XLVI CONGRESSO PAULO LEAL DE FÍSICA

The Standard Model of particle physics (an introduction)

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Lagrangian Reading Guide:

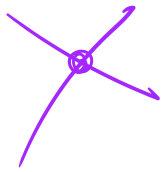
$$\mathcal{L} = \overbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - m^2\phi\phi}^{\text{Free (Quadratic on Field)}} - \overbrace{\frac{\lambda}{4!}\phi^4}^{\text{Interaction}}$$

$$T - V$$

$$V = m^2\phi\phi + \frac{\lambda}{4!}\phi^4$$

Lagrangian Reading Guide:

$$\mathcal{L} = \overbrace{\underbrace{\frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi}_{\text{PROPAGATING PARTICLE}} \underbrace{- m^2 \phi \phi}_{\text{WITH MASS } m}}^{\text{FREE (QUADRATIC ON FIELD)}} \underbrace{\overbrace{- \frac{\lambda}{4!} \phi^4}^{\text{INTERACTION}}}_{}$$



Lagrangian Reading Guide:

FIXED BY SYMMETRY (OPERATORS)

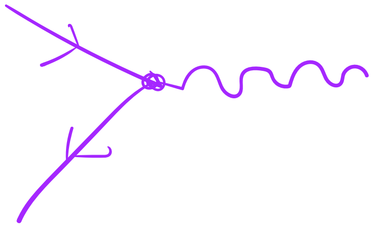
$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - m^2 \phi \phi - \frac{\lambda}{4!}\phi^4$$

MEASURED IN EXPERIMENTS (COEFFICIENTS)

Lagrangian Reading Guide:

$$\mathcal{L} = \overbrace{i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{FREE}} \overbrace{\underbrace{- e\bar{\psi}\gamma^{\mu}\psi A_{\mu}}}^{\text{INTERACTION}}$$

$$m_{\gamma} = 0$$



Abelian vs. Non-Abelian Gauge Bosons

Abelian:

$$\psi \xrightarrow{U(1)} e^{i\theta(x)}\,\psi \qquad\qquad A_\mu \xrightarrow{U(1)} A_\mu - \frac{1}{e}\,\partial_\mu\,\theta(x)$$

Non-Abelian:

$$\psi(x) \xrightarrow[\text{LOCAL}]{\tilde{\text{N}}\text{-ABEL.}} \underbrace{U(x)}_{\text{EXP}\left(i\alpha_a t^a\right)}\,\psi(x)$$

$$A^a_\mu t^a \xrightarrow[\text{LOCAL}]{\tilde{\text{N}}\text{-ABEL.}} U(x)\cdot A^a_\mu t^a \cdot U^\dagger(x) = \left(A^a_\mu + \frac{1}{g}(\partial_\mu \alpha^a) + f_{abc}\,A^b_\mu\,\alpha^c\right)t^a + O(\alpha^2)$$

Abelian vs. Non-Abelian Gauge Bosons

Abelian:

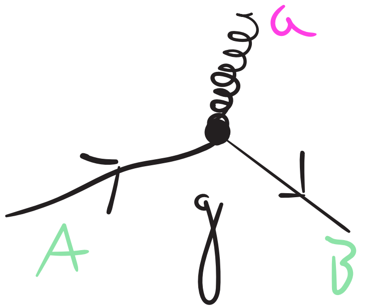
$$D_\mu = \partial_\mu + ie A_\mu$$

$$\bar{\Psi}(i\gamma^\mu D_\mu)\Psi = \bar{\Psi}(i\gamma^\mu \partial_\mu)\Psi - e\bar{\Psi}\gamma^\mu \Psi A_\mu$$

Non-Abelian:

$$\bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2$$

$$D_\mu = \partial_\mu - ig \underbrace{A^a_\mu t^a}$$



Abelian vs. Non-Abelian Gauge Bosons

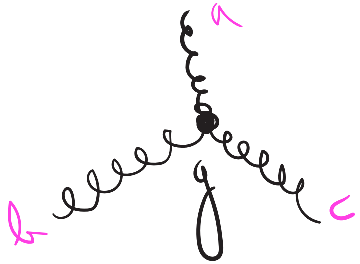
Abelian:

