XLVI CONGRESSO PAULO LEAL
DE FÍSICA

The Standard Model of particle physics
(an introduction)

Ricardo D’Elia Matheus

The main ingredients

THE THEORY: QFT (**Q**uantum **F**ield **T**heory)

RELATIVISTIC

QUANTUM

The main ingredients

THE THEORY: QFT (**Q**uantum **F**ield **T**heory)

RELATIVISTIC

QUANTUM

THE MODEL: SM (**S**tandard **M**odel of Particle Physics)

PARTICLE CONTENT

SYMMETRIES
(AND REDUNDANCIES)

SPONTANEOUS SYMMETRY BREAKING
(SSB)

The main ingredients

MATTER CONTENT

(LEPTONS QUARKS GAUGE BOSONS HIGGS)

SYMMETRIES

(AND THEIR BREAKING)

$$SU(3)\times \underbrace{SU(2)_L \times U(1)_Y}_{\hookrightarrow U(1)_{EM}}$$

The main ingredients

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(AND THEIR BREAKING)

$$\underbrace{SU(3) \times SU(2)_L \times U(1)_Y}_{\hookrightarrow U(1)_{EM}}$$

$\xRightarrow[\text{FIXES}]{\text{UNIQUELY}}$ $\mathcal{L}_{SM}$ (LAGRANGIAN DENSITY)

The main ingredients

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(AND THEIR BREAKING)

$$\underbrace{SU(3) \times SU(2)_L \times U(1)_Y}_{\hookrightarrow U(1)_{EM}}$$

UNIQUELY FIXES $\mathcal{L}_{SM}$ (LAGRANGIAN DENSITY)

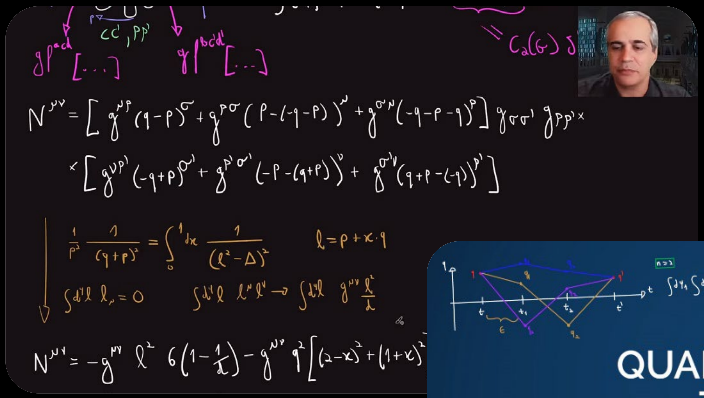
QFT

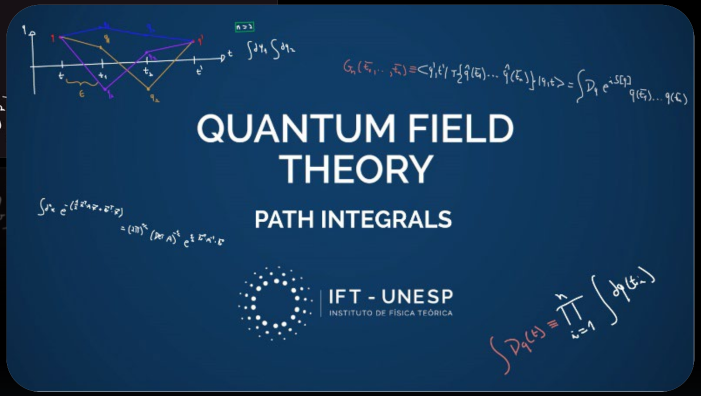
PREDICTIONS
- SPECTRA
- CROSS-SECTIONS
- BOUND STATES

Quantum Field Theory on YouTube

Playlist links:

QFT I:





QFT II:

Motivation for Quantum Field Theory

RELATIVISTIC QUANTUM MECHANICS DOES NOT WORK!

