PPC 2016

N-Relaxion:
Large field excursions from a few site relaxion model

Ricardo D'Elia Matheus

IFT - Instituto de Física Teórica - UNESP

The Hierarchy Problem

Think about the Standard Model (SM) as an EFT with a cut-off at M_p:

$$V(H) = m_H^2(\alpha, \beta)H^2 + \lambda h^4 + \mathcal{O}(1/M_p^2)$$

$$\langle H \rangle = v$$

The only mass scale is M_p!

⇨ All dimensionless Wilson coefficients should be of order one.

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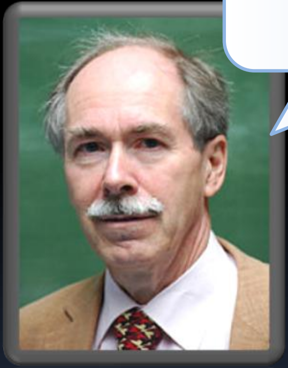

Technical Naturalness

$$m_H^2 \equiv \rho\, M_P^2$$

$$Dim[\rho] = Dim[\lambda] = 0$$

$$\rho \approx \lambda \approx 1$$

Important exception: taking a coefficient to zero increases symmetry. In that case it can be arbitrarily small.

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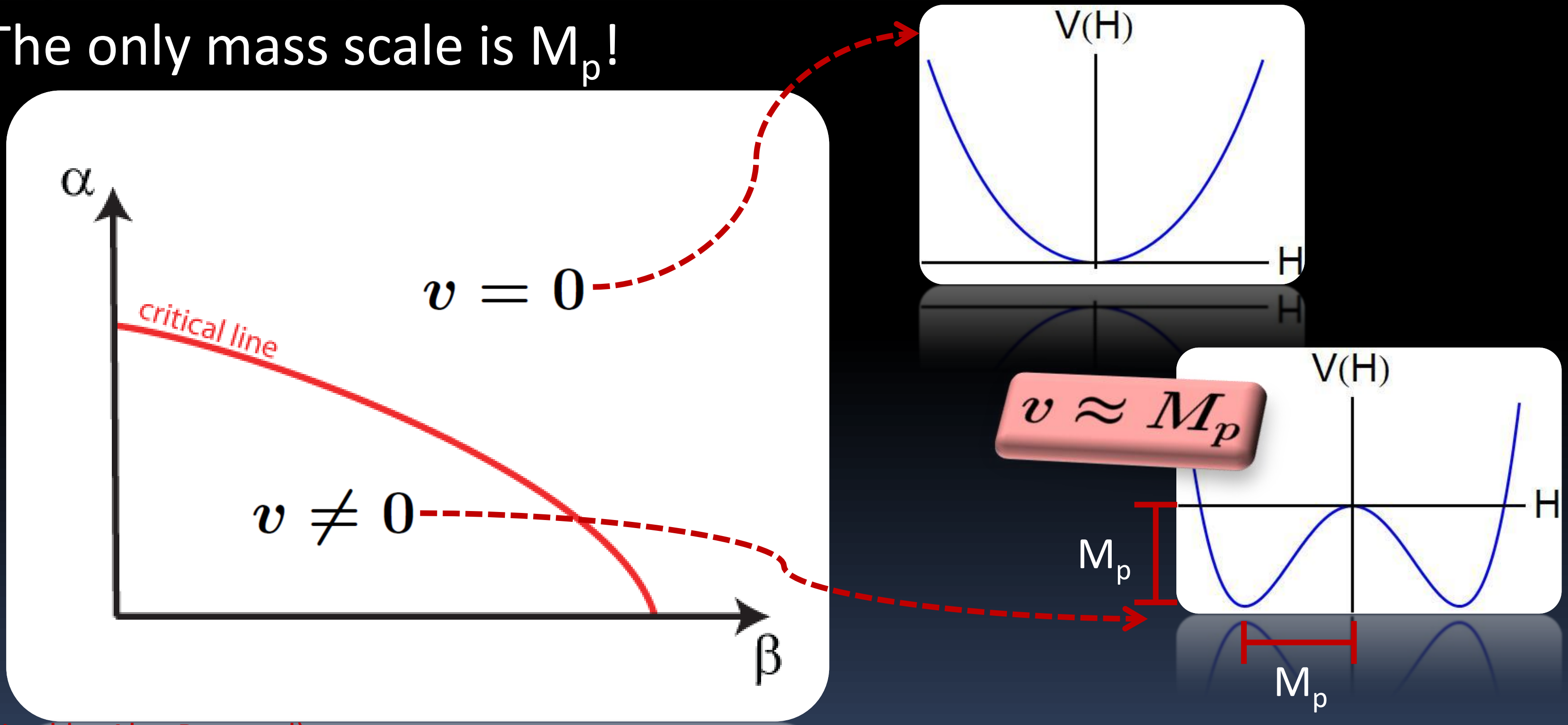

(inspired by Alex Pomarol)

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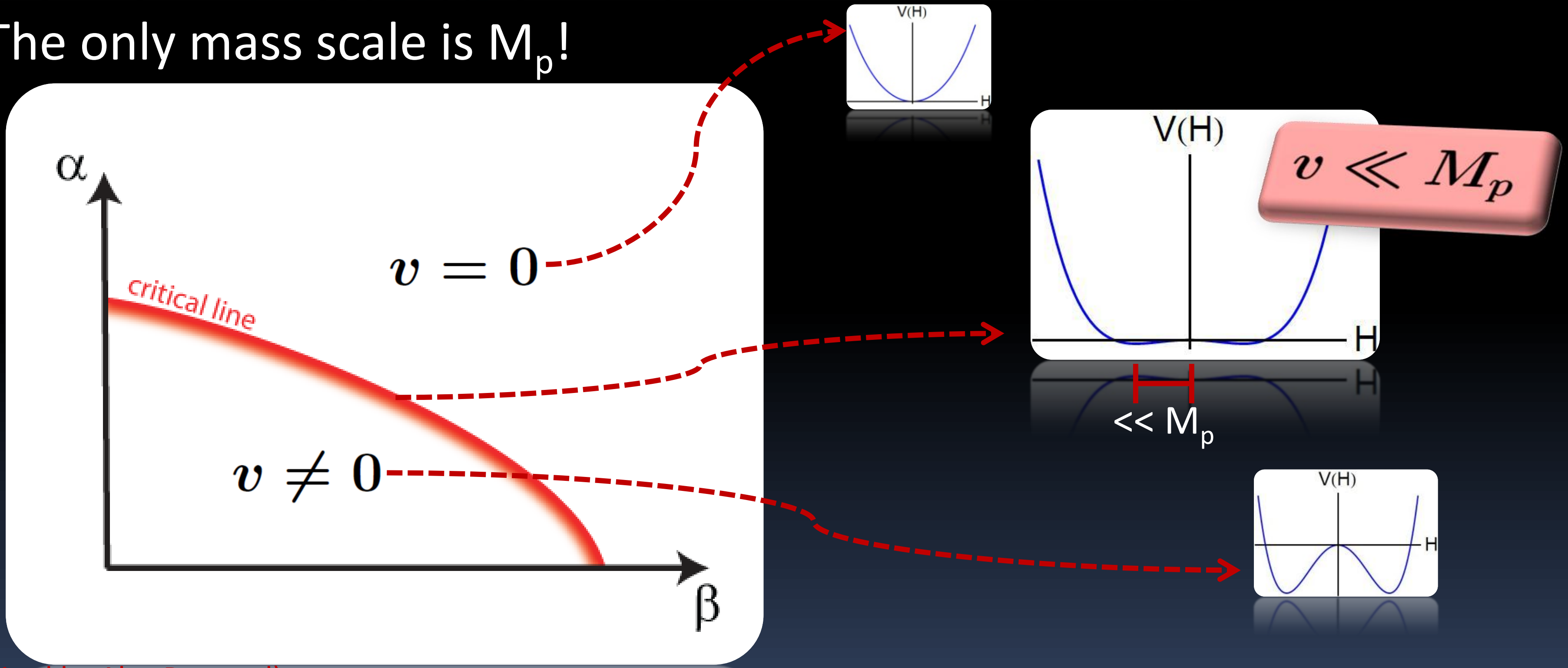


