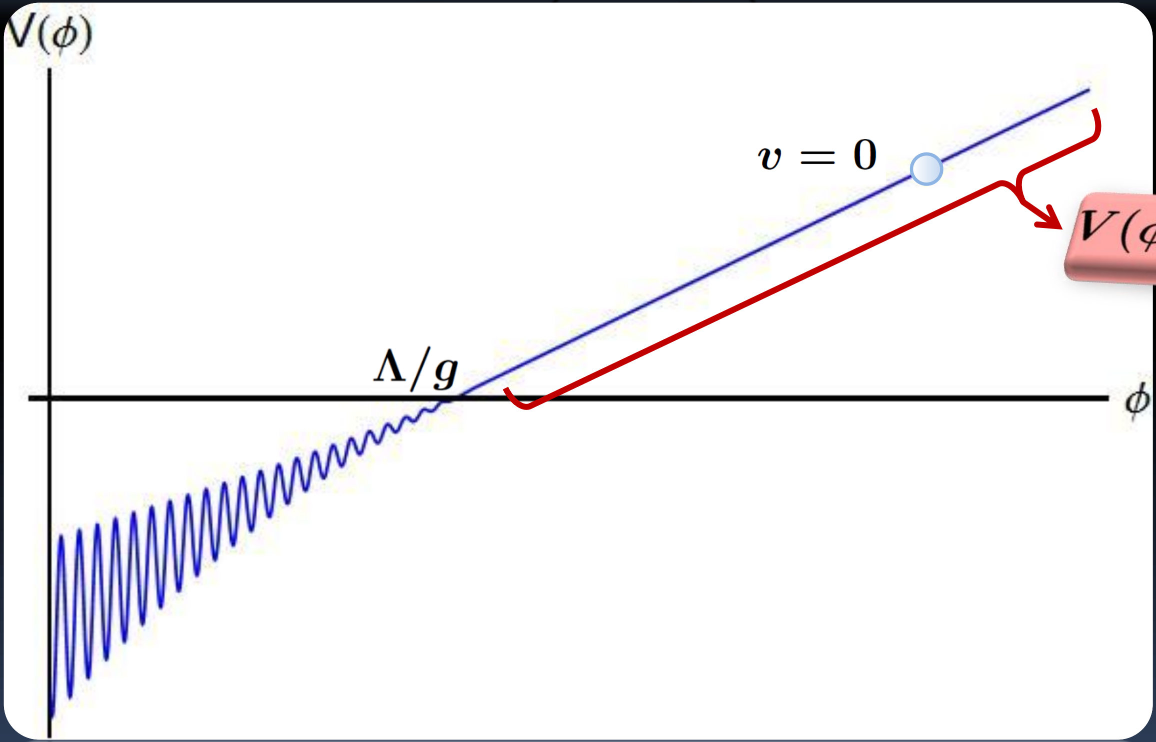

$$V(\phi, H = 0) = \Lambda^3 g\phi$$

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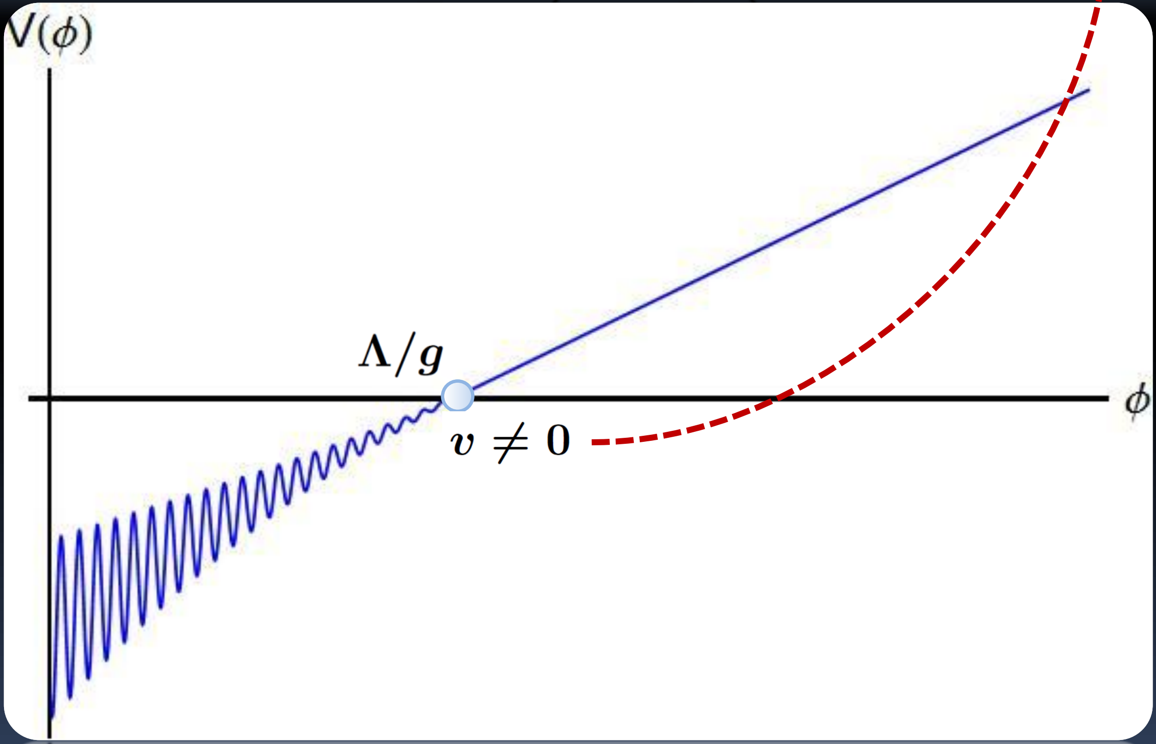

Becomes more important as *v* grows

Espinosa et al.
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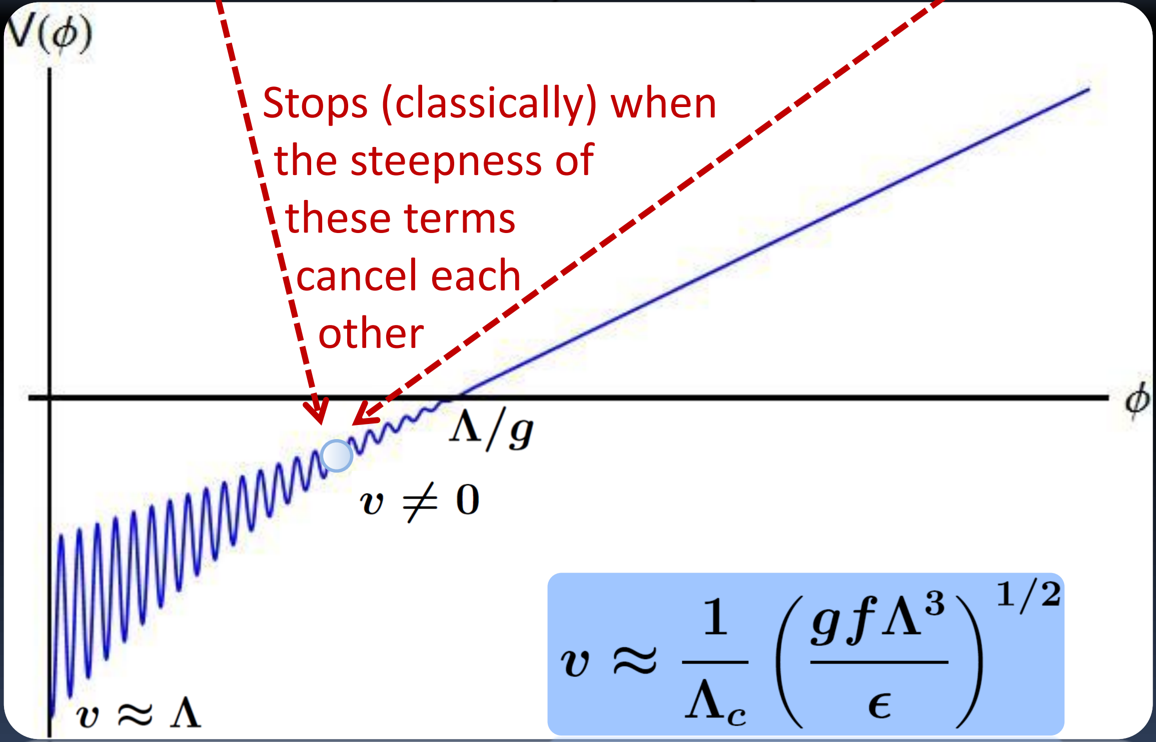