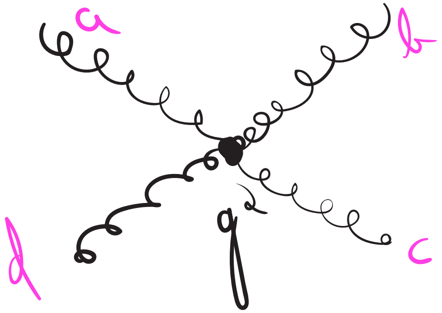
$$F_{\mu\nu} = \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)$$

Non-Abelian:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$-\frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \Longrightarrow$$





Noether's Theorem

$$\left.\begin{aligned}
\phi &\xrightarrow[\alpha]{SYM} \phi + \alpha \Delta\phi \\
S &\xrightarrow[\alpha]{SYM} S \\
\mathcal{L} &\xrightarrow[\alpha]{SYM} \mathcal{L} + \alpha \underbrace{\partial_\mu J^\mu}_{\Delta\mathcal{L}}
\end{aligned}\right\} \quad \Delta S = 0$$

$$\partial_\mu j^\mu = 0$$

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - J^\mu$$

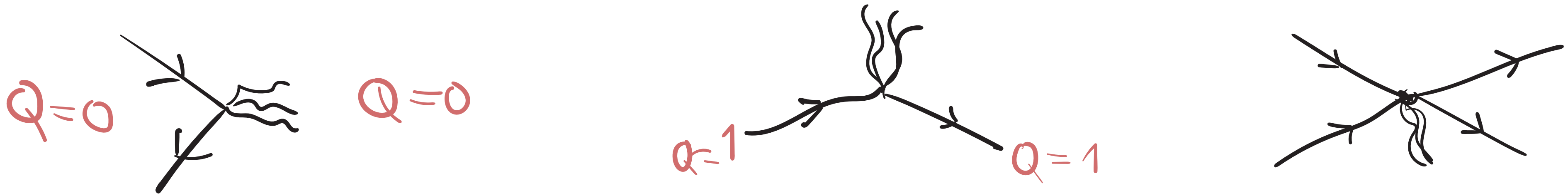
Noether's Theorem

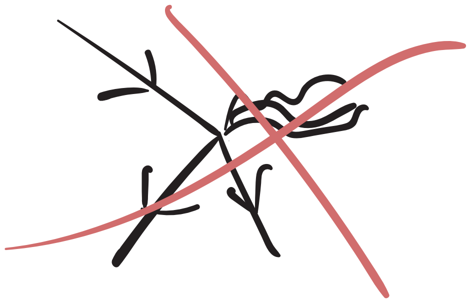
SYMMETRY		"CHARGE"
TIME TRANS.:	$t \to t + a$	E
TRANSLATION:	$\vec{x} \to \vec{x} + \vec{a}$	$\vec{P}$
ROTATION:	$x_i \to R_{ij} x_j$	$\vec{L}$
U(1) PHASE:	$\phi(x_\mu) \to e^{i\theta} \phi(x_\mu)$	Q