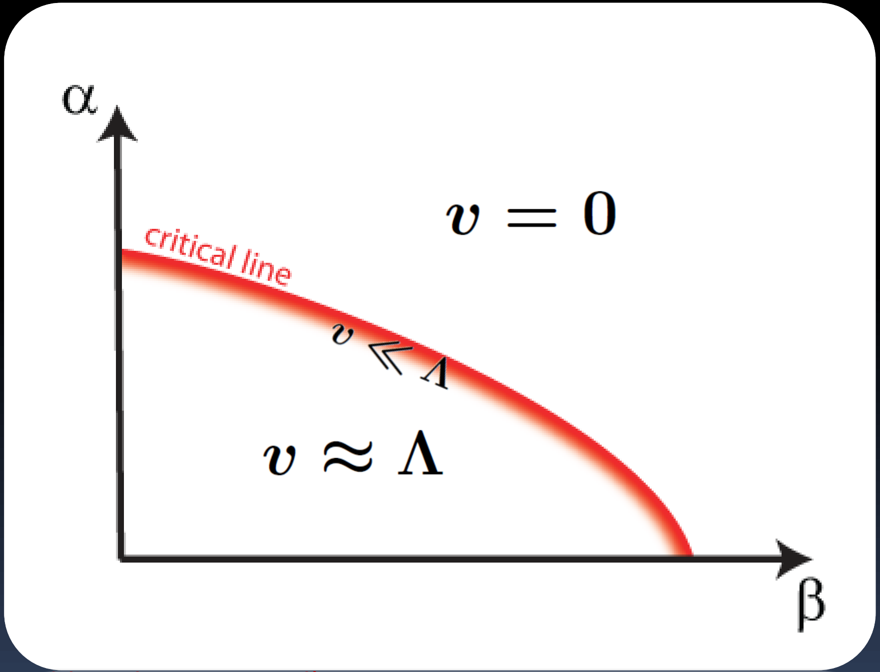
(inspired by Alex Pomarol)

Solving the Hierarchy Problem

Question: how come we live so close to the line?

Two answers: (1) Some symmetry forces it! (SUSY)

(2) The cut-off Λ, is not really M_p. In fact $\Lambda << M_p$ and $\Lambda \sim 1$ TeV
(Composite Models, Extra Dimensions et al.)



Both DEMAND new physics @ ~TeV

(inspired by Alex Pomarol)

A New Hope

Question: how come we live so close to the line?

The Third Way: (3) History! Make α and β dynamical (fields in fact)

(stupid) Example: $m_H^2(\alpha,\beta)H^2 \rightarrow \alpha\beta H^2$ $m_H^2 = \langle\alpha\rangle\langle\beta\rangle$

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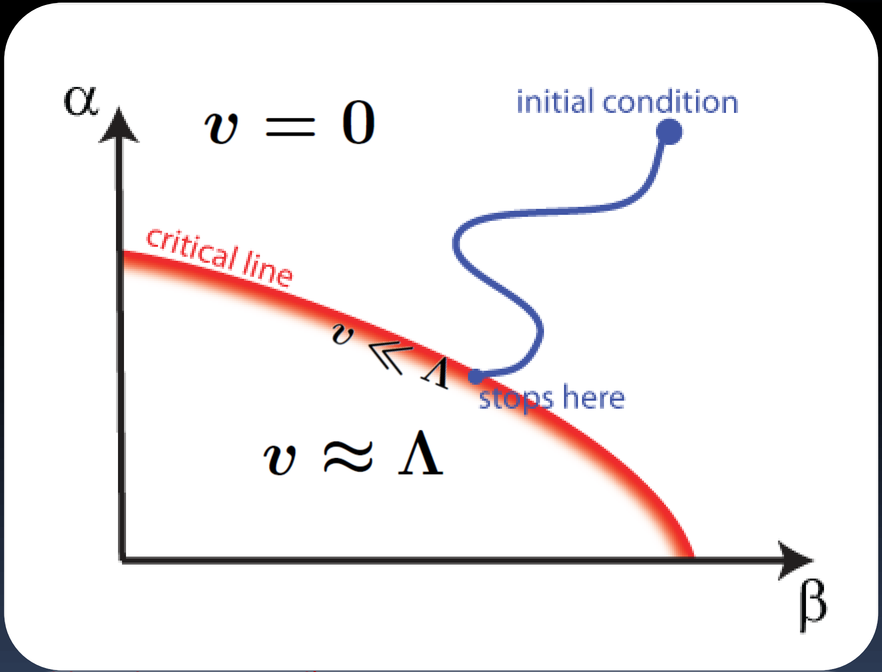

But how does the evolution stop?