Stops (classically) when the steepness of these terms cancel each other

$$v \approx \frac{1}{\Lambda_c}\left(\frac{gf\Lambda^3}{\epsilon}\right)^{1/2}$$

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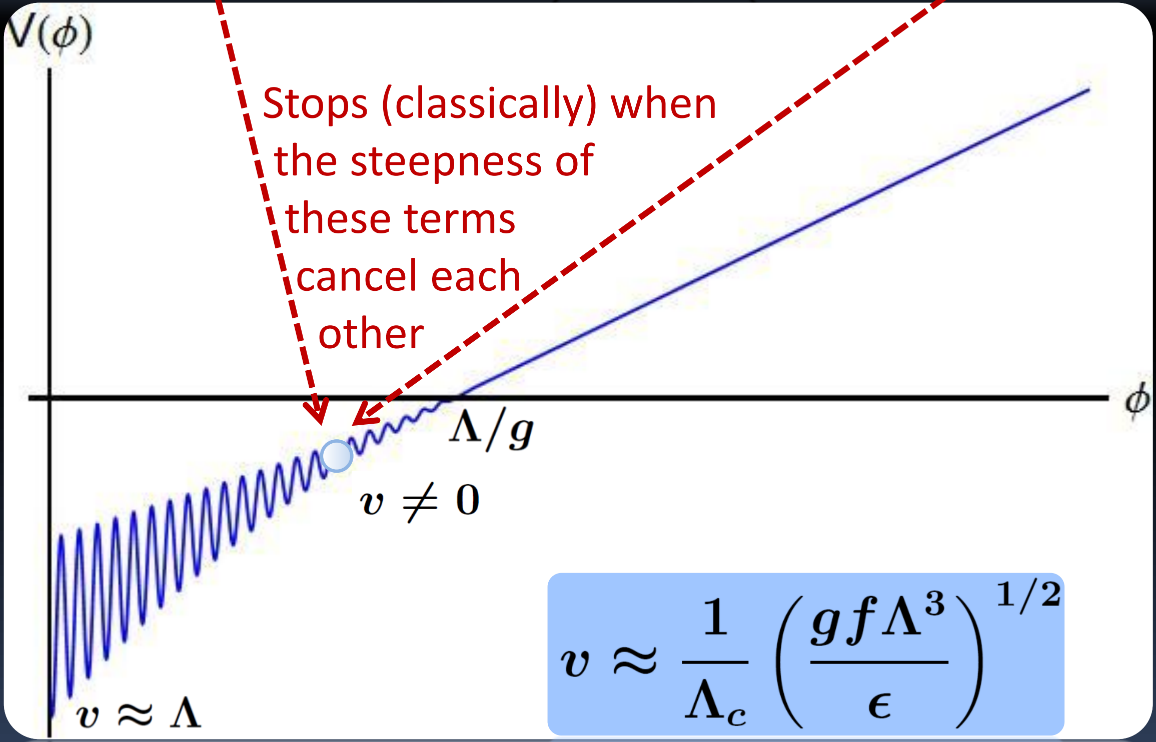

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The overall slope is controlled by *g*.

$$v \ll \Lambda$$

⇕

$$g \ll 1$$

Technically Natural!

NO NEW PHYSICS

close to v

Espinosa et al.
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$$V(\phi, H) \sim m_u(H)\langle q\bar{q}\rangle \cos(\phi/f)$$

$$\Lambda_c = \Lambda_{QCD} \quad \epsilon = Y_u$$

Does not work, mainly because Λ_c too low leads to low Λ, also: $\theta_{QCD} \sim 1$

Λ/g

$v \neq 0$

The overall slope is controlled by *g*.

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⇕

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Espinosa et al.
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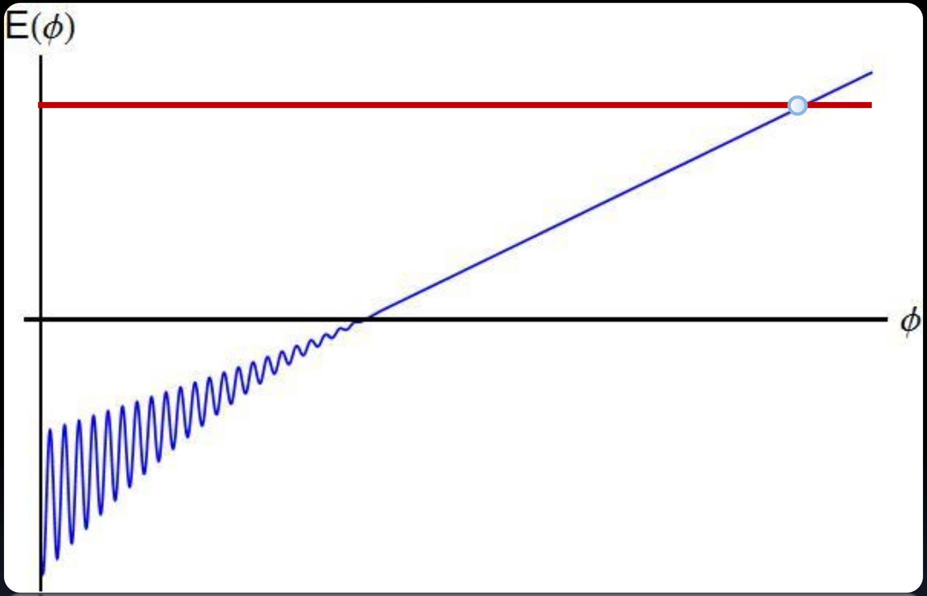
So far, so good. Now on to the little dirty details:

Do we risk overshooting? Do we need to start close to ϕ_c?



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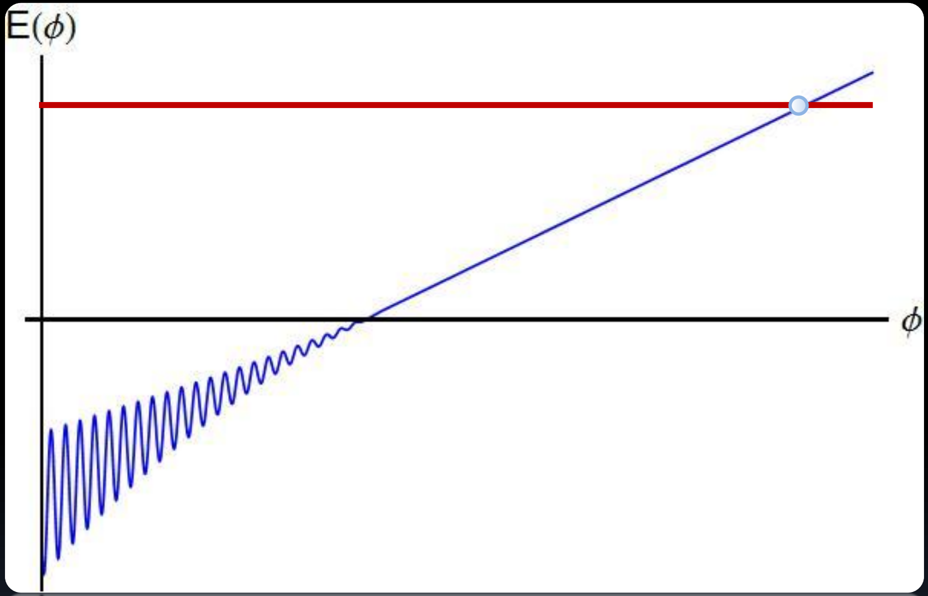

NO, if slow rolling (during an inflationary epoch). Inflation introduces Hubble friction:

$$\cancel{\ddot{\phi}} + 3H_I\dot{\phi} = -\partial_\phi V(\phi)$$

(ϕ is not the inflaton)

Consequence: homework for cosmologists as a long period of inflation is needed ($N_e \sim 10^{40}$, smaller if H_I is not constant – Patil, Schwaller, arXiv:1507.08649)

(Optional approach: no inflation, temperature dependent potential. E. Hardy, arXiv:1507.07525)

Espinosa et al.
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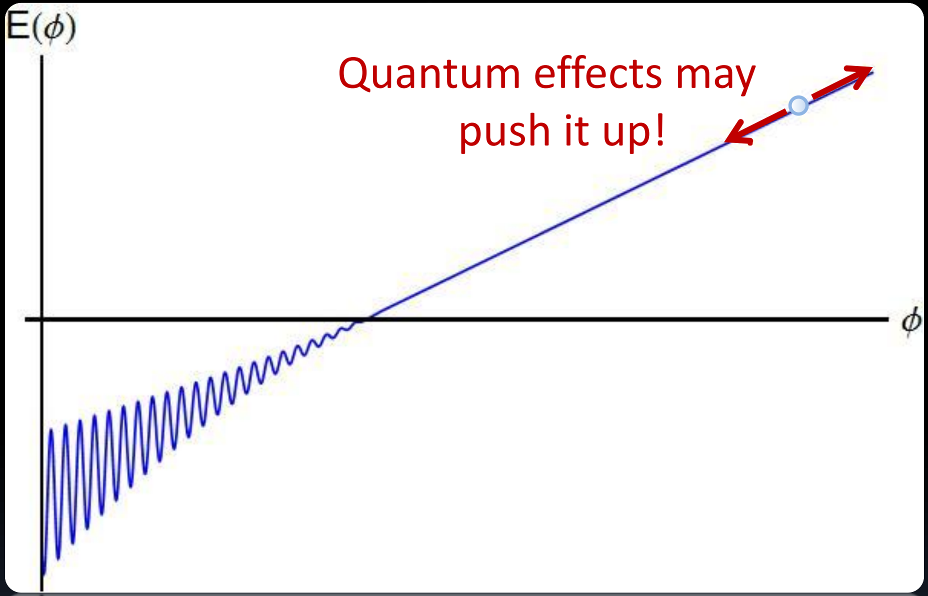
Limitations: Inflation ⇨ de Sitter space ⇨ Temperature (from Horizon)



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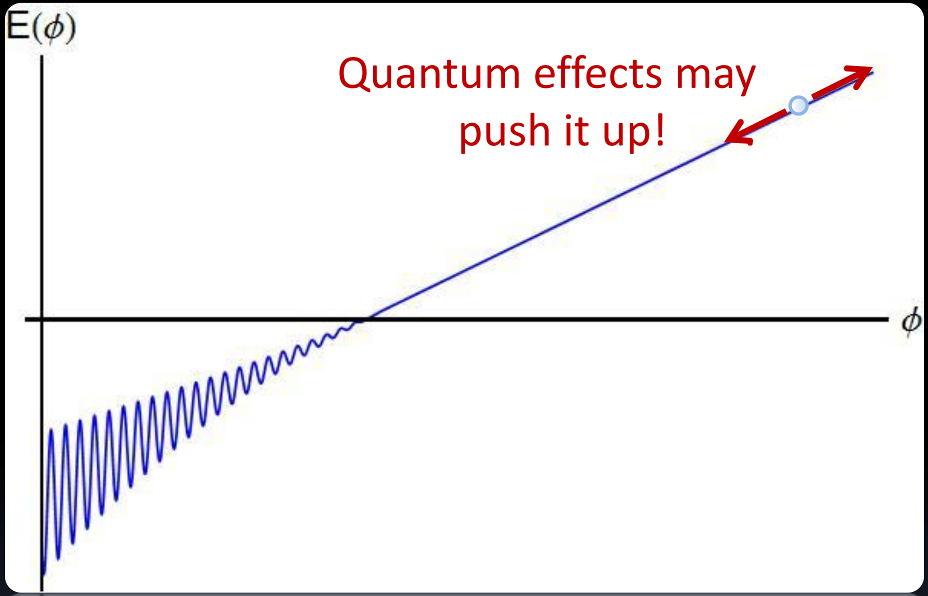