- NOT CAUSAL! (SEE QFT I - VIDEO I)
- WAVE FUNCTION DESCRIBES THE PROPAGATION OF ONE PARTICLE STATES (MORE PRECISELY: STATES WITH A DEFINITE NUMBER OF PARTICLES)

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NUMBER OF PARTICLES IS NOT FIXED ANYMORE!

Quantum Field Theory

INSPIRED BY: $\vec{E}, \vec{B} \rightleftharpoons$ NUMBER OF PHOTONS WE WILL QUANTIZE A FIELD THEORY:

Quantum Field Theory

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MECHANICS

QUANTUM MECHANICS

$$q_i(t), p_i(t) \xrightarrow{\text{QUANT.}} \hat{q}_i(t), \hat{p}_i(t)$$

Quantum Field Theory

INSPIRED BY: $\vec{E}, \vec{B} \rightleftharpoons$ NUMBER OF PHOTONS WE WILL QUANTIZE A FIELD THEORY

MECHANICS

$$q_i(t),\ p_i(t)$$

$$\{p_i, q_j\}_{P.B.} = -\delta_{ij}$$

$$\dot{q}_i(t) = \{q_i, H\}_{P.B.} = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i(t) = \{p_i, H\}_{P.B.} = -\frac{\partial H}{\partial q_i}$$

$$\{f, g\}_{P.B.} = \sum_i \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

$\xrightarrow{\text{QUANT.}}$

$$\{\ ,\ \}_{P.B.} \Rightarrow -\frac{i}{\hbar}[\ ,\ ]$$

$$\hbar \to 1$$

QUANTUM MECHANICS

$$\hat{q}_i(t),\ \hat{p}_i(t)$$

$$[\hat{p}_i, \hat{q}_j] = -i\hbar\, \delta_{ij}$$

$$\frac{d\hat{q}_i}{dt} = -i\hbar\, [\hat{q}_i, \hat{H}]$$

$$\frac{d\hat{p}_i}{dt} = -i\hbar\, [\hat{p}_i, \hat{H}]$$

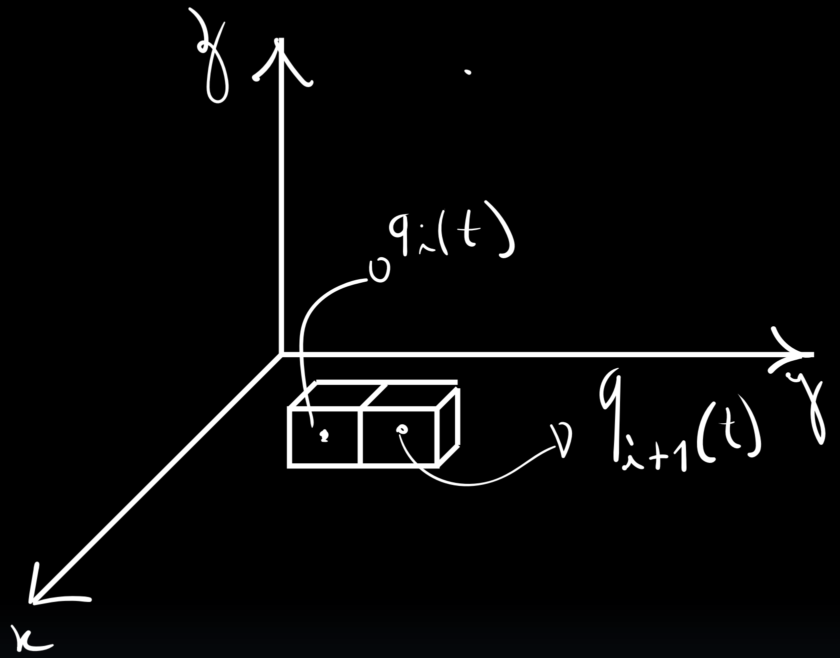
Note how space and time are treated differently!