(inspired by Alex Pomarol)

A New Hope

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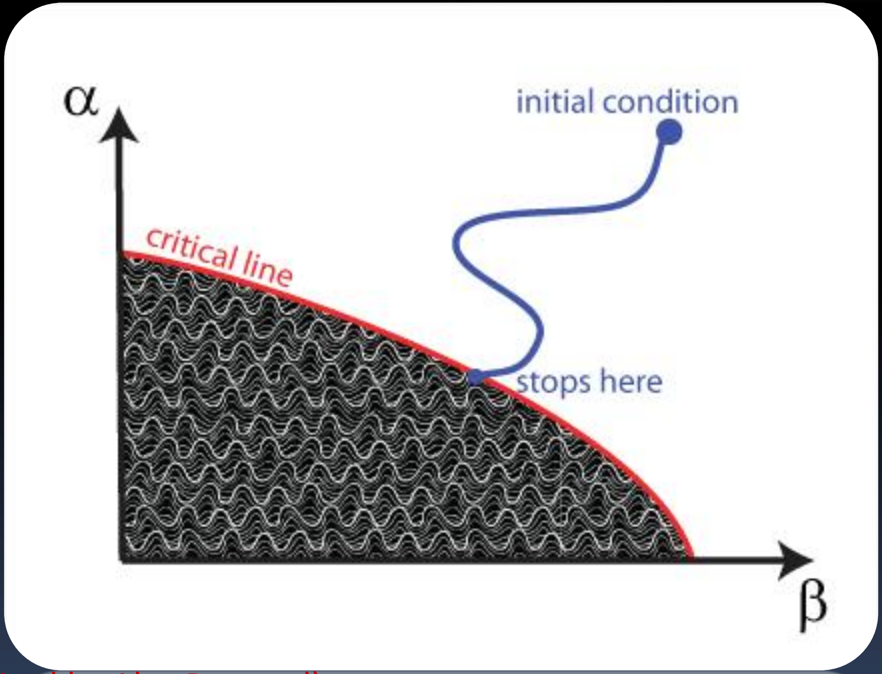


But how does the evolution stop?

Local Minima! A whole LOT OF local minima!

Can it be done in a (technically) natural way?

(spoiler: yes! But…)

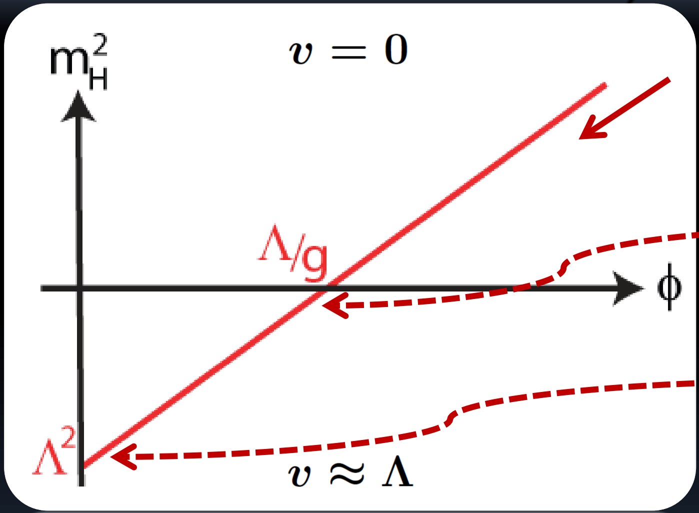
(inspired by Alex Pomarol)

Graham, Kaplan, Rajendran
arXiv:1504.07551

The Relaxion

Introduce one scalar field ϕ, and:

$$m_H^2 \rightarrow m_H^2(\phi) = -\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)$$



“rolls” down

Must stop here...

... not here

Large Field Excursions!

$\phi_c \equiv \Lambda/g$ If $g \ll 1$ $\rightarrow$ $\phi \approx \Lambda/g \gg \Lambda$

Espinosa et al.
arXiv: 1506.09217

The Relaxion

The minimal model:

$$V(\phi,H)=\underbrace{\Lambda^3 g\phi}_{\text{Linear slope for }\phi}\underbrace{-\frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)}_{\frac{1}{2} m_H^2}H^2+\underbrace{\epsilon\Lambda_c^2H^2\cos(\phi/f)}_{\text{Local minima in }\phi}$$

Both *g* and ε break shift symmetries (more about that later) and can be naturally small !

Λ is the cut-off for the SM

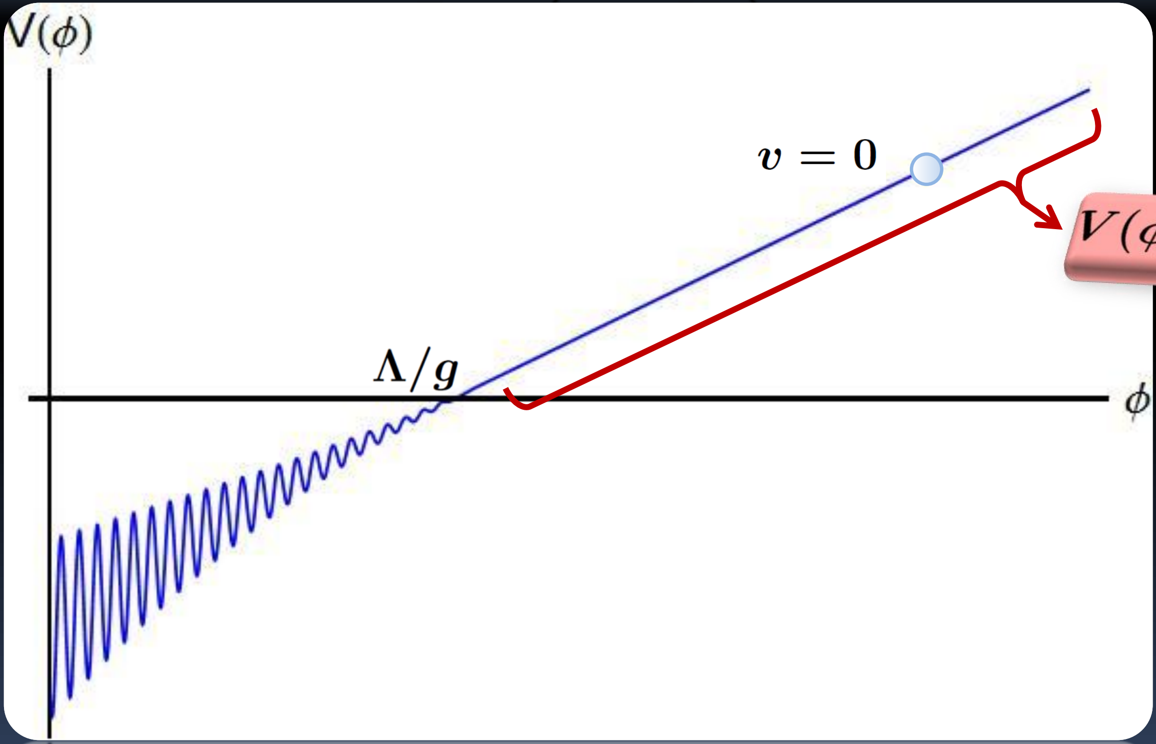
Λ_c is the scale at which the periodic potential is generated