Quantum effects may push it up!

$$\Delta\phi_{\rm class} \sim \frac{V'(\phi)}{H_I^2} = \frac{g\Lambda^3}{H_I^2}$$

$$\Delta\phi_{\rm quant} \sim H_I$$

$$\Delta\phi_{\rm class} > \Delta\phi_{\rm quant}$$

$$g > (H_I/\Lambda)^3$$

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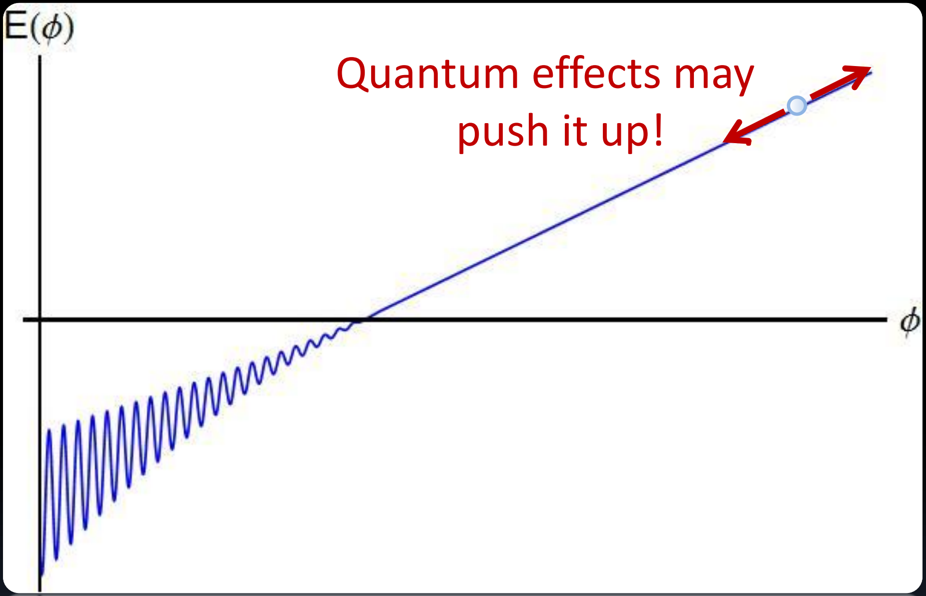

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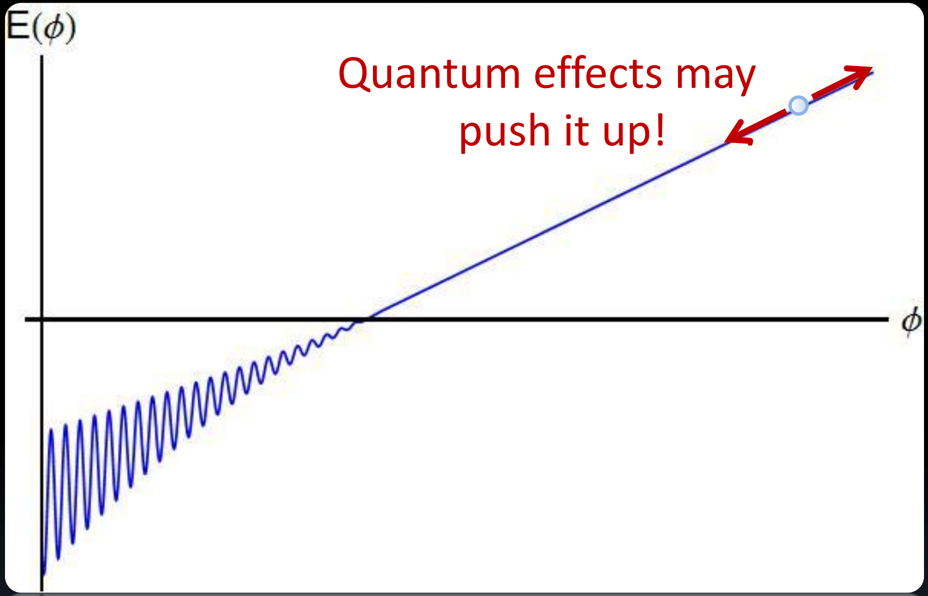

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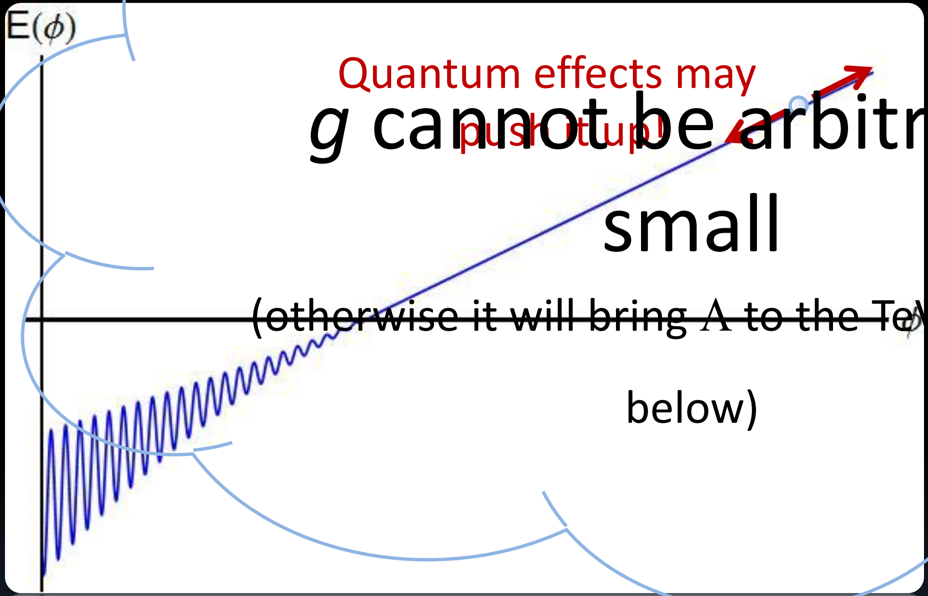

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g cannot be arbitrarily small

(otherwise it will bring Λ to the TeV scale and below)

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Espinosa et al.
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Is this potential “all it can be”?

$$V(\phi, H) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right) H^2 + \epsilon\Lambda_c^2 H^2 \cos(\phi/f)$$

Even with *g*, ε << 1 we still have to guarantee the potential is radiatively stable. Similar question to: have I included all terms allowed by symmetry?

Espinosa et al.
arXiv: 1506.09217

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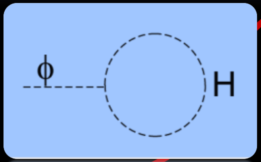

$$g\frac{\Lambda^3}{16\pi^2}\phi$$

Small correction to first term

Espinosa et al.
arXiv: 1506.09217

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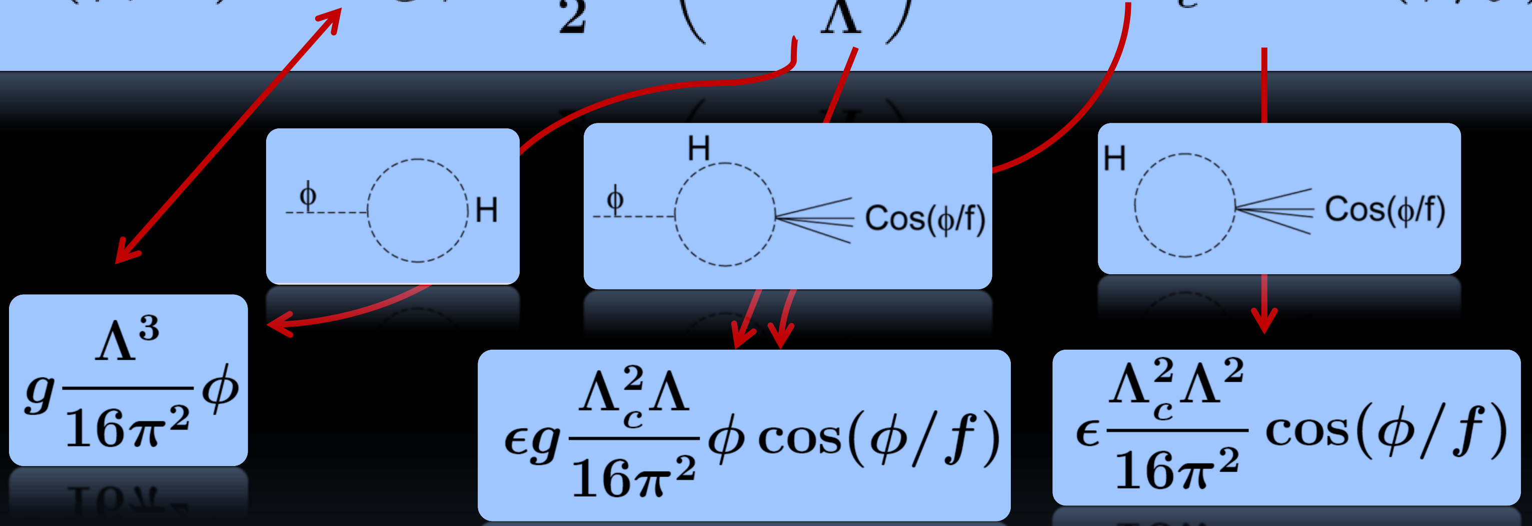

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DANGER! Local minima everywhere, even when *v = 0*.

