THE $SU(2)\times U(1)$ COVARIANT DERIVATIVE:

$$D_\nu = \partial_\nu - i g W_\nu^a t^a - i g' Y B_\nu =$$

$$= \partial_\nu - i \frac{g}{\sqrt{2}}\left(W_\nu^+ t^+ + W_\nu^- t^-\right) - i \frac{1}{\sqrt{g^2+g'^2}} Z_\nu \left(g^2 t^3 - g'^2 Y\right) - i \frac{g g'}{\sqrt{g^2+g'^2}} A_\nu \left(t^3 + Y\right)$$

K_2, K_1: DIAGONAL IN 2x2 SU(2) SPACE

$$\underbrace{t^\pm \equiv \left(t^1 \pm i t^2\right)}_{\text{NON DIAGONAL}}$$

L-QUARK: $\partial_\nu - i g W_\nu^a t^a - i g' Y B_\nu - i g_s G_\nu^a t_G^a$

R-QUARK: $\partial_\nu - i g' Y B_\nu - i g_s G_\nu^a t_G^a$

L-LEPTON: $\partial_\nu - i g W_\nu^a t^a - i g' Y B_\nu$

R-LEPTON: $\partial_\nu - i g' Y B_\nu$

INTERACTION WITH THE "MASSLESS A_ν"

$$\left(-i \frac{g g'}{\sqrt{g^2+g'^2}} A_\nu \underbrace{\left(t^3 + Y\right)}_{\text{DEPENDENT}}\right)\psi$$

INDEPENDENT OF THE FERMION

$$t^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$Y = Y \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

DOUBLET "UP STATE": $\begin{pmatrix} u \\ 0 \end{pmatrix} \Rightarrow \left(t^3 + Y\right)\begin{pmatrix} u \\ 0 \end{pmatrix} = \left(\frac{1}{2} + Y\right)\begin{pmatrix} u \\ 0 \end{pmatrix}$

DOUBLET "DOWN STATE": $\begin{pmatrix} 0 \\ d \end{pmatrix} \Rightarrow \left(t^3 + Y\right)\begin{pmatrix} 0 \\ d \end{pmatrix} = \left(-\frac{1}{2} + Y\right)\begin{pmatrix} 0 \\ d \end{pmatrix}$

SINGLETS: $\left(\cancel{t^3} + Y\right) = Y$

So:

$$\left(-i \underbrace{\frac{g g'}{\sqrt{g^2+g'^2}}}_{e} A_\mu \underbrace{(t^3+y)}_{Q}\right)\psi = -i e Q A_\mu$$

In the "broken" phase, we can exchange:

$$\{g, g'\} \to \{e, \theta_w\} \qquad \cos\theta_w \equiv \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_w \equiv \frac{g'}{\sqrt{g^2+g'^2}}$$

Then:

$$D_\mu = \dots - i \frac{e}{s\theta_w \, c\theta_w} Z_\mu \left(t^3 - s^2\theta_w Q\right) - i e A_\mu Q$$

Masses in Fermionic Sector:

$$\mathcal{L}_\psi = \sum_\psi i \bar\psi \gamma^\mu D_\mu \psi + \overbrace{\cancel{m\bar\psi\psi}}^{\cancel{m\bar\psi_L\psi_R} + \cancel{m\bar\psi_R\psi_L}} = \sum_\psi \left(i \bar\psi_L \gamma^\mu D_\mu \psi_L + i \bar\psi_R \gamma^\mu D_\mu \psi_R \right)$$

$$\mathcal{L}_{H\psi} = -\underbrace{y_\psi}_{\text{Yukawa Coupling}} \bar\psi_L H \psi_R \xrightarrow{SU(2)_L \times U(1)_Y} -y_\psi \bar\psi_L e^{\cancel{-i t_a \theta_a} - i(Y_L - Y_R - Y_H)\theta + \cancel{i t_a \theta_a}} H \psi_R$$

OK as long as $Y_L - Y_R - \underbrace{Y_H}_{1/2} = 0 \Rightarrow \boxed{Y_L - Y_R = 1/2}$

$$\mathcal{L}_{\tilde H\psi} = -y_\psi \bar\psi_L \tilde H \psi_R \xrightarrow{SU(2)_L \times U(1)_Y} -y_\psi \bar\psi_L e^{\cancel{-i t_a \theta_a} - i(Y_L - Y_R + Y_H)\theta + \cancel{i t_a \theta_a}} \tilde H \psi_R$$

$$\tilde H \equiv i\sigma^2 H^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} H^* = \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} v + h - i\phi_3 \\ \phi_1 - i\phi_2 \end{pmatrix}}_{\text{No new DoFs!}}$$

$$\left.\begin{aligned} \tilde H &\xrightarrow{SU(2)} e^{i t_a \theta_a} \tilde H \\ \tilde H &\xrightarrow{U(1)} e^{-i Y_H \theta} \tilde H \end{aligned}\right\} \text{Exercise 12}$$

Now $\boxed{Y_L - Y_R = -\frac{1}{2}}$

If we check the numbers we see that one condition is satisfied by "up-right" fermions and the other for "down-right". So:

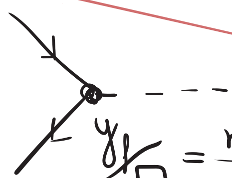
$$\mathcal{L}_{H\psi} + \mathcal{L}_{\tilde{H}\psi} = \sum_f \left(-y_f \, \bar{Q} H d_R - y_f \, \bar{L} H \nu_R - y_f \, \bar{Q} \tilde{H} u_R - y_f \, \bar{L} \tilde{H} e_R \right) + \text{c.c.}$$

$$H = \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}} + \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ i\phi_3 \end{pmatrix}$$

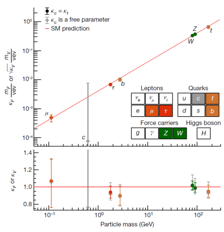
$$\longrightarrow \; -\frac{y_f}{\sqrt{2}} \left(\bar{u}_L \;\; \bar{d}_L \right) \cdot \begin{pmatrix} 0 \\ v + h \end{pmatrix} d_R =$$

$$= \underbrace{-\frac{y_f v}{\sqrt{2}}} \underbrace{\bar{d}_L d_R}_{\bar{\psi}_L \psi_R} - \frac{y_f}{\sqrt{2}} \underbrace{\bar{d}_L d_R h}$$

$$m_f \equiv \frac{y_f v}{\sqrt{2}}$$



$$\frac{y_f}{\sqrt{2}} = \frac{m_f}{v}$$



The Higgs Sector

$$\mathcal{L}_H = |D_\mu H|^2 + \underbrace{\mu^2 H^\dagger H - \lambda (H^\dagger H)^2} =$$

$$\longrightarrow V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + \ldots \longrightarrow V(H) = -\mu^2 h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4$$

$$m_h \equiv \sqrt{2}\mu = \sqrt{2\lambda}\, v$$

$$v = \left(\frac{\mu^2}{\lambda} \right)^{1/2}$$

$-\lambda v h^3 \Longrightarrow$ 

$-\frac{1}{4}\lambda h^4 \Longrightarrow$ 

UNTESTED PREDICTIONS

(AS OF 2023)