Espinosa et al.
arXiv: 1506.09217

The Relaxion

The minimal model:

$$V(\phi, H) = \underbrace{\Lambda^3 g\phi}_{} - \frac{1}{2}\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right) H^2 + \epsilon\Lambda_c^2 H^2 \cos(\phi/f)$$



$$V(\phi, H = 0) = \Lambda^3 g\phi$$

Espinosa et al.
arXiv: 1506.09217

The Relaxion

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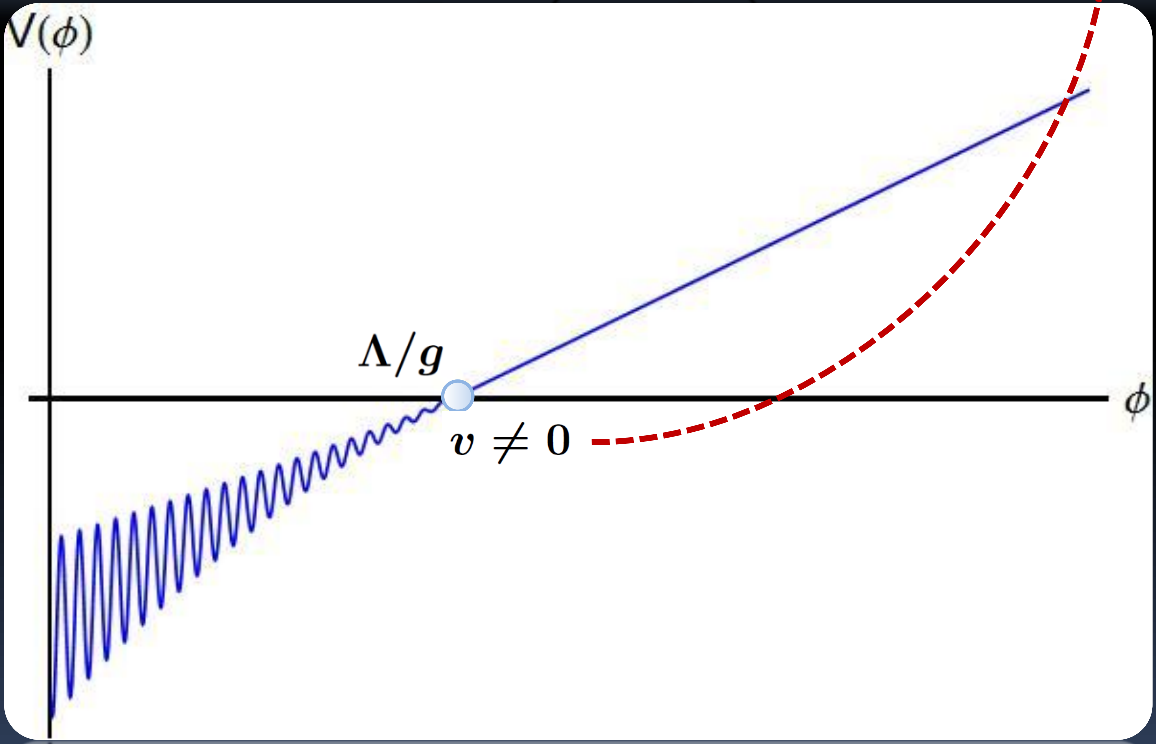


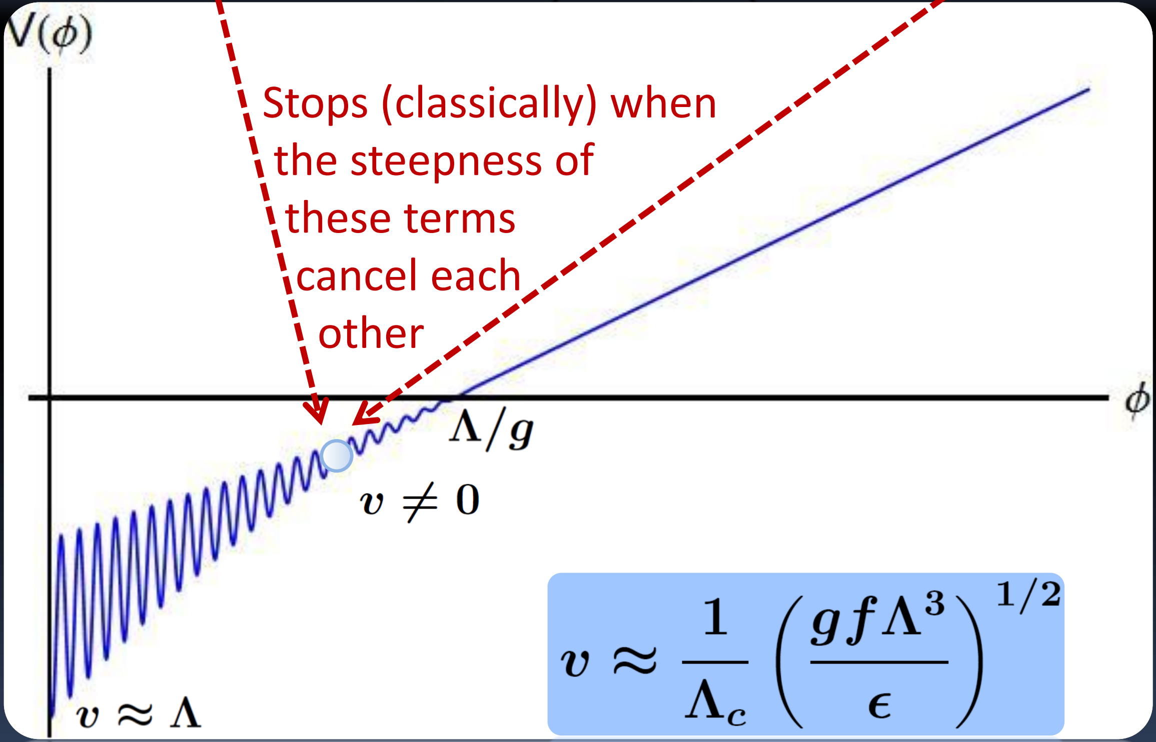
Becomes more important as *v* grows

Espinosa et al.
arXiv: 1506.09217

The Relaxion

The minimal model:

$$V(\phi,H)=\underbrace{\Lambda^3 g\phi}-\frac{1}{2}\Lambda^2\left(1-\frac{g\phi}{\Lambda}\right)H^2+\underbrace{\epsilon\Lambda_c^2H^2\cos(\phi/f)}$$



Stops (classically) when the steepness of these terms cancel each other

$$v\approx\frac{1}{\Lambda_c}\left(\frac{gf\Lambda^3}{\epsilon}\right)^{1/2}$$

The overall slope is controlled by *g*.

$v\ll\Lambda$

⇕

$g\ll 1$

Technically Natural!