R.D.Matheus

Quantum Field Theory

Easy generalization to Fields:

$\phi(\vec{x},t)$:



$$\{q_i(t)\} \sim \{\phi_i(t)\} \underset{\text{DISCRETE}}{\overset{\text{CONTINUUM}}{\rightleftarrows}} \phi(\vec{x},t)$$

$$\{p_i(t)\} \sim \{\Pi_i(t)\} \rightleftarrows \Pi(\vec{x},t)$$

Quantum Field Theory

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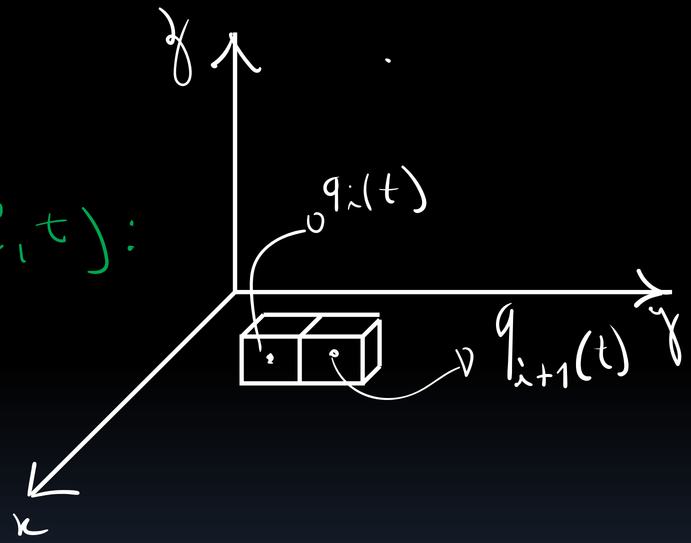

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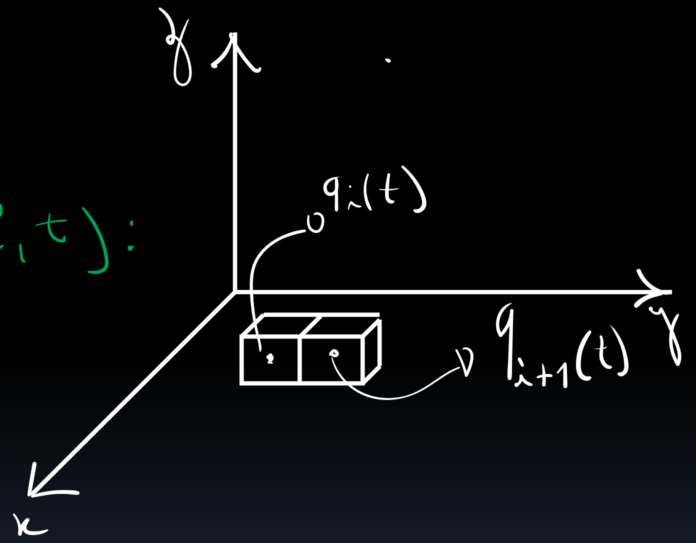

$$\left.\begin{aligned}
\{q_i(t)\} \sim \{\phi_i(t)\} &\underset{\text{DISCRETE}}{\overset{\text{CONTINUUM}}{\rightleftarrows}} \phi(\vec{x},t) \\
\{p_i(t)\} \sim \{\Pi_i(t)\} &\rightleftarrows \Pi(\vec{x},t)
\end{aligned}\right\} \text{ClFT}$$

$$\{\phi_i, \Pi_j\}_{P.B.} \Rightarrow -\frac{i}{\hbar}[\hat{\phi}_i, \hat{p}_j] \qquad \Big\Downarrow \ \text{QUANT.}$$

$$[\hat{\phi}_i, \hat{\Pi}_j] = i\delta_{ij} \rightleftarrows \underbrace{[\hat{\phi}(\vec{x},t), \hat{\Pi}(\vec{y},t)] = i\delta^3(\vec{x}-\vec{y})}_{\text{QFT}}$$

Quantum Field Theory

We will demand locality on the CℓFT:

$$S = \int dt\, L = \int dt \int d^3x \, \underbrace{\mathcal{L}(\phi, \partial_\mu \phi)}_{\text{Lagrangian Density}} = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\pi(\vec{x}, t) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

$$H = \int d^3x \left[\pi(\vec{x}, t)\, \dot{\phi}(\vec{x}, t) - \mathcal{L} \right] = \int d^3x \, \mathcal{H}(\phi, \pi)$$