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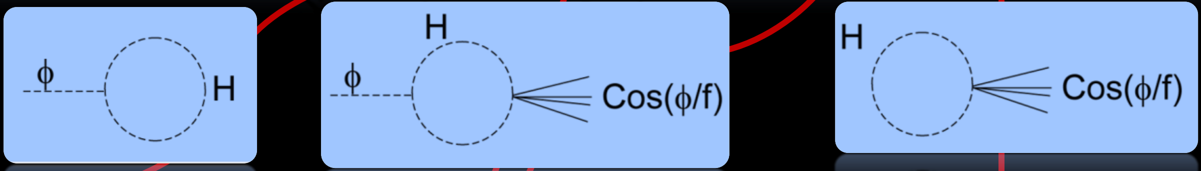

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Also: $g^n\epsilon^m\Lambda^{4-m-2m}\Lambda_c^{2m}\phi^n\cos^m(\phi/f)\left(1+\frac{1}{2}\frac{H^2}{\Lambda^2}+\ldots\right)$

Espinosa et al.
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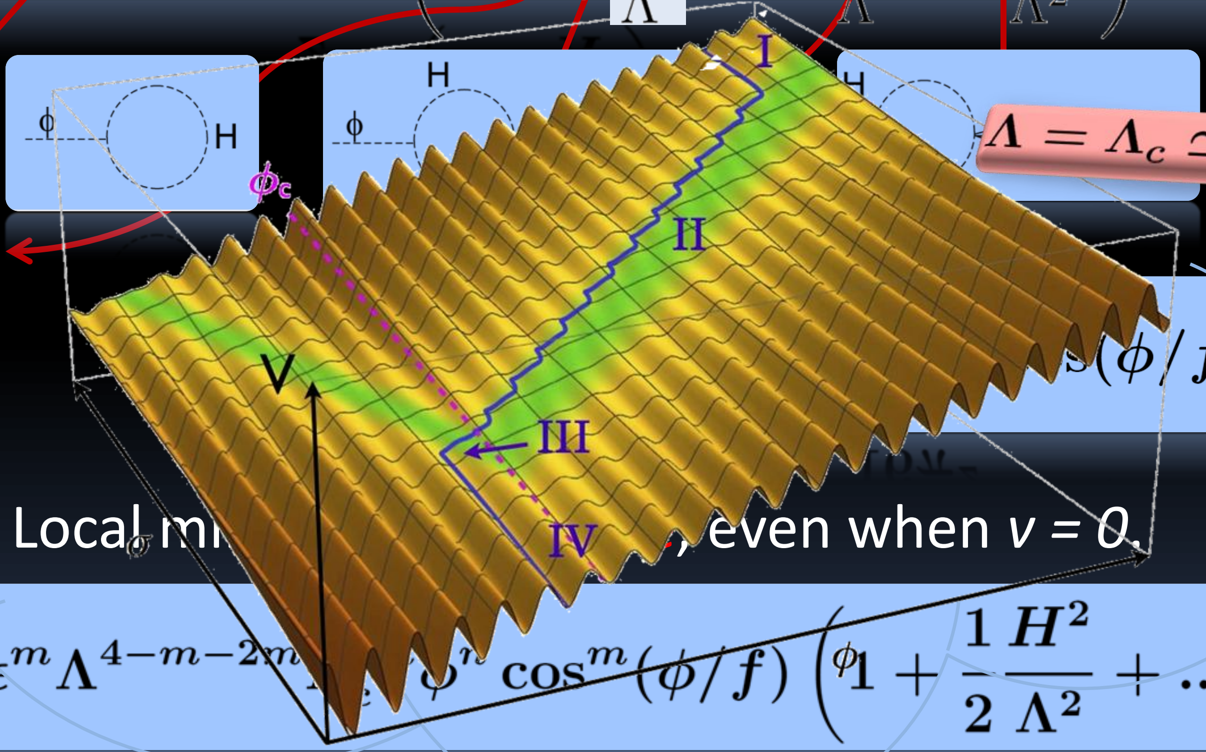
The Relaxion

Is this potential "all it can be"?

Double scanner mechanism: $A\cos(\phi/f)$

$$V(\phi,H)=\Lambda^3 g\phi-\frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)H^2+\epsilon\Lambda^2H^2\cos(\phi/f)\quad+\ldots$$

$$A(\phi,\sigma,H)\equiv\epsilon\Lambda^4\left(\beta+c_\phi\frac{g\phi}{\Lambda}-c_\sigma\frac{g_\sigma\sigma}{\Lambda}+\frac{|H|^2}{\Lambda^2}\right)$$



$$g\frac{\Lambda^3}{16\pi^2}\phi$$

$$\Lambda=\Lambda_c\simeq10^9GeV$$

(ϕ/f)

DANGER! Local mi , even when *v = 0*.

Also: $g^n\epsilon^m\Lambda^{4-m-2m}\ \phi^n\cos^m(\phi/f)\left(1+\frac{1}{2}\frac{H^2}{\Lambda^2}+\ldots\right)$

Symmetries

What are the symmetries involved? Is there a UV completion to this thing?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2 - \epsilon\Lambda_c^2 H^2\cos(\phi/f)$$

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$g = 0$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \epsilon\Lambda_c^2 H^2\cos(\phi/f)$$

↳ symmetric under $\phi \rightarrow \phi + 2n\pi f$

Discrete shift symmetry

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symmetric under $\phi \rightarrow \phi + 2n\pi f$

Discrete shift symmetry

$\epsilon = 0$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

symmetric under $\phi \rightarrow \phi + c, \forall c$

Continuous shift symmetry

Naturalness → $g \ll \epsilon$

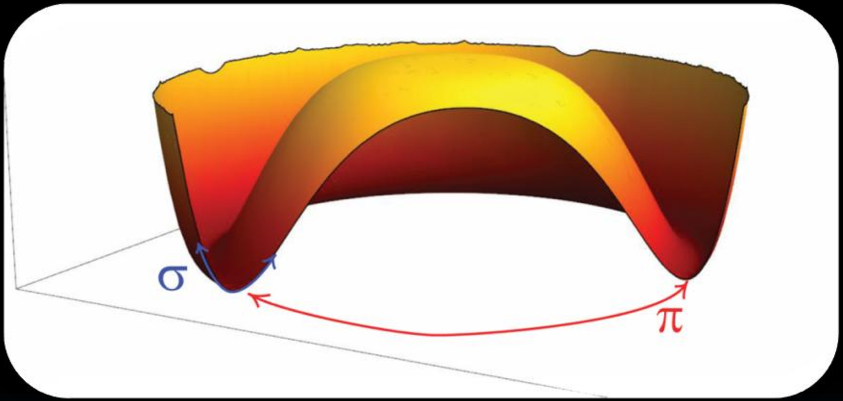
Symmetries

Shift symmetries eh? Where do we normally find those?

Symmetries

Shift symmetries eh? Where do we normally find those?

Spontaneous breaking of a Global Symmetry



Nambu-Goldstone Bosons (NGB)

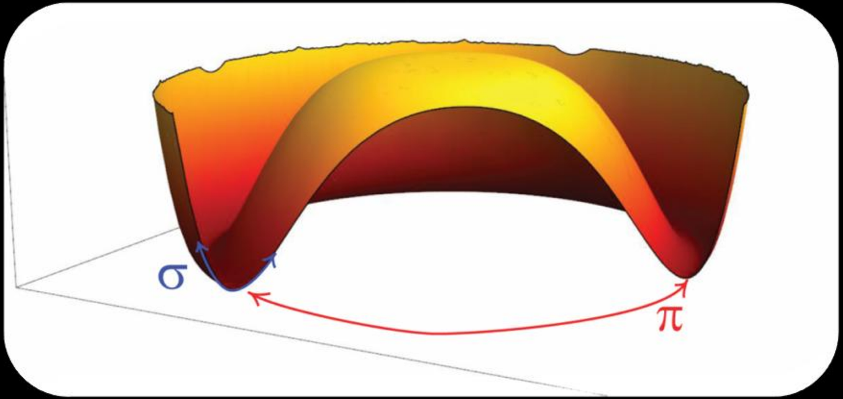
$\mathcal{L}(\pi) = \mathcal{L}(\pi + c), \forall c$ $V(\pi) = 0$

Compact Field Space (2πf)

Continuous shift symmetry

Symmetries

Shift symmetries eh? Where do we normally find those?



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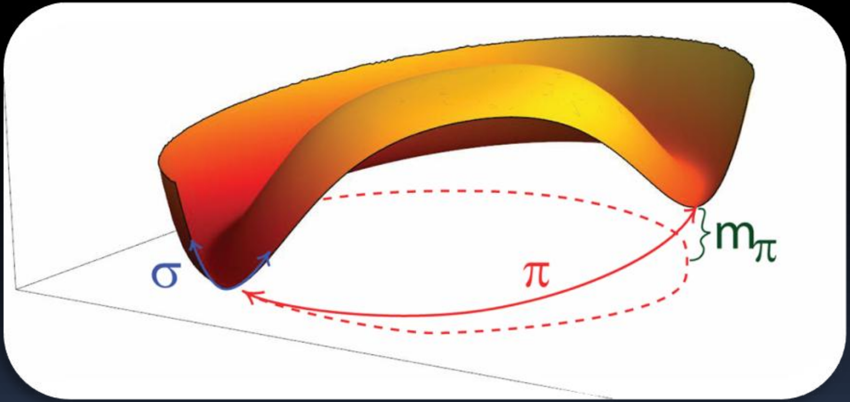


Pseudo-NGB (pNGP)

$\mathcal{L}(\pi) = \mathcal{L}(\pi + 2n\pi f)$

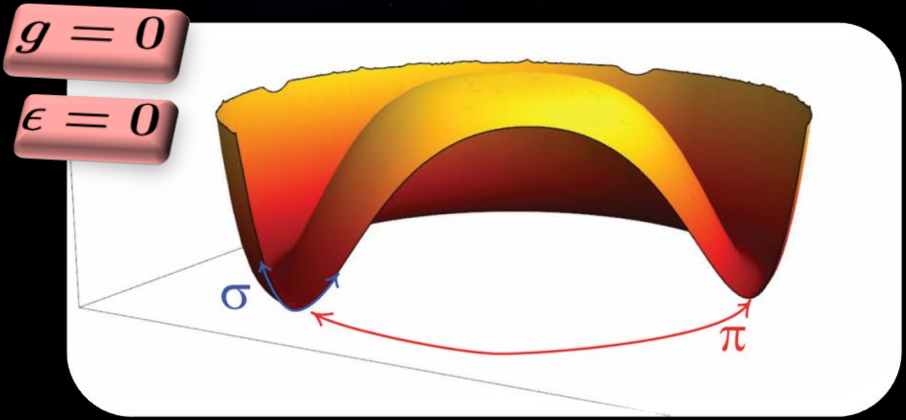
Compact Field Space (2πf)

Discrete shift symmetry

Allowed potential MUST be periodic in the field!