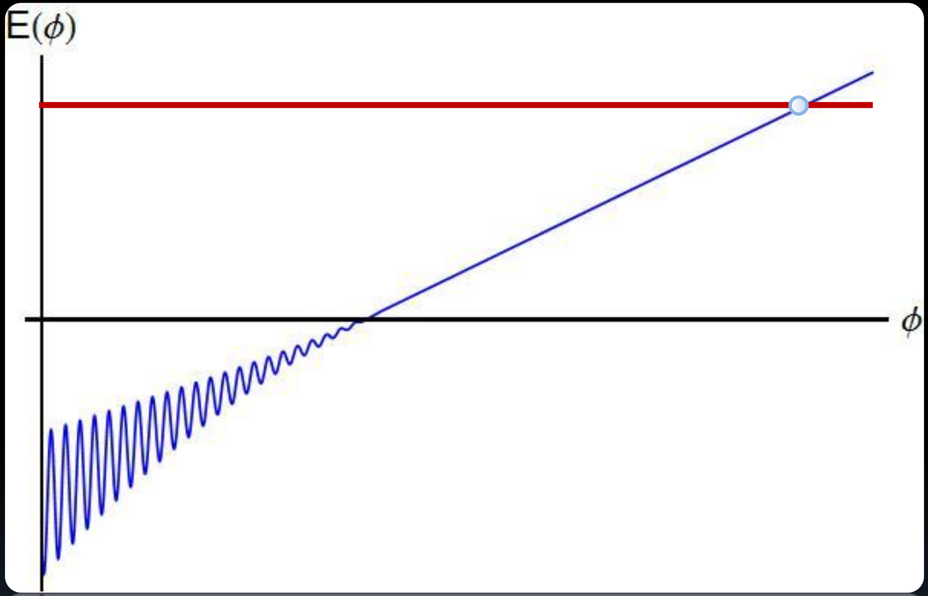
NO NEW PHYSICS close to *v*

Espinosa et al.
arXiv: 1506.09217

The Relaxion

So far, so good. Now on to the little dirty details:

Do we risk overshooting? Do we need to start close to ϕ_c?



NO, if slow rolling (during an inflationary epoch). Inflation introduces Hubble friction:

$$\cancel{\ddot{\phi}} + 3H_I\dot{\phi} = -\partial_\phi V(\phi)$$

(ϕ is not the inflaton)

Consequence: homework for cosmologists as a long period of inflation is needed ($N_e \sim 10^{40}$, smaller if H_I is not constant – Patil, Schwaller, arXiv:1507.08649)

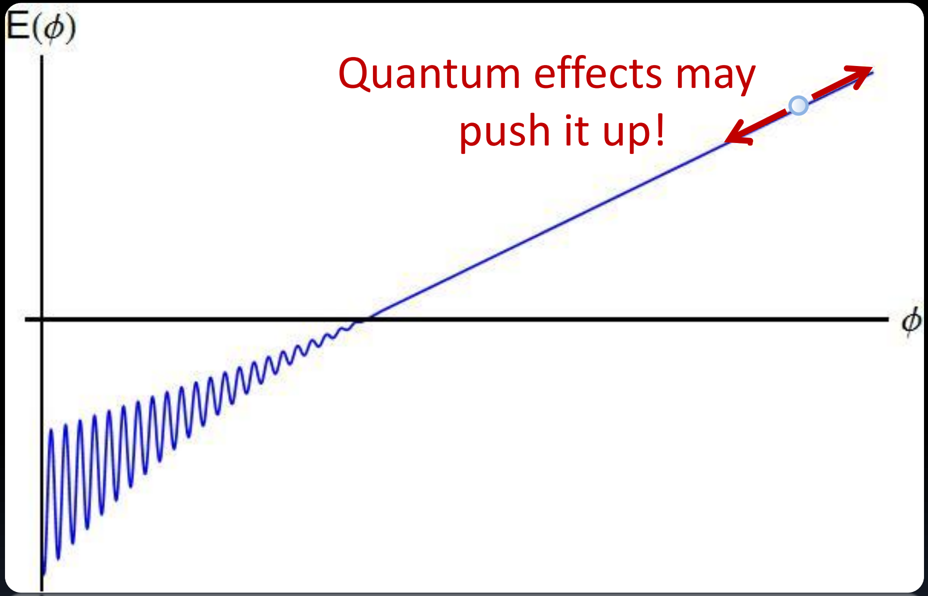
(Optional approach: no inflation, temperature dependent potential. E. Hardy, arXiv:1507.07525)

Espinosa et al.
arXiv: 1506.09217

The Relaxion

So far, so good. Now on to the little dirty details:

Limitations: Inflation → de Sitter space → Temperature (from Horizon)



Quantum effects may push it up!

$$\Delta\phi_{\text{class}} \sim \frac{V'(\phi)}{H_I^2} = \frac{g\Lambda^3}{H_I^2}$$

$$\Delta\phi_{\text{quant}} \sim H_I$$

$$\Delta\phi_{\text{class}} > \Delta\phi_{\text{quant}}$$

$$g > (H_I/\Lambda)^3$$

ϕ is not the inflaton

$$\left.\begin{matrix} V(\phi \sim \Lambda/g) \approx \Lambda^4 \\ V(\phi) < V_I \approx H_I^2 M_p^2 \end{matrix}\right\} H_I > \frac{\Lambda^2}{M_p}$$

$$g > (\Lambda/M_p)^3$$

Espinosa et al.
arXiv: 1506.09217

The Relaxion

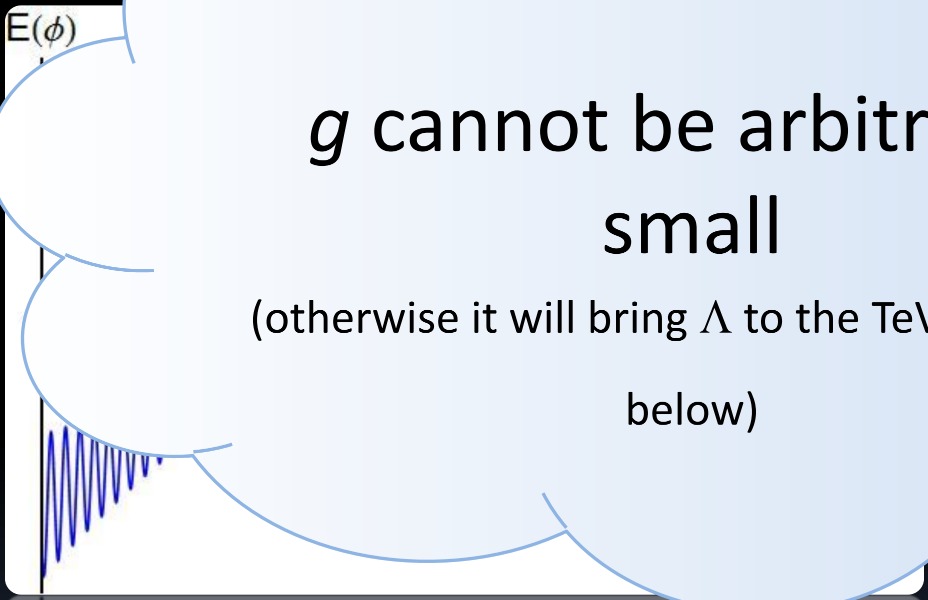
So far, so good. No

Limit (from Horizon)