$\mathcal{H}$ ⟶ Hamiltonian Density

R.D.Matheus

Building Lagrangians

It's all about symmetries, starting with Lorentz invariance:

Lorentz Group SO(1,3)

$$x_\mu \xrightarrow{\text{LORENTZ}} x'_\mu = \Lambda_\mu{}^\nu x_\nu$$

$$s = x^\mu x_\mu \xrightarrow{\text{LORENTZ}} s = x'^\mu x'_\mu$$

SCALARS: $\phi(x) \xrightarrow{\text{LORENTZ}} \phi'(x') = \phi(x)$ (spin 0 particles)

SPINORS: $\psi(x) \xrightarrow{\text{LORENTZ}} \psi'(x') = M_D(\Lambda)\,\psi(x)$ (spin 1/2 particles)

VECTORS: $A_\mu(x) \xrightarrow{\text{LORENTZ}} A'_\mu(x') = \Lambda_\mu{}^\nu A_\nu(x)$ (spin 1 particles)

Building Lagrangians

It's all about symmetries, starting with Lorentz invariance:

Lorentz Group SO(1,3)

$$x_\mu \xrightarrow{\text{Lorentz}} x'_\mu = \Lambda_\mu{}^\nu x_\nu$$

$$s = x^\mu x_\mu \xrightarrow{\text{Lorentz}} s = x'^\mu x'_\mu$$

Scalars: $\phi(x) \xrightarrow{\text{Lorentz}} \phi'(x') = \phi(x)$

Spinors: $\psi(x) \xrightarrow{\text{Lorentz}} \psi'(x') = M_D(\Lambda)\,\psi(x)$

Vectors: $A_\mu(x) \xrightarrow{\text{Lorentz}} A'_\mu(x') = \Lambda_\mu{}^\nu A_\nu(x)$

$$\Rightarrow (\phi)^n, \quad A_\mu A^\mu, \quad \underbrace{\overline{\psi}}\psi, \quad A^\mu \overline{\psi}\gamma_\mu\psi, \quad F_{\mu\nu}F^{\mu\nu}, \quad \ldots$$

$\hookrightarrow \psi^\dagger\gamma^0$: $\overline{\psi}(x) \xrightarrow{\text{Lorentz}} \overline{\psi}(x) M_D^{-1}(\Lambda)$

Example of Quantization:

(we will be back so symmetries and Lagr. building soon)

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2$$

(Exercise) Show that Euler-Lagrange leads to: $(\partial_\mu\partial^\mu + m^2)\phi = 0$

Solutions: $e^{\pm i(kx - \omega_k t)}$; $\omega_k = \sqrt{k^2 + m^2}$

General Solution:

$$\phi(x,t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left[a_k e^{i(kx-\omega_k t)} + a_k^\dagger e^{-i(kx-\omega_k t)}\right]$$

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$$\left[\hat{\phi}(\vec{x},t), \hat{\Pi}(\vec{x}',t)\right] = i\delta^3(\vec{x}-\vec{x}') \Longrightarrow \left[\underbrace{\hat{a}_k}_{\text{ANNIHILATION}}, \underbrace{\hat{a}^\dagger_{k'}}_{\text{CREATION}}\right] = (2\pi)^3\delta^3(k-k')$$

Example of Quantization:

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - \frac{m^2}{2}\phi^2$$

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3} \omega_k \left[\underbrace{\hat{a}^+_k \hat{a}_k}_{} + \frac{1}{2} \right] = \overbrace{\int \frac{d^3k}{(2\pi)^3} \omega_k \hat{N}_k}^{:\hat{H}:} + \text{VACUUM ENERGY}$$

$$\hat{P}^i = \int d^3x \, T^{0i} = \int \frac{d^3k}{(2\pi)^3} k^i \hat{N}_k$$

$$\boxed{E_p = \sqrt{p^2 + m^2}}$$

Particles are eigenstates of $\hat{H}, \hat{\vec{P}}, \hat{N}$ with $n_p E_p, n_p \vec{p}, n_p$