Conservation Constrain Decays

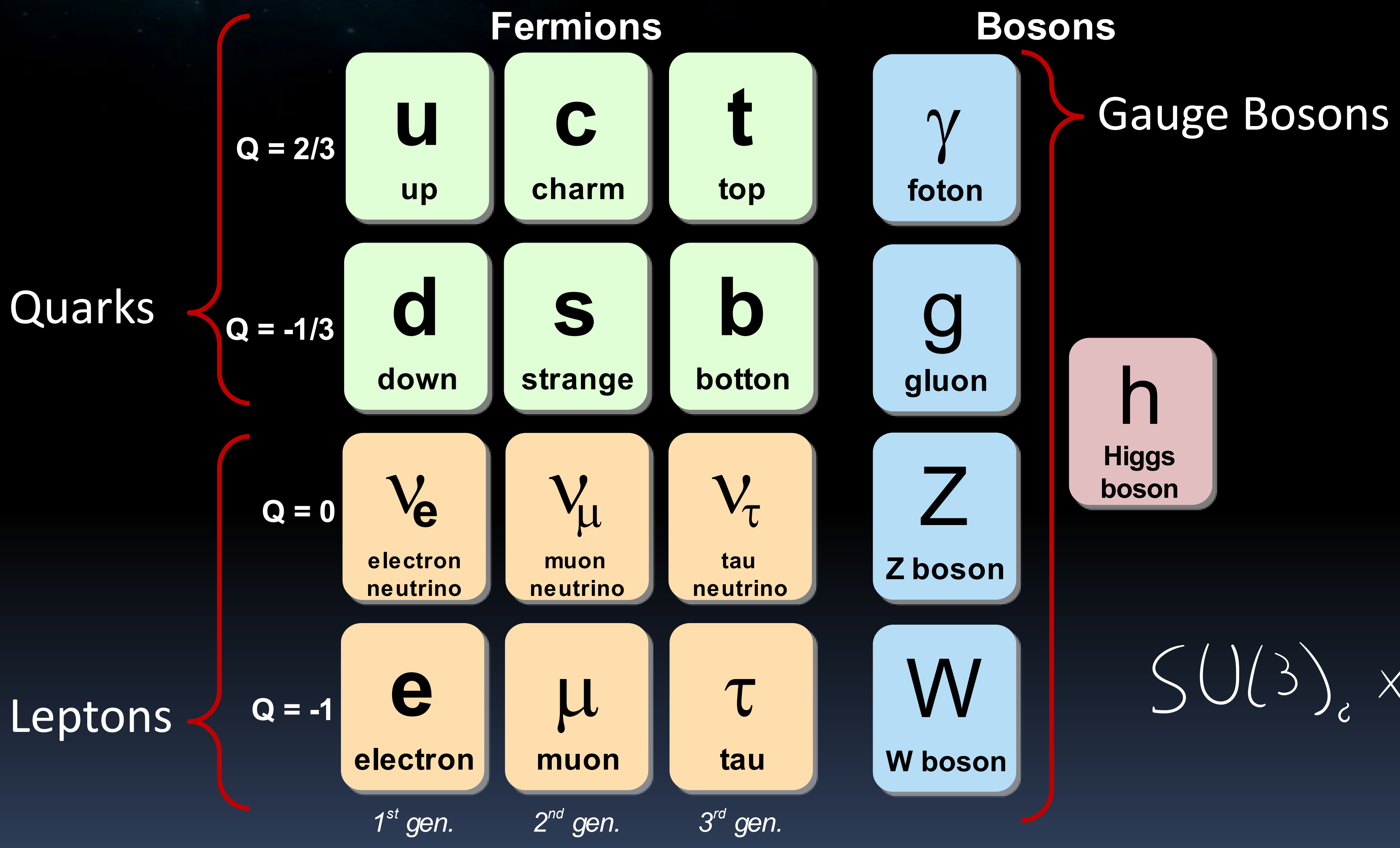
U(1): $\phi^* \phi \subset \mathcal{L}$

$\bar{\psi} \psi$





The Standard Model



$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Chirality Matters

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad (\gamma_5)^2 = \hat{1}$$

$$P_L \equiv \frac{1-\gamma_5}{2} \qquad P_R \equiv \frac{1+\gamma_5}{2}$$

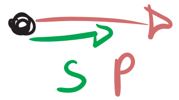
Exercise 5: show

$$\begin{cases} P_L + P_R = \hat{1} \\ P_L^2 = P_L \qquad P_R^2 = P_R \\ P_L P_R = P_R P_L = 0 \end{cases}$$

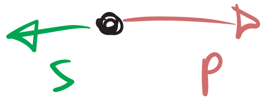
$$\Longrightarrow \Psi = \underbrace{P_L \Psi}_{\Psi_L} + \underbrace{P_R \Psi}_{\Psi_R}$$