$g < 1 \rightarrow \Lambda$ is not M_p

g cannot be arbitrarily small

(otherwise it will bring Λ to the TeV scale and below)



$\frac{g\Lambda^3}{H_I^2}$

$\Delta\phi_{\text{class}} > \Delta\phi_{\text{quant}}$

$g > (H_I/\Lambda)^3$

ϕ is not the inflaton

$V(\phi \sim \Lambda/g) \approx \Lambda^4$

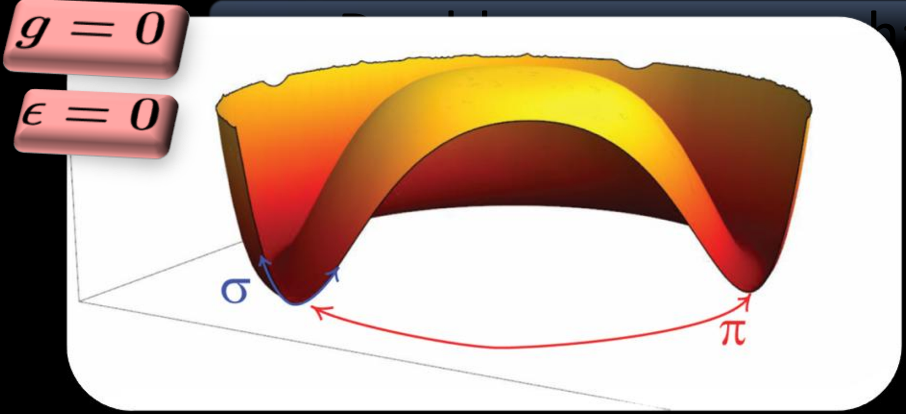
$V(\phi) < V_I \approx H_I^2 M_p^2$

$H_I > \frac{\Lambda^2}{M_p}$

$g > (\Lambda/M_p)^3$

Symmetries

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2 - \epsilon\Lambda_c^2 H^2\cos(\phi/f)$$

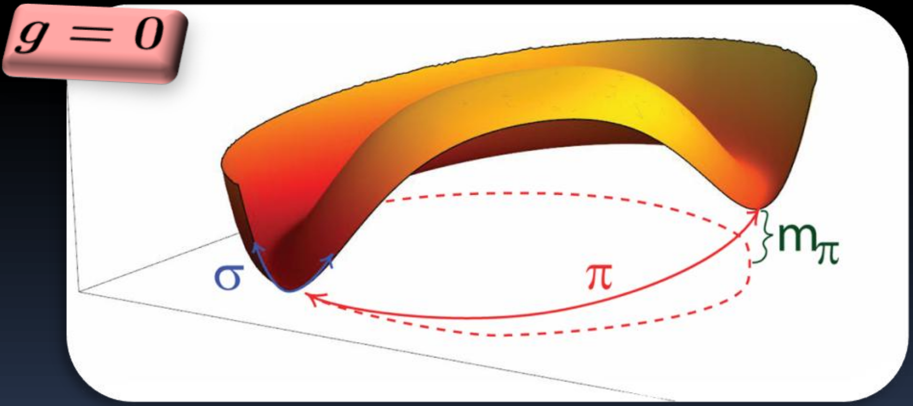


Nambu-Goldstone Bosons (NGB)

$\mathcal{L}(\pi) = \mathcal{L}(\pi + c), \forall c$ $V(\pi) = 0$

Compact Field Space (2πf)

Continuous shift symmetry



Pseudo-NGB (pNGP)

$\mathcal{L}(\pi) = \mathcal{L}(\pi + 2n\pi f)$

Compact Field Space (2πf)

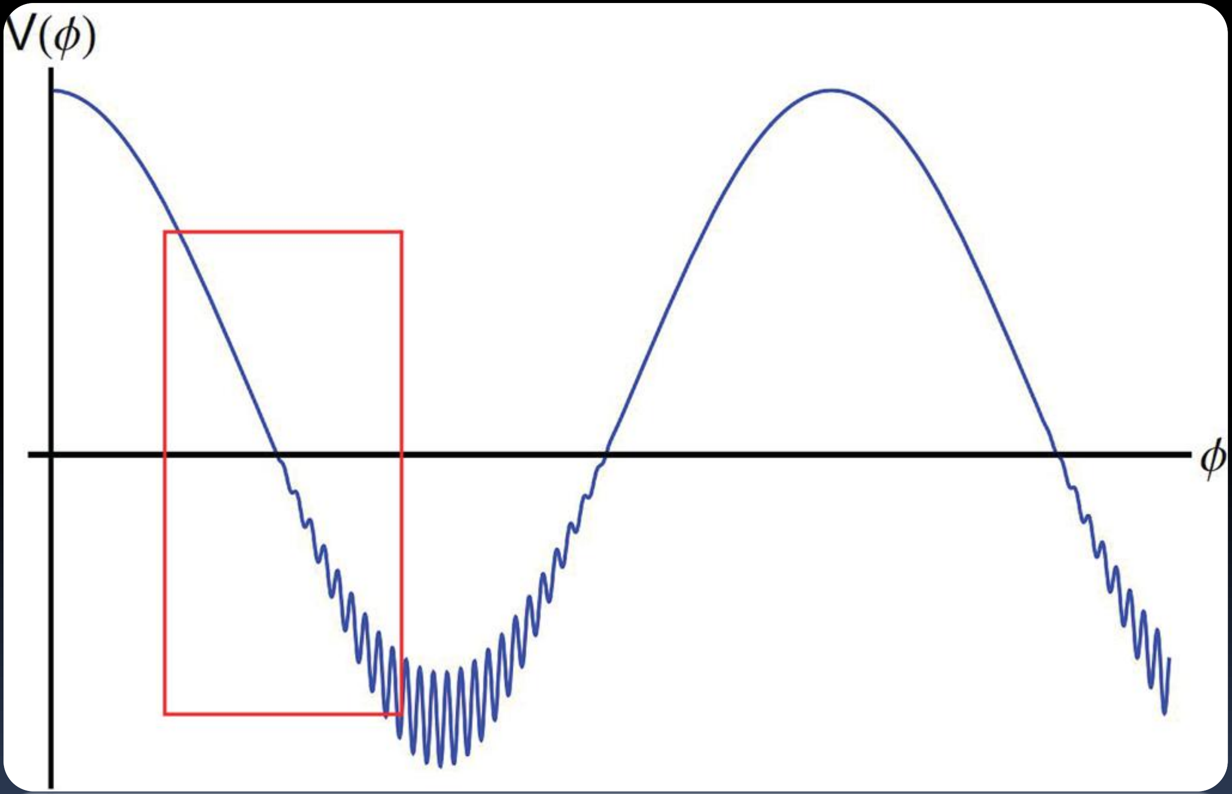
Discrete shift symmetry

Allowed potential MUST be periodic in the field!