Symmetries

Shift symmetries eh? Where do we normally find those?



$g = 0$

$\epsilon = 0$

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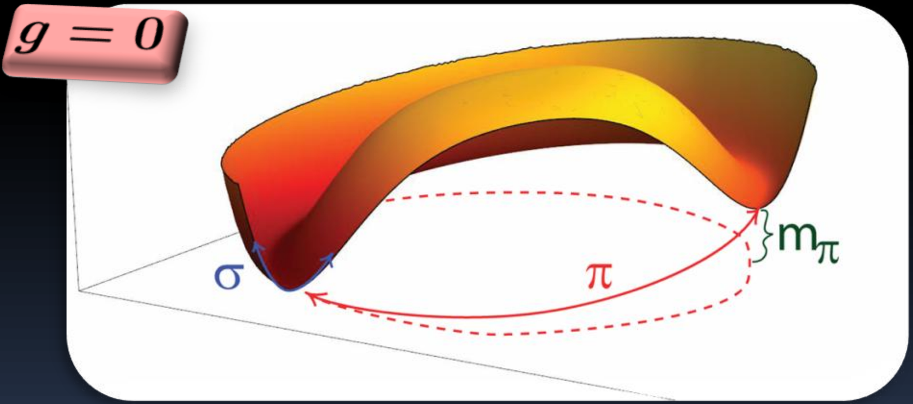

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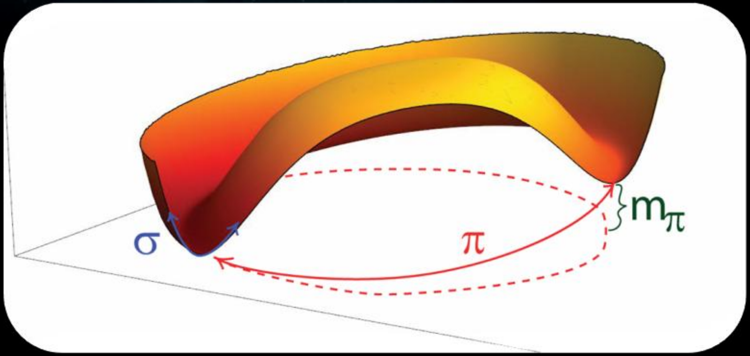


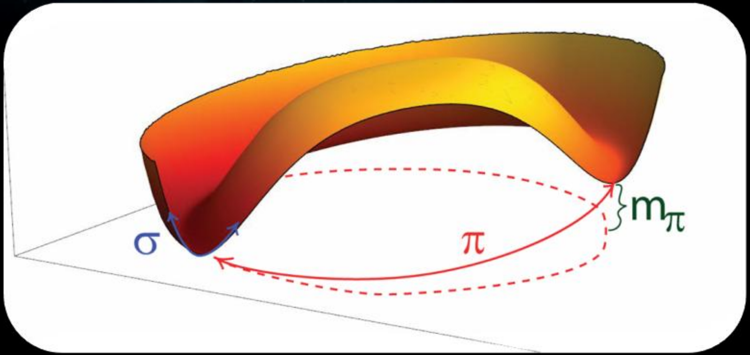
pNGP ➡ $m_\pi < m_\sigma$

Effective theory below m$_\sigma$: non-linear sigma model

$$\Sigma = e^{i\frac{T^a\pi^a}{f}} = \cos\left(\frac{\pi}{f}\right) + i\frac{T^a\pi^a}{\pi}\sin\left(\frac{\pi}{f}\right)$$

$$\pi = \sqrt{\pi^a\pi^a}$$

Symmetries



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Effective theory below m_σ: non-linear sigma model

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$$\pi = \sqrt{\pi^a\pi^a}$$

What about *g ≠ 0*? (non-periodic terms) $-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$

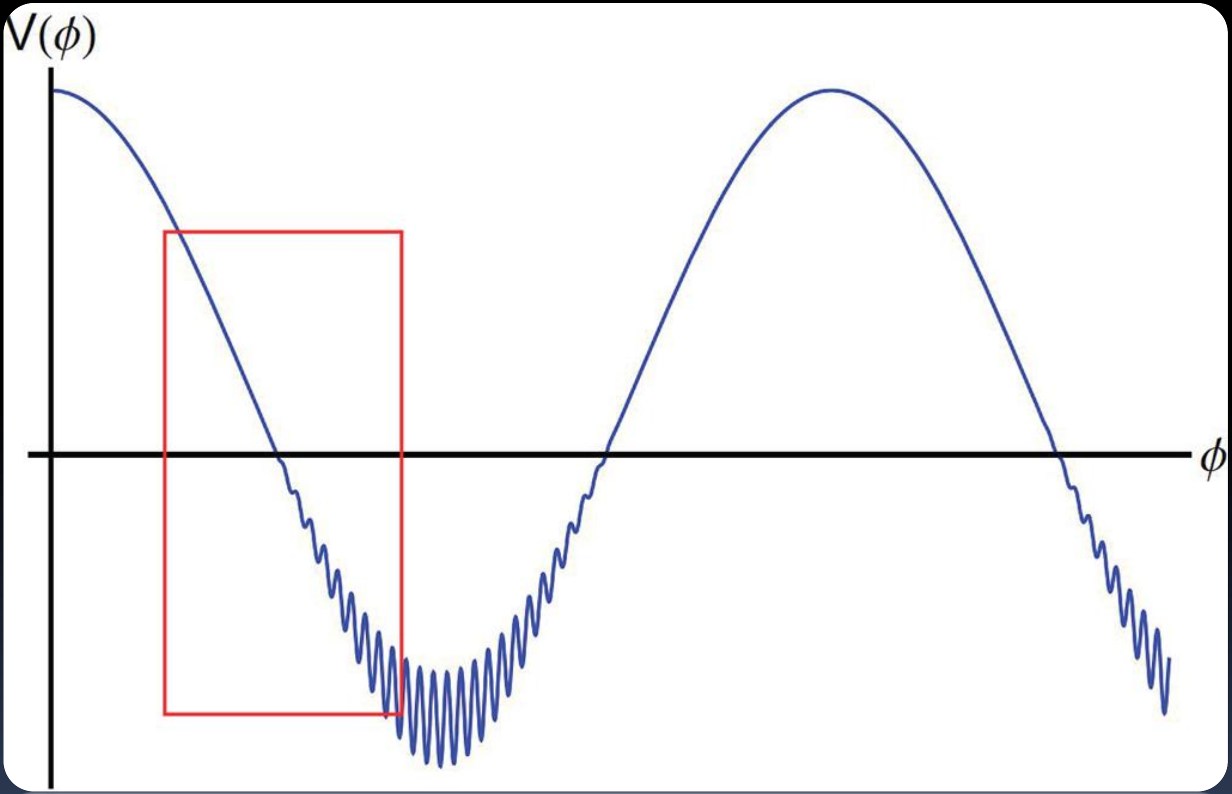
- Makes the field space non-compact
- The discrete shift symmetry cannot be broken by local operators
  (it is a redundancy in the description, a gauge symmetry)

Symmetries

$$V(\pi, H) \sim \kappa_1(H^2)\cos\left(\frac{\pi}{F}\right) + \kappa_2(H^2)\cos\left(\frac{\pi}{f}\right)$$

$F \gg f$

appropriate functions

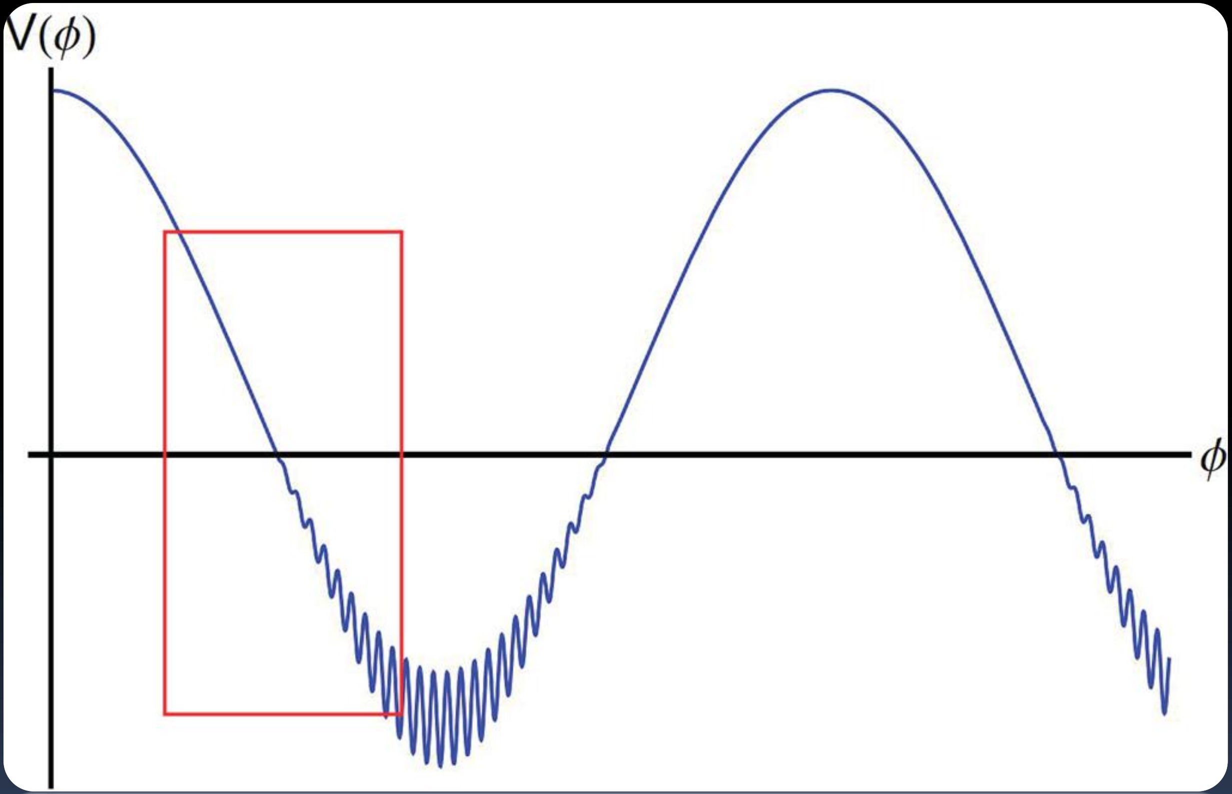


Symmetries

$$V(\pi, H) \sim \kappa_1(H^2)\cos\left(\frac{\pi}{F}\right) + \kappa_2(H^2)\cos\left(\frac{\pi}{f}\right)$$

$$F \gg f$$

But how can we get the same pNGB to have two very different periods (compact field spaces) ?