Chirality Matters

MASSLESS PARTICLES: CHIRALITY-STATES = HELICITY-STATES



$H=+ \Rightarrow \psi=\psi_R$



$H=- \Rightarrow \psi=\psi_L$

MASSIVE PARTICLES: CHIRALITY-STATES $\xrightarrow{\text{NOT}}$ HELICITY-STATES

$\xrightarrow{\text{NOT}}$ MASS-STATES

Chirality Matters

Exercise 6: show

$$\bar{\psi} \equiv \psi^\dagger \gamma_0$$

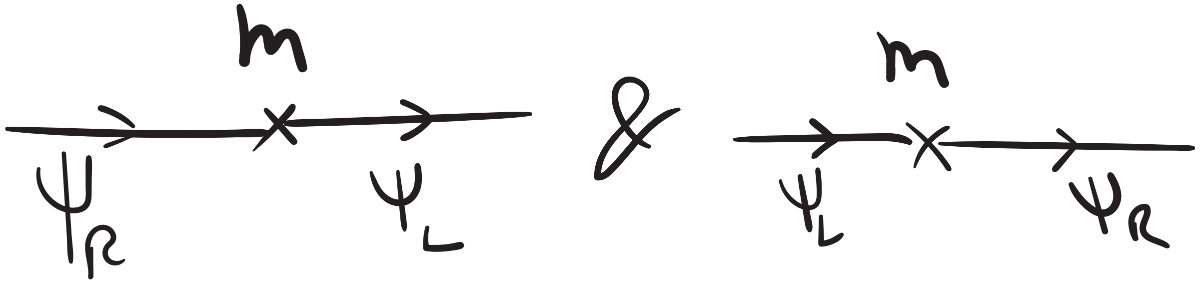
$$\bar{\psi}_{L/R} \equiv (P_{L/R}\,\psi)^\dagger \gamma_0$$

$$\gamma_5^\dagger = \gamma_5$$

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_L\gamma_\mu\psi_L + \bar{\psi}_R\gamma_\mu\psi_R$$

$$\underbrace{m\bar{\psi}\psi}_{\text{MASS BASIS}} = m\,\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

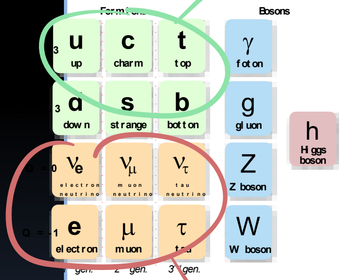
CHIRALITY BASIS



Chirality Matters for $U(1)_Y$

$U(1)$ "HYPERCHARGES"

$$U(1)_Y : \psi \xrightarrow{U(1)_Y} e^{i y \theta(x)} \psi$$



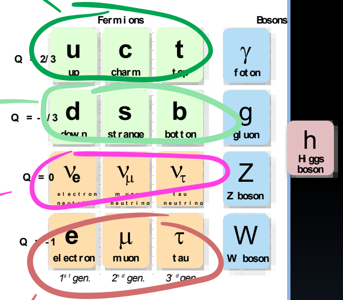
	LEFT-HANDED	RIGHT-HANDED
QUARKS:	$y = +1/6$	$y = 2/3$
		$y = -1/3$
LEPTONS:	$y = -1/2$	y_R? $y = 0$
		$y = -1$

"UP"

"DOWN"

"UP"

"DOWN"



Chirality Matters for $SU(2)_L$

$SU(2)_L$: $\psi_L \xrightarrow{SU(2)_L} e^{i\frac{\sigma_a}{2}\theta_a(x)}\,\psi_L = e^{i t_a \theta_a}\,\psi_L$

$\psi_R \xrightarrow{SU(2)_L} \psi_R$

SU(2) DOUBLETS:

$$\underbrace{L^e = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad L^\mu = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \qquad L^\tau = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}}_{L^\ell\,,\quad \ell = 1,2,3}$$

$$Q^1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad Q^2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \qquad Q^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

Parity Violation makes Fermion Masses "Tricky"

$$m\,\overline{\psi_L^e}\,\psi_R^e \xrightarrow{SU(2)_L\times U(1)_Y} m\,\overline{\psi_L^e}\,e^{-it_a\theta_a - i(y_L^e - y_R^e)\theta}\,\psi_R^e$$

FORBIDDEN BY $U(1)$: $\underbrace{y_L^e}_{-1} \neq \underbrace{y_R^e}_{-2}$

FORBIDEN BY $SU(2)$

And since we are talking about mass...

GAUGE SECTOR:

THEORY

EXPERIMENT

$U(1)$: B_μ

$SU(2)$: $W^1_\mu, W^2_\mu, W^3_\mu$

$SU(3)$: $G^1_\mu, \ldots, G^8_\mu$

ALL MASSLESS

$(F^a_{\mu\nu})^2$ ✓

$\cancel{A^a_\mu A^{a\mu}}$

γ MASSLESS P-PRESERVING

W^+, W^-, Z MASSIVE

?!