Symmetries

$$V(\pi, H) \sim \kappa_1(H^2)\cos\left(\frac{\pi}{F}\right) + \kappa_2(H^2)\cos\left(\frac{\pi}{f}\right)$$

$F \gg f$



But how can we get the same pNGB to have two very different periods (compact field spaces) ?

A Clockwork Axion
Kaplan, Rattazzi,
arXiv:1511.01827

Clockwork Relaxion

Key element: many pNGBs with the same decay constant *f*:

$$\mathcal{L}_{pNGB} = \underbrace{f^2 \sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j}_{U(1)^{N+1}} + \underbrace{\left(\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3 + h.c. \right)}_{U(1)^{N+1} \to U(1)} + \cdots$$

$$U_j \equiv e^{i\pi_j/(\sqrt{2}f)}$$

$$Q_{j+1} = Q_j/3$$

$$V^{(2)} = \frac{1}{2}\epsilon f^2 \sum_{j=0}^{N} (q\pi_{j+1} - \pi_j)^2$$

$$\pi^{(0)} \sim \left(\pi_0 + \frac{1}{3}\pi_1 + \frac{1}{9}\pi_2 + \ldots + \frac{1}{3^N}\pi_N \right)$$

$$V(\pi^{(0)}) \sim \Lambda_N^4 \cos(\pi^{(0)}/F) + \Lambda_0^4 \cos(\pi^{(0)}/f)$$

$$F = 3^N f$$

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$\mathcal{L}_{pNGB}$

$f)$

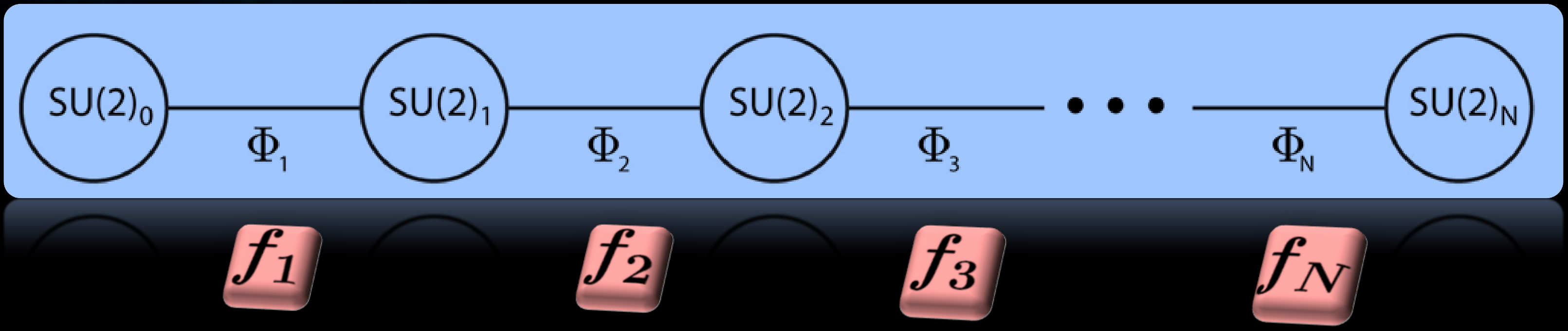
$\pi^{($

$3^N f$

$V(\pi^{(0)}) \sim \Lambda$

The N-site model & extra dimensions

Arkani-Hamed, Cohen, Georgi, arXiv:hep-th/0104005v1



$$S_4 = \int d^4x \left\{ -\frac{1}{2} \sum_{j=0}^{N} \mathrm{Tr}[F_{\mu\nu,j} F_j^{\mu\nu}] + \sum_{j=1}^{N} \mathrm{Tr}[(D_\mu \Phi_j)^\dagger (D^\mu \Phi_j)] - V(\Phi) \right\}$$

Large *N* limit: SU(2) gauge theory in five dimensions.

Choice of scales determines metric:

$f_j = f, \quad \forall j$ ➡ Flat extra dimension

$f_j = f q^j, \quad 0 < q < 1$ ➡ AdS_5

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

Kaplan-Rattazzi clockwork axion:

$$\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3$$

$$Q_{j+1} = Q_j/3$$

$$\Downarrow$$

$$F = 3^N f$$

No continuum limit!

Goals:

- Find a model closer to a dimensional deconstruction that: (i) has a relaxion and (ii) provides a effective scale *F* much greater than *f.*

- Emulate the discretization of AdS$_5$ (which is motivated by dualities to strongly coupled theories). This is tricky, since we are taking $f_j = f$

- Generalize to non-abelian symmetries

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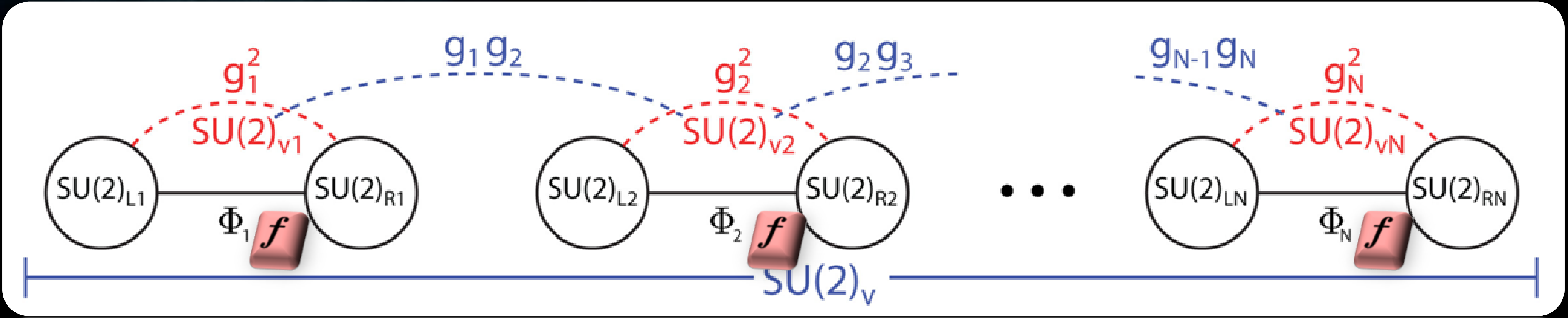
N-Relaxion



$$\sum_{j=1}^{N} \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger\, \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N})\, g_j^2 \left(\Phi_j+\Phi_j^\dagger\right)\right] - \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi_{j+1}^\dagger)\right]$$

Small symmetry breaking parameters

(will be hierarchical to emulate AdS$_5$)

AdS$_5$

$f_j = f q^j$

$g_j \to q^j, \quad 0 < q < 1$

$$q = \frac{g_{j+1}}{g_j}$$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \dots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \dots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \dots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

(massless at tree level, loops induce: $m = f^2 q^{2N}$)

Same as the Wilson Line in AdS$_5$!