A Clockwork Axion
Kaplan, Rattazzi,
arXiv:1511.01827

Clockwork Relaxion

Key element: many pNGBs with the same decay constant *f*:

$$\mathcal{L}_{pNGB} = f^2 \underbrace{\sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j}_{U(1)^{N+1}} + \left(\underbrace{\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3 + h.c.}_{U(1)^{N+1} \to U(1)} \right) + \cdots$$

$$U_j \equiv e^{i\pi_j/(\sqrt{2}f)}$$

$$Q_{j+1} = Q_j/3$$

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$$V^{(2)} = \frac{1}{2} \epsilon f^2 \sum_{j=0}^{N} (q\pi_{j+1} - \pi_j)^2$$

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$$V^{(2)} = \frac{1}{2}\epsilon f^2 \sum_{j=0}^{N} (q\pi_{j+1} - \pi_j)^2 \Rightarrow \pi^{(0)} \sim \left(\pi_0 + \frac{1}{3}\pi_1 + \frac{1}{9}\pi_2 + \ldots + \frac{1}{3^N}\pi_N\right)$$

$$V(\pi^{(0)}) \sim \Lambda_N^4 \cos(\pi^{(0)}/F) + \Lambda_0^4 \cos(\pi^{(0)}/f) \rightarrow \boxed{F = 3^N f}$$

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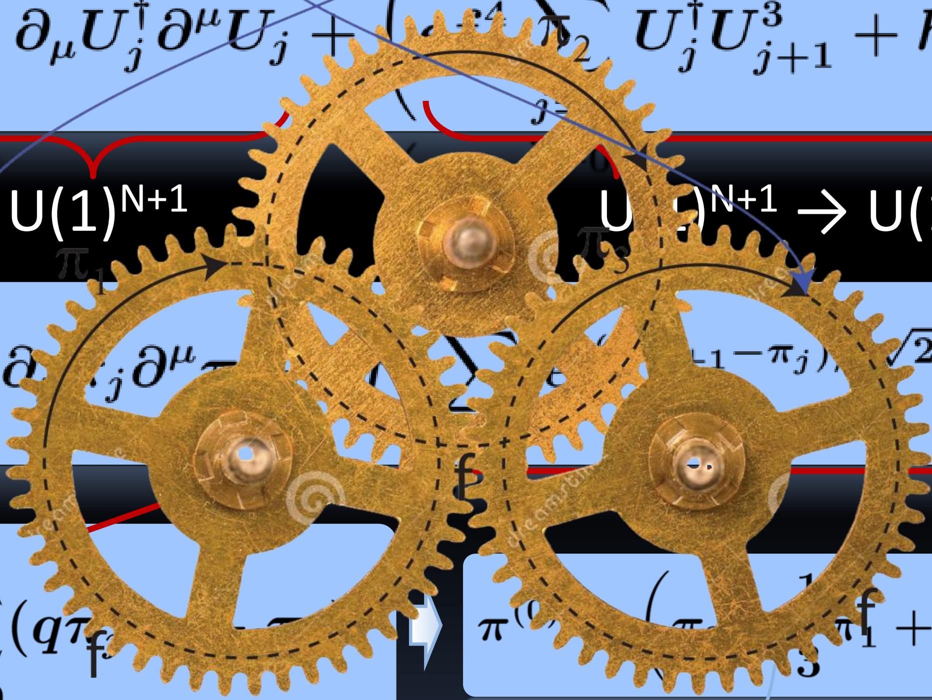
$U_j \equiv e^{i\pi_j/(\sqrt{2}f)}$

U(1)$^{N+1}$

U(1)$^{N+1}$ → U(1)

$Q_{j+1} = Q_j/3$



$$\mathcal{L}_{pNGB} = \frac{1}{2} \sum_{j=0}^{N} \partial \quad {}_j \partial^\mu \quad \sum \quad {}_{+1}-\pi_j) \quad \sqrt{2}f) + h.c. + \cdots$$

$$V^{(2)} = \frac{1}{2} \epsilon f^2 \sum_{j=0}^{N} (q\tau$$

$$\pi^{(0} \Big(\quad \frac{1}{3}\pi_1 + \frac{1}{9}\pi_2 + \ldots + \frac{1}{3^N}\pi_N \Big)$$

$$V(\pi^{(0)}) \sim \Lambda_N^4 \cos(\pi^{(0)}/F) + \Lambda_0^4 \cos(\pi^{(0)}/f)$$

$$F = 3^N f$$

Quick Recap

- pNGBs have the low energy potential needed to realize the relaxion mechanism

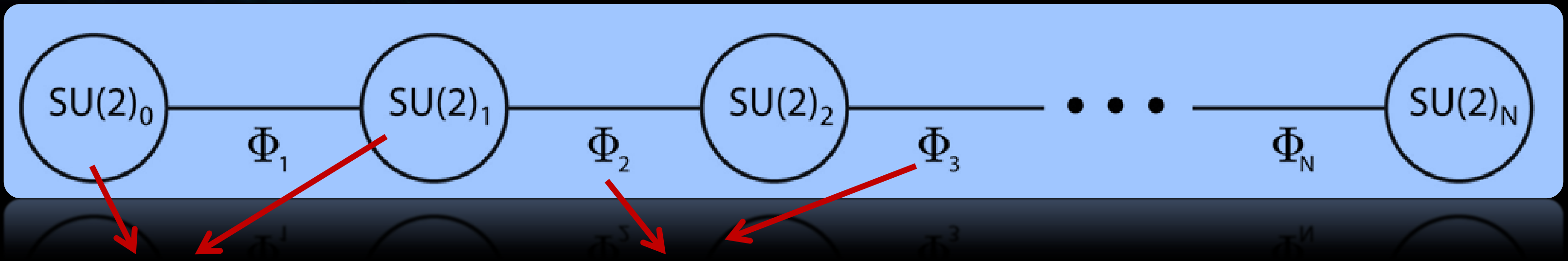
- radiative stability demands at least two fields (π and σ) to ensure no oscillations trap the relaxion field before the critical line (double scanner scenario)

- more copies of the two fields are needed to generate oscillations of longer period F from a theory with scale f, but the relation between F and f is exponential.

  ↳ Also makes the theory compatible with the needed Large Field Excursions, and the compact space for the field is now $2\pi F$

The N-site model & extra dimensions

Arkani-Hamed, Cohen, Georgi, arXiv:hep-th/0104005v1



Gauge groups

Non-linear sigma bi-doublets

$$S_4 = \int d^4x \left\{ -\frac{1}{2}\sum_{j=0}^{N} \mathrm{Tr}[F_{\mu\nu,j}F_j^{\mu\nu}] + \sum_{j=1}^{N} \mathrm{Tr}[(D_\mu\Phi_j)^\dagger(D^\mu\Phi_j)] - V(\Phi) \right\}$$

This is exactly the same as discretizing a 5th dimension (same as lattice field theory, with the Φ being the link variables).

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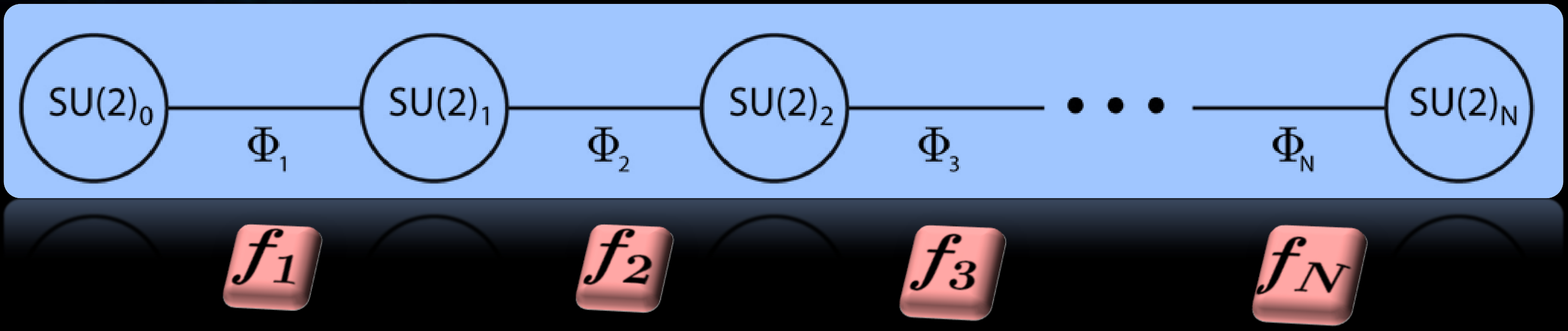
Arkani-Hamed, Cohen, Georgi, arXiv:hep-th/0104005v1



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Large *N* limit: SU(2) gauge theory in five dimensions.