$G^1_\mu, \ldots, G^8_\mu$ MASSLESS ✓

Spontaneous Symmetry Breaking

U(1) EXAMPLE:

$$\begin{cases} 1\ \text{SCALAR: } \phi \xrightarrow{U(1)} e^{i\theta(x)}\phi \\ 1\ \text{GAUGE-BOSON: } A_\mu(x) \xrightarrow{U(1)} A_\mu(x) - \frac{1}{e}\partial_\mu \theta(x) \end{cases}$$

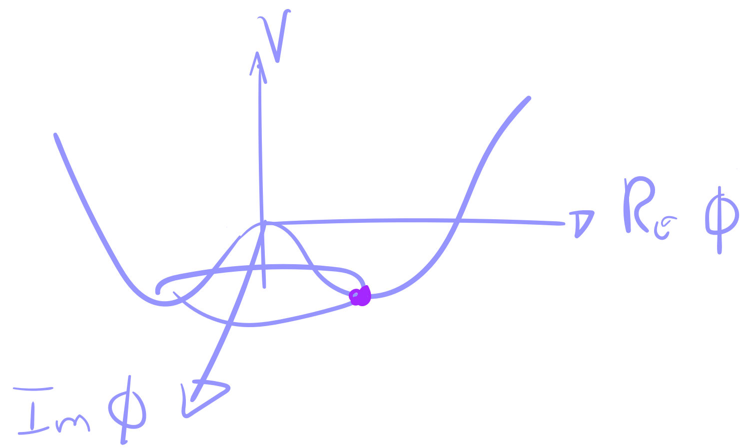
$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 \underbrace{+ \mu^2 \phi^*\phi - \frac{\lambda}{2}(\phi^*\phi)^2}$$

$$V(\phi) = -\mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$$

Spontaneous Symmetry Breaking

PARTICLES ARE EXCITATIONS AROUND THE VACUUM

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \implies |\phi| = \underbrace{\phi_0}_{} = \left(\frac{\mu^2}{\lambda}\right)^{1/2}$$



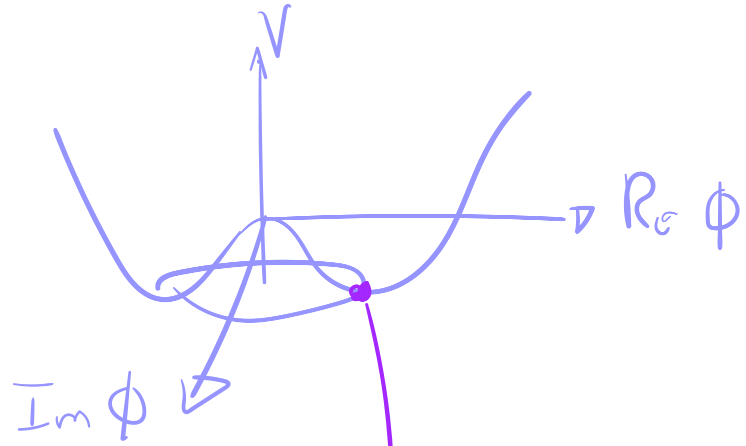
$\langle \phi \rangle = \phi_0 \rightarrow$ VACUUM EXPECTATION VALUE (VEV)

Spontaneous Symmetry Breaking

PARTICLES ARE EXCITATIONS AROUND THE VACUUM

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \Longrightarrow |\phi| = \phi_0 = \left(\frac{\mu^2}{\lambda}\right)^{1/2}$$

$\langle \phi \rangle = \phi_0$ → VACUUM EXPECTATION VALUE (VEV)



$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$$

Spontaneous Symmetry Breaking

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^* \phi)^2$$

$$\Downarrow \quad \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2} 2\mu^2 \phi_1^2 + \mathcal{O}(\phi_i^3)$$ *(Exercise)*

Spontaneous Symmetry Breaking

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^* \phi)^2$$

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2}\,2\mu^2 \phi_1^2 + \mathcal{O}(\phi_1^3)$$ *(Exercise)*

ϕ_1 EXCITATIONS: MASS $\sqrt{2}\,\mu$

ϕ_2 EXCITATIONS: MASSLESS (NO ϕ_2^2 TERM)