R.D.Matheus

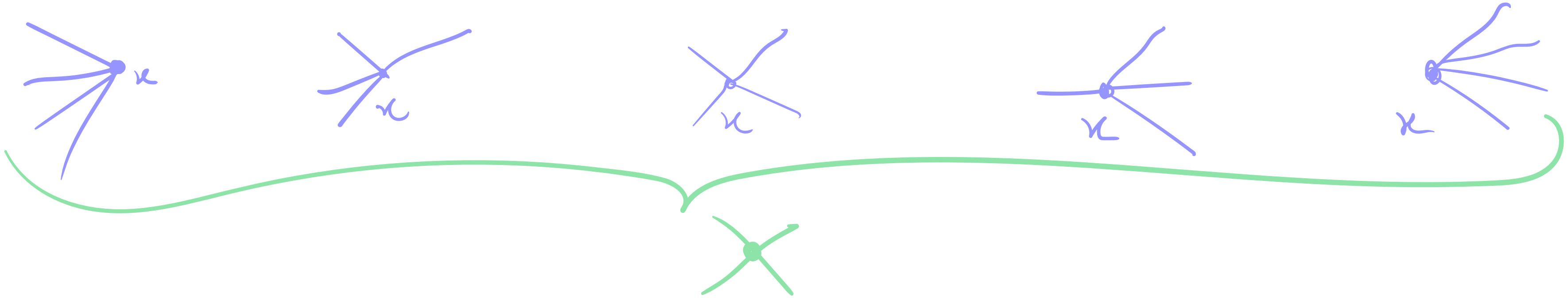
Free Theories

$$\mathcal{L}_\psi = i\,\bar{\Psi}\gamma^\mu\partial_\mu\psi - m\bar{\Psi}\psi$$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Interactions

Eg: $\lambda \phi^4(x) \sim aaaa + a^\dagger aaa + a^\dagger a^\dagger aa + a^\dagger a^\dagger a^\dagger a + a^\dagger a^\dagger a^\dagger a^\dagger$

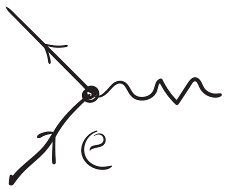


Coupling Constant

Interactions

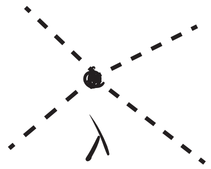
QED:

$$e\,\bar{\psi} A_\mu \gamma^\mu \psi \Rightarrow$$



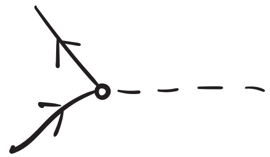
$\lambda\phi^4$:

$$\frac{\lambda}{4!}\phi^4 \Rightarrow$$



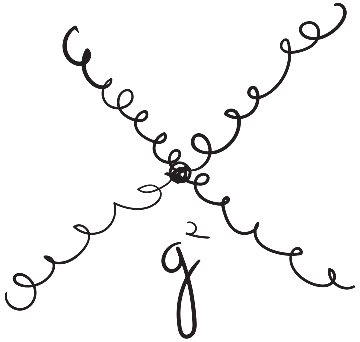
Yukawa:

$$y\,\phi\,\bar{\psi}\,\psi \Rightarrow$$



QCD:

$$-\frac{g^2}{4} f^{abc} f^{ab'c'} G^b_\mu G^c_\nu G^{b'\mu} G^{c'\nu} \Rightarrow$$

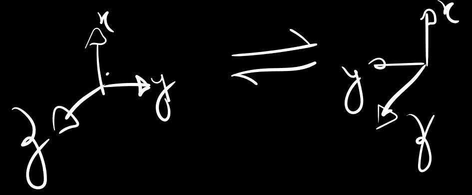


More Symmetries

(Back to Lagrangian building!)