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

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New scales for oscillation

$$f_j \equiv f q^{j-N} c_N$$

$$c_N \approx 1$$

(small q or big N)

$$f_N \approx f$$

$$F = f_1 \approx f / q^{N-1}$$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
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N-Relaxion

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \cdots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \cdots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \cdots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

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$$f_j \equiv f q^{j-N} c_N$$

$$c_N \approx 1$$

$$f_N \approx f$$

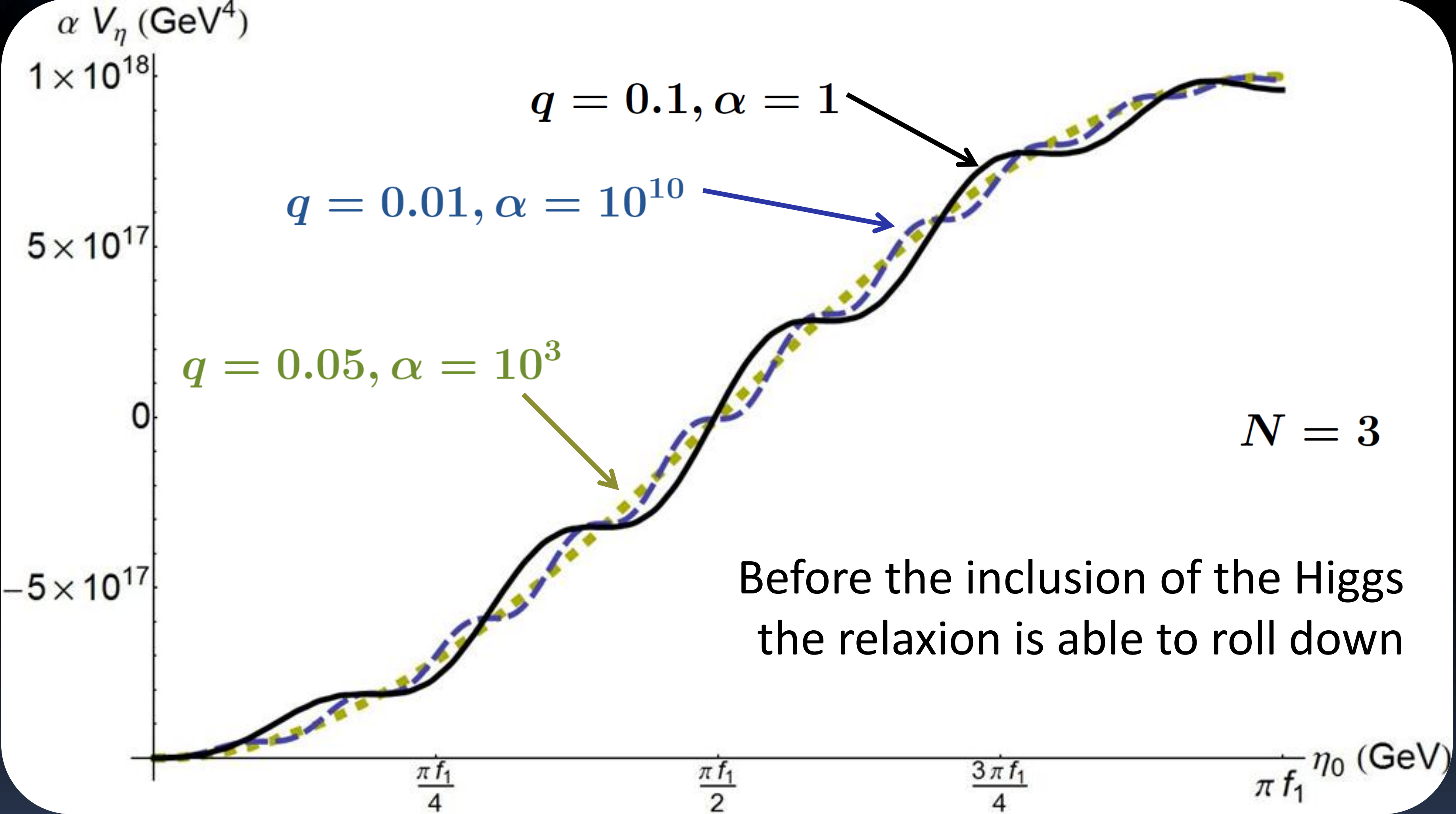
$$F = f_1 \approx f / q^{N-1}$$

Amplitudes are also controlled by *q*
Bigger frequencies ↔ smaller amplitudes
(only the first few really matter)

$V(\eta_0)$ gets flat for $q \ll 1$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion



Before the inclusion of the Higgs
the relaxion is able to roll down

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \underbrace{\left(1+\frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta}_{\text{Most general thing you can do}} + \underbrace{|D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4}_{V_H^{SM}} + \underbrace{\epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N+\Phi_N^\dagger]|H|^2}_{\text{New explicit breaking at site N}}$$

Generates the linear terms

$$-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$$

$$\epsilon f^2|H|^2\cos\frac{\eta_0}{f_N}$$

Generates high frequency oscillations once $v \neq 0$

- Also generates high frequency oscillations everywhere, double scanner needed!
- Modification of AdS_5 near the infrared brane (IR), enforces that the SM Higgs should be IR localized

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
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N-Relaxion

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

$$\frac{\Lambda^6}{f^3 M_p^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

Constraints:

"not the inflaton"

"classical rolling vs quantum fluctuations"

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3}$$

$$q \lesssim 10^{-23/(N+1)}$$

"suppressing terms like ε *Cos^2*"

$$\epsilon < v^2/f^2$$

$$f \lesssim 10^8 \ \mathrm{GeV}$$

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Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

$$\frac{\Lambda^6}{f^3 M_p^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

$$q \lesssim 10^{-23/(N+1)}$$

$$f \lesssim 10^8\ \mathrm{GeV}$$

Co

$q = 10^{-24/(N+1)}$ & $\epsilon = 10^{-12}$

$$f \approx 10^8\ \mathrm{GeV}$$

$$N = 2 \rightarrow m_{\eta_0} \approx 10^{-7}\ \mathrm{eV}$$

$$N = 3 \rightarrow m_{\eta_0} \approx 10^{-11}\ \mathrm{eV}$$

Conclusions

- The relaxation models are a proof of concept. If we come to the conclusion that they are self-consistent, then the hierarchy problem ceases to be an argument for new physics at the TeV scale.

- We manage to build an N-site relaxion model with a well defined continuum limit. Some improvements are needed and/or interesting:
  - To build the double scanner sector (or another solution to the high frequency oscillations induced by the Higgs)
  - To explore other symmetry breaking patterns. Can any of the possible patterns allow us to increase the cut-off? Or do away with the double scanner?
  - What about the continuum limit? What theory do we get in AdS_5?

Thank You!