Choice of scales determines metric:

$f_j = f, \quad \forall j$ ➡ Flat extra dimension

$f_j = f q^j, \quad 0 < q < 1$ ➡ AdS$_5$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

Kaplan-Rattazzi clockwork axion:

$$\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3$$

$$Q_{j+1} = Q_j/3$$

$$\Downarrow$$

$$F = 3^N f$$

No continuum limit!

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Goals:

- Find a model closer to a dimensional deconstruction that: (i) has a relaxion and (ii) provides a effective scale *F* much greater than *f.*

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- Generalize to non-abelian symmetries

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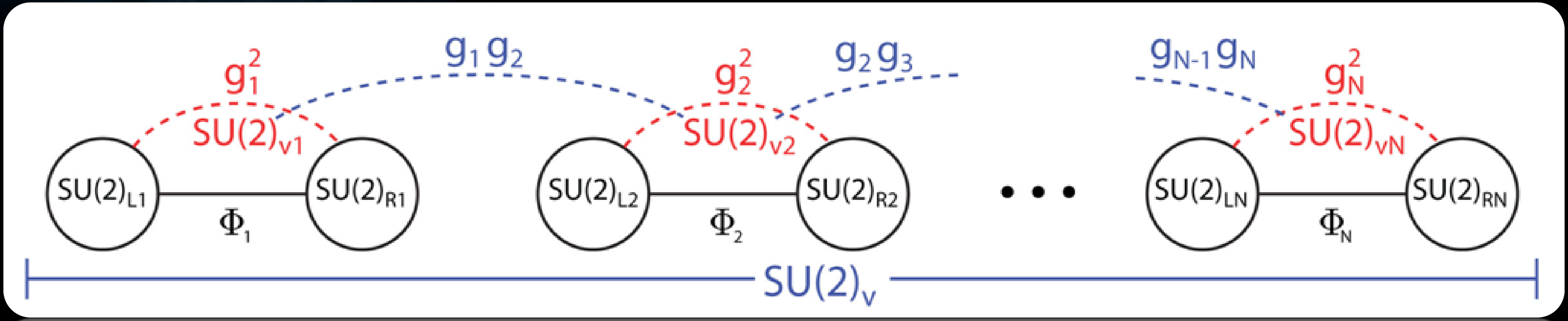
N-Relaxion



$$\sum_{j=1}^{N} \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger\, \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N})\, g_j^2 \left(\Phi_j+\Phi_j^\dagger\right)\right] - \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1}\, \mathrm{Tr}\left[(\Phi_j-\Phi_j^\dagger)(\Phi_{j+1}-\Phi_{j+1}^\dagger)\right]$$

Small symmetry breaking parameters

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
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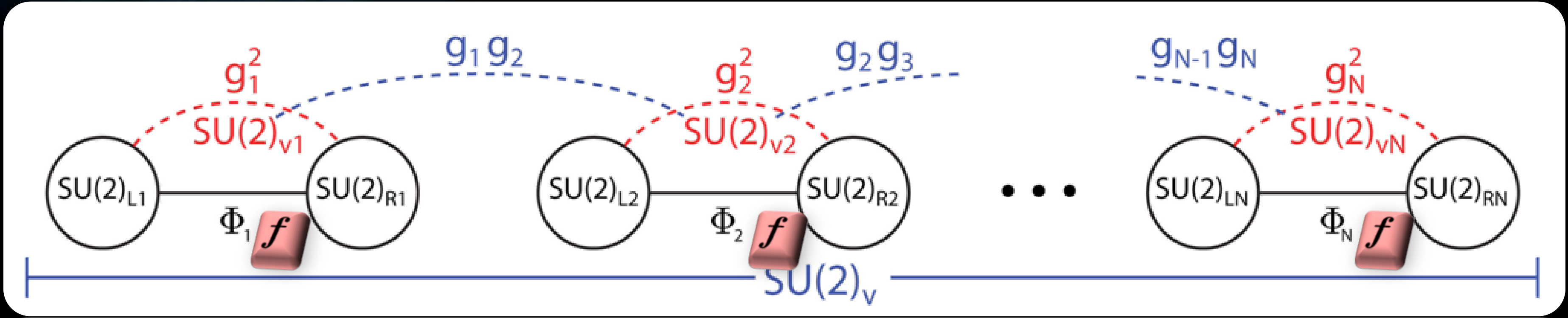
N-Relaxion



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AdS$_5$

$f_j = f q^j$

$g_j \to q^j, \quad 0 < q < 1$

$$q = \frac{g_{j+1}}{g_j}$$

Small symmetry breaking parameters

(will be hierarchical to emulate AdS$_5$)

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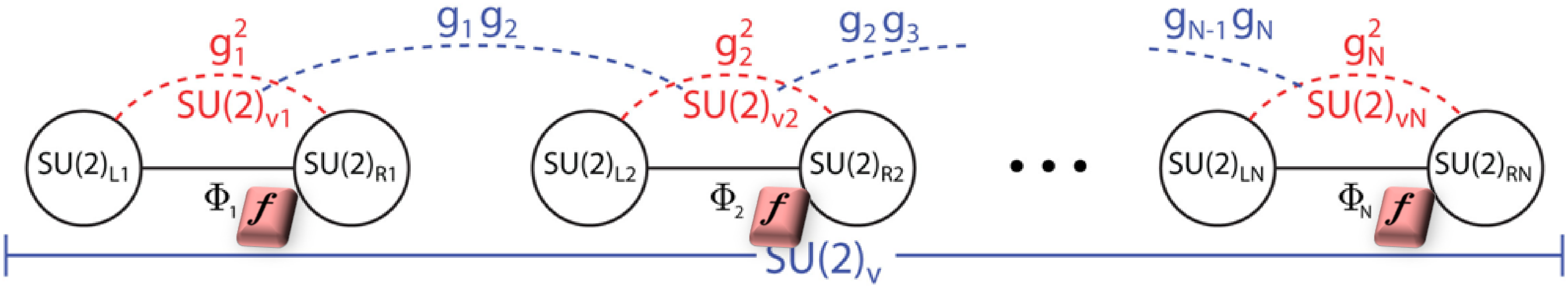
N-Relaxion



$$\sum_{j=1}^{N} \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger\, \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \left(\Phi_j+\Phi_j^\dagger\right)\right] - \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1}-\Phi_{j+1}^\dagger)\right]$$

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$g_j \rightarrow q^j, \quad 0 < q < 1$

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Small symmetry breaking parameters

(will be hierarchical to emulate AdS$_5$)

$$\sum_{j=1}^{N}\left[\frac{1}{2}\partial_\mu\vec{\pi}_j\cdot\partial^\mu\vec{\pi}_j + f^4(2-\delta_{j,1}-\delta_{j,N})g_j^2\cos\left(\frac{\pi_j}{f}\right)\right] + f^4\sum_{j=1}^{N-1} g_j g_{j+1}\frac{\vec{\pi}_j\cdot\vec{\pi}_{j+1}}{\pi_j\pi_{j+1}}\sin\left(\frac{\pi_j}{f}\right)\sin\left(\frac{\pi_{j+1}}{f}\right)$$

Quadratic (mass) terms everywhere, diagonalization needed

N-Relaxion

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \dots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \dots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \dots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

(massless at tree level, loops induce: $m = f^2 q^{2N}$)

Same as the Wilson Line in AdS$_5$!

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

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N-Relaxion

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⇨

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

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New scales for oscillation

$$f_j \equiv f q^{j-N} c_N$$

$c_N \approx 1$ (small q or big N)

$$f_N \approx f$$

$$F = f_1 \approx f / q^{N-1}$$

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$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \dots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \dots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \dots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

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(massless at tree level, loops induce: $m = f^2 q^{2N}$)

Same as the Wilson Line in AdS$_5$!

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

Amplitudes are also controlled by *q*

Bigger frequencies ↔ smaller amplitudes

(only the first few really matter)