Discrete part of the Lorentz Group:

$$s^2 = x_\mu x^\mu \longrightarrow s^2 = x'_\mu x'^\mu$$

- C: PARTICLES $\rightleftharpoons$ ANTIPARTICLES
- P: CHIRALITY 
- T: TIME REVERSAL

But also Internal Symmetries:

COMPLEX FIELD: $\phi = \phi_1 + i\,\phi_2 \qquad \phi_{1,2} \in \mathbb{R}$

$$\phi \xrightarrow{U} \underbrace{e^{i\theta}}_{U}\,\phi \qquad \text{(GLOBAL } U(1)\text{)}$$

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(GLOBAL $U(1)$)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi + \cancel{m^2 \phi\phi} + \cancel{\frac{\lambda_3}{2} \phi\phi\phi^*} + \lambda_n (\phi^*\phi)^n$$

More Symmetries

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$$\phi \xrightarrow{U(1)} e^{i\theta(x)}\, \phi \qquad \text{(LOCAL OR GAUGE } U(1)\text{)}$$

More Symmetries

But also Internal Symmetries:

$$\phi \xrightarrow{U(1)} e^{i\theta(x)}\phi \quad (\text{LOCAL OR GAUGE } U(1))$$

$$A_\mu \xrightarrow{U(1)} A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi \xrightarrow{U(1)} e^{i\theta(x)} D_\mu \phi$$

Scalar QED

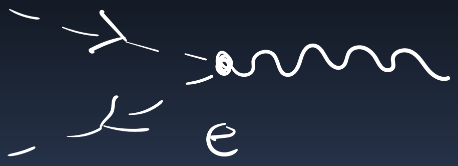
$\phi \xrightarrow{U(1)} e^{i\theta(x)}\phi$ (LOCAL OR GAUGE $U(1)$)

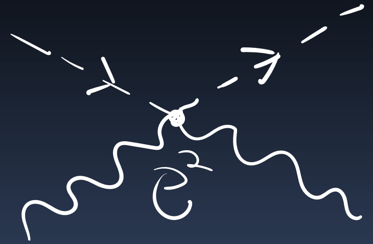
$A_\mu \xrightarrow{U(1)} A_\mu - \frac{1}{e}\partial_\mu \theta(x)$

$$\mathcal{L} = |D_\mu \phi|^2 + m^2 \phi^* \phi - \underbrace{\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{} + \cancel{m_\gamma A_\mu A^\mu} =$$

(Exercise) Show that this is invariant

$$= |\partial_\mu \phi|^2 + m \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + ie A^\mu (\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) + e^2 A_\mu A^\mu \phi^* \phi$$



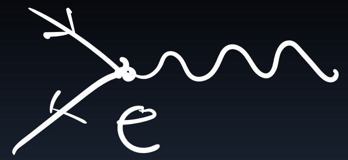


QED!

$$\psi \xrightarrow{U(1)} e^{i\theta(x)}\psi$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} =$$

$$= \bar{\psi}(i\gamma^\mu \partial_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma^\mu\psi A_\mu$$



Non-abelian symmetry groups

(isospin) $N = \begin{pmatrix} p \\ n \end{pmatrix}_A \xrightarrow[\text{GLOBAL}]{SU(2)} N' = e^{i\alpha_a \frac{\sigma^a_{AB}}{2}} \begin{pmatrix} p \\ n \end{pmatrix}_B$

$A = 1, 2$ $a = 1, 2, 3$

(QCD) $q = \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix}_A \xrightarrow[\text{LOCAL}]{SU(3)} q' = e^{i\alpha_a t^a_{AB}} \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix}_B$

$A = 1, 2, 3$ $a = 1, \ldots, 8$

Gell-Mann Matrices

Generators for the fundamental representation of SU(3)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Non-abelian symmetry groups

The local version also allows/demands Gauge fields:

$$\psi(x) \xrightarrow[\text{LOCAL}]{\tilde{\text{N}}\text{-ABEL.}} \underbrace{U(x)}_{\to\ \text{EXP}(i\alpha_a t^a)}\ \psi(x)$$

$$D_\mu = \partial_\mu - i g \underbrace{A_\mu^a\, t^a}_{\text{MATRIX}} \longrightarrow \text{ONE FOR EACH "DIRECTION" (GENERATOR)}$$

$$A_\mu^a = (A_\mu^1, A_\mu^2, A_\mu^3)$$

$$A_\mu^M = \frac{1}{2}\begin{pmatrix} A_\mu^3 & A_\mu^1 - i A_\mu^2 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 \end{pmatrix}$$