$$f_j \equiv f q^{j-N} c_N$$

$$c_N \approx 1$$

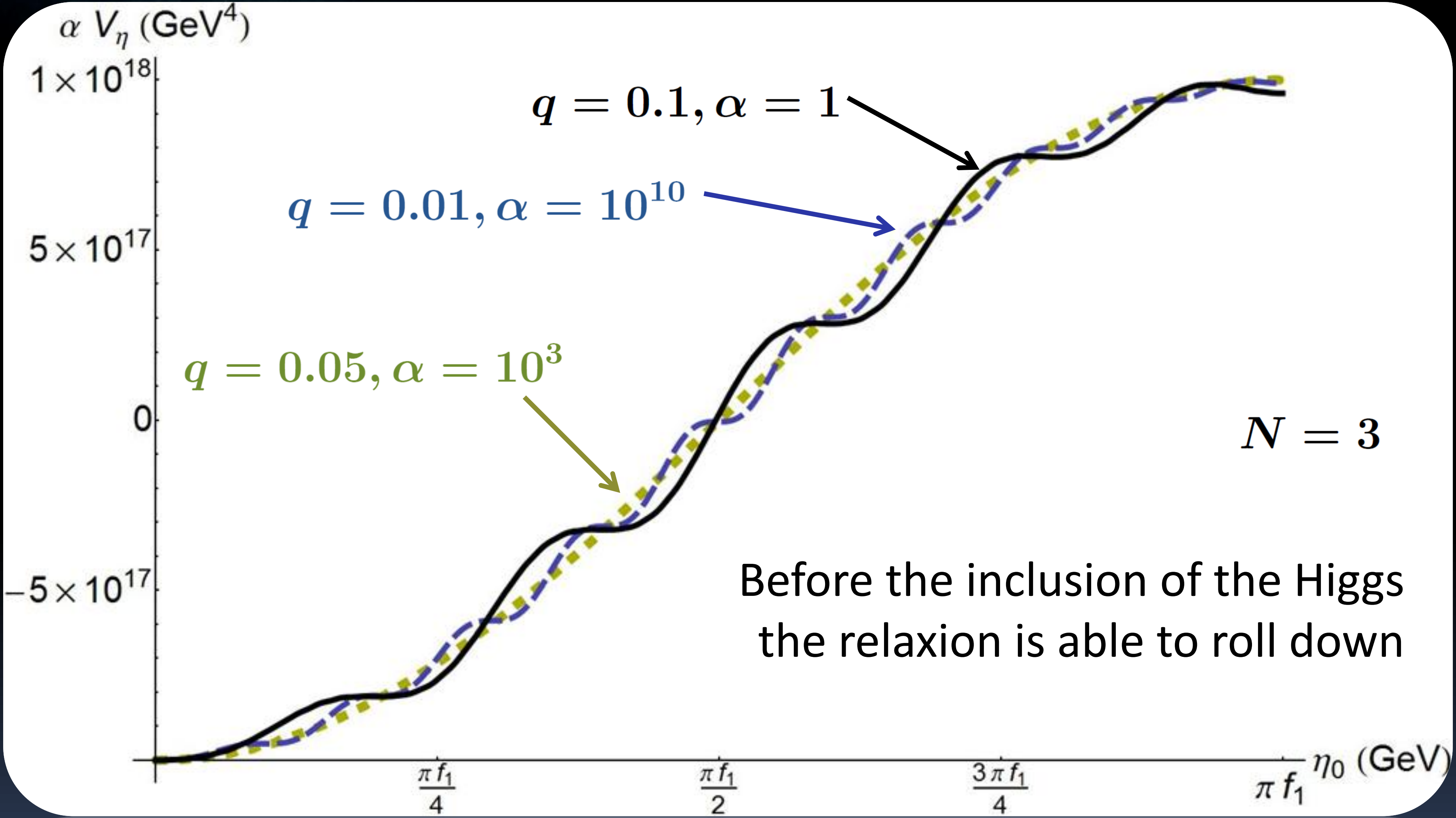
$$f_N \approx f$$

$$F = f_1 \approx f / q^{N-1}$$

$V(\eta_0)$ gets flat for $q \ll 1$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion



Before the inclusion of the Higgs the relaxion is able to roll down

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \underbrace{\left(1+\frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta}_{\text{Most general thing you can do}} + \underbrace{|D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4}_{V_H^{SM}} + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N+\Phi_N^\dagger]|H|^2$$

Generates the linear terms

$$-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$$

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Most general thing you can do ⇓ Generates the linear terms

$$-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$$

New explicit breaking at site N

$$\epsilon f^2|H|^2\cos\frac{\eta_0}{f_N}$$

Generates high frequency oscillations once *v ≠ 0*

→ Also generates high frequency oscillations everywhere, double scanner needed!

→ Modification of AdS_5 near the infrared brane (IR), enforces that the SM Higgs should be IR localized

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon}q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

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$q^{N+1} < \epsilon < 1$

Constraints:

"not the inflaton" ⇨ $H_I M_p > \Lambda^2$

"classical rolling vs quantum fluctuations" ⇨ $q^{N+1} > H_I^3 / f^3$

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N-Relaxion

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Constraints:

"not the inflaton"

"classical rolling vs quantum fluctuations"

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3}$$

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N-Relaxion

Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \left(1+\frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N+\Phi_N^\dagger]|H|^2$$

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$q^{N+1} < \epsilon < 1$

Constraints:

"not the inflaton"

"classical rolling vs quantum fluctuations"

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3}$$

"suppressing terms like ε *Cos^2*" $\epsilon < v^2/f^2$

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N-Relaxion

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon}q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

$$\Downarrow$$

Constraints:

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3} \Rightarrow \frac{\Lambda^6}{f^3 M_p^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

$$q \lesssim 10^{-23/(N+1)}$$

$$\epsilon < v^2/f^2 \Rightarrow f \lesssim 10^8\ \mathrm{GeV}$$

N-Relaxion

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Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

$$\frac{\Lambda^6}{f^3 M_p^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

$$q \lesssim 10^{-23/(N+1)}$$

$$f \lesssim 10^8 \text{ GeV}$$

Co

$$q = 10^{-24/(N+1)} \quad \& \quad \epsilon = 10^{-12}$$

$$f \approx 10^8 \text{ GeV}$$

$$N = 2 \rightarrow m_{\eta_0} \approx 10^{-7} \text{ eV}$$

$$N = 3 \rightarrow m_{\eta_0} \approx 10^{-11} \text{ eV}$$

Conclusions

- The relaxation models are a proof of concept. If we come to the conclusion that they are self-consistent, then the hierarchy problem ceases to be an argument for new physics at the TeV scale.

- We manage to build an N-site relaxion model with a well defined continuum limit. Some improvements are needed and/or interesting:
  - To build the double scanner sector (or another solution to the high frequency oscillations induced by the Higgs)
  - To explore other symmetry breaking patterns. Can any of the possible patterns allow us to increase the cut-off? Or do away with the double scanner?
  - What about the continuum limit? What theory do we get in AdS_5?

Thank You!

UV completion

$$
\begin{aligned}
\mathcal{L}_{UV} = \sum_{j=1}^{N} & \left\{ \bar{\psi}_j \not{p} \psi_j + \bar{\chi}_j \not{p} \chi_j \right\} \\
& + \sum_{j=1}^{N-1} \left\{ \bar{\psi}_{Lj} \left[\lambda_j \phi_j + \lambda_{j+1} \phi_{j+1} - \lambda'_j f \right] \psi_{Rj} \right. \\
& \left. + \bar{\chi}_{Lj} \left[\tilde{\lambda}_j \phi_j - \tilde{\lambda}_{j+1} \phi^\dagger_{j+1} - \tilde{\lambda}'_j f \right] \chi_{Rj} + \text{h.c.} \right\}
\end{aligned}
$$

Integrate out the fermions

$$
\begin{aligned}
\mathcal{L}_\Phi = \sum_{j=1}^{N} & \left\{ \text{Tr}\left[(\partial_\mu \Phi_j)^\dagger \partial^\mu \Phi_j \right] + \right. \\
& \left. + \frac{f^3}{2} (2 - \delta_{j,1} - \delta_{j,N}) g_j^2 \text{Tr} \left[\Phi_j + \Phi_j^\dagger \right] \right\} \\
& - \frac{f^2}{2} \sum_{j=1}^{N-1} g_j g_{j+1} \text{Tr} \left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi^\dagger_{j+1}) \right]
\end{aligned}
$$

Extra breaking for the Higgs comes from:

$$
\mathcal{L}'_{UV} = \xi^\dagger \not{p} \xi + \zeta \not{p} \zeta^\dagger + \xi(\epsilon \phi_N - m)\zeta + \text{h.c.}
$$

Deconstructing AdS$_5$

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$S_5^A = \int d^4x \int_0^{\pi R} dy \sqrt{-g}\left\{-\frac{1}{2g_5^2}\mathrm{Tr}\left[F_{MN}^2\right]\right\}$$

$$= \int d^4x \int_0^{\pi R} dy \left\{-\frac{1}{2g_5^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{g_5^2}e^{-2ky}\,\mathrm{Tr}\left[(\partial_5 A_\mu - \partial_\mu A_5)^2\right]\right\}.$$

$$\int_0^{\pi R} dy \to \sum_{j=0}^{N} a,$$

$$\partial_5 A_\mu \to \frac{A_{\mu,j} - A_{\mu,j-1}}{a}$$

$$S_5^A = \frac{a}{g_5^2}\int d^4x \left\{-\frac{1}{2}\sum_{j=0}^{N}\mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N}\frac{e^{-2kaj}}{a^2}\mathrm{Tr}\left[\left(A_{\mu,j} - A_{\mu,j-1} - a\partial_\mu A_{5,j}\right)^2\right]\right\}.$$

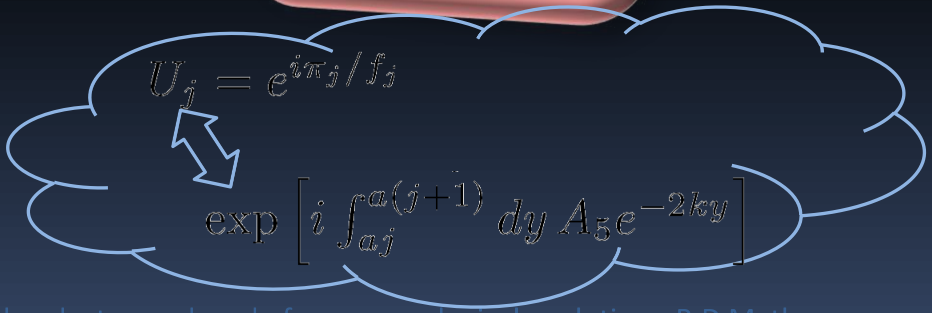
Which is the same as the gauged pNGB to quadratic level:

$$S_4^A = \frac{1}{g^2}\int d^4x \left\{-\frac{1}{2}\sum_{j=0}^{N}\mathrm{Tr}\left[F_{\mu\nu,j}\,F_j^{\mu\nu}\right] + \sum_{j=1}^{N} f^2 g^2 q^{2j}\,\mathrm{Tr}\left[\left(A_{\mu,j} - A_{\mu,j-1} - \partial_\mu \frac{\pi_j}{f_j}\right)^2\right]\right\},$$

$$\frac{g_5^2}{a} \leftrightarrow g^2,$$

$$f \leftrightarrow \frac{1}{\sqrt{a g_5}} = \frac{1}{ag},$$

$$q \leftrightarrow e^{-ka},$$



$$U_j = e^{i\pi_j/f_j}$$

$$\Updownarrow$$

$$\exp\left[i\int_{aj}^{a(j+1)} dy\, A_5 e^{-2ky}\right]$$

Phenomenology?

- Very light particle with weaker than gravity interaction. NOTHING AT THE LHC!

- Classical Oscillations can affect gravitational potential: pulsar timing (astro-ph.CO/1309.5888) and structure formation (astro-ph.CO/1410.2896)

- Late decay of relaxions can show up in CMB and diffuse gamma ray background

- Fifth force (too weak for present day precision)