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$$\psi(x) \xrightarrow[\text{LOCAL}]{\tilde{\text{N}}\text{-ABEL.}} \underbrace{U(x)}_{\exp(i\alpha_a t^a)} \psi(x)$$

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$$A_\mu^a t^a \xrightarrow[\text{LOCAL}]{\tilde{\text{N}}\text{-ABEL.}} U(x) \cdot A_\mu^a t^a \cdot U^\dagger(x) = \left(A_\mu^a + \frac{1}{g}(\partial_\mu \alpha^a) + f_{abc} A_\mu^b \alpha^c \right) t^a + O(\alpha^2)$$

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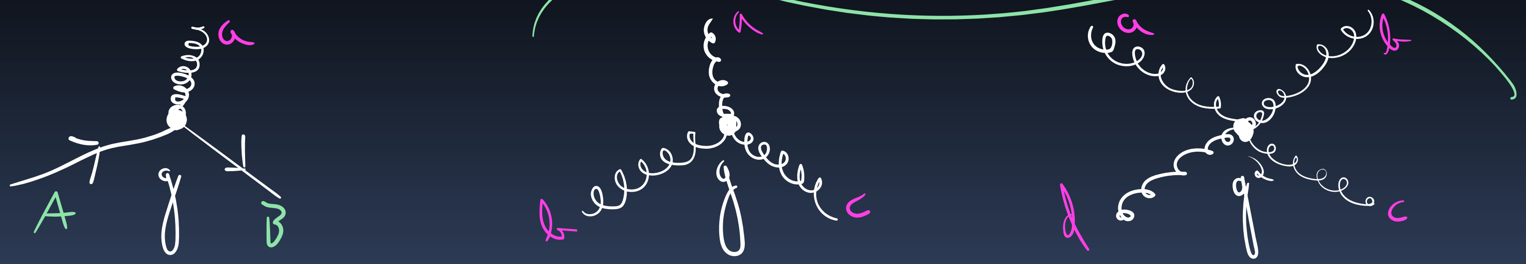
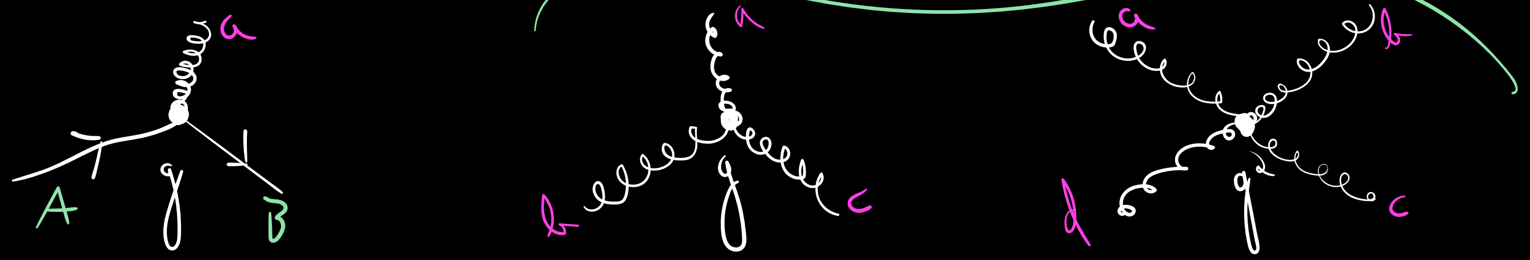
$$\mathcal{L}_{YM} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}\left(F_{\mu\nu}^a\right)^2 \longrightarrow F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$



Non-abelian symmetry groups

The local version also allows/demands Gauge fields:

$$\mathcal{L}_{ym} = \underbrace{\bar{\Psi}(i\not{D} - m)\Psi}_{} - \frac{1}{4}\underbrace{(F^a_{\mu\nu})^2}_{} \to F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$



IMPORTANT POINT: $\cancel{m^2 A^a_\mu A^{\mu a}}$