Spontaneous Symmetry Breaking

WHAT HAPPENS TO $D_\mu \phi$?

$$|D_\mu \phi|^2 = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \sqrt{2}\, e\, \phi_0 A_\mu \partial^\mu \phi_2 + \underbrace{e^2 \phi_0^2 A_\mu A^\mu}_{!!!} + \underbrace{\ldots}_{\text{CUBIC AND QUARTIC TERMS}}$$

(Exercise)

Spontaneous Symmetry Breaking

WHAT HAPPENS TO $D_\mu \phi$?

$$|D_\mu \phi|^2 = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \sqrt{2}\, e\, \phi_0\, A_\mu \partial^\mu \phi_2 + \underbrace{e^2 \phi_0^2 A_\mu A^\mu}_{!!!} + \underbrace{\ldots}_{\text{CUBIC AND QUARTIC TERMS}}$$

(Exercise)

$$e^2 \phi_0^2 A_\mu A^\mu = \frac{1}{2} m_A^2 A_\mu A^\mu \qquad m_A = \sqrt{2}\, \underbrace{e}_{\text{COUPLING WITH THE SCALAR}}\, \underbrace{\phi_0}_{\text{VEV OF THE SCALAR}}$$

Spontaneous Symmetry Breaking

GOLDSTONE THEOREM:

GLOBAL $G \xrightarrow{\text{SSB}}$ GLOBAL $H \Rightarrow$ 1 MASSLESS $\underbrace{\text{"GOLDSTONE BOSON"}}_{\text{NGB}}$ FOR EACH GENERATOR BROKEN

E.G. $U(1) \xrightarrow{\text{SSB}}$ NOTHING $\Rightarrow \phi_2$ MASSLESS

HIGGS MECHANISM:

GLOBAL $G \xrightarrow{\text{SSB}}$ GLOBAL $H \Rightarrow$ NGBs MIX WITH GAUGE BOSONS TO GIVE THEM MASS

(IN THE PRESENCE OF GAUGE G)

E.G. $U(1) \xrightarrow{\text{SSB}}$ NOTHING $\Rightarrow \phi_2$ AND A_μ MIX:

$\sqrt{2}\, e\, \phi_0\, A_\nu\, \partial^\nu \phi_2 \Rightarrow$

(A BASIS COULD BE CHOSEN SO THAT $\cancel{\phi_2}$, A_ν HAS 3 POLARIZATIONS)

Spontaneous Symmetry Breaking

A SECOND EXAMPLE: $\phi = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}$ $\quad \phi_i \in \mathbb{R}$ $\quad |\phi|^2 = \phi^i\phi^i$

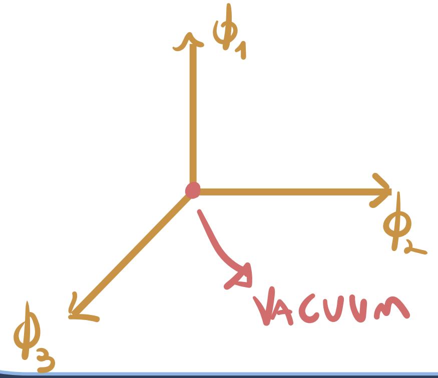
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi^i)(\partial_\mu\phi^i) - V(|\phi|^2)$$

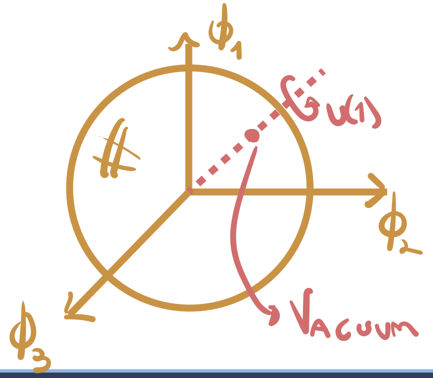
Symmetric under $SO(3)$

$$\phi^i \xrightarrow{SO(3)} R^{ij}\phi^j$$

$$R^T R = R R^T = \mathbb{1}_{3\times 3}$$

$$V(|\phi|^2) = \frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}(|\phi|^2)^2 \qquad V(|\phi|^2) = -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}(|\phi|^2)^2$$





$$SO(3) \longrightarrow U(1)$$

3 ROTATIONS $\qquad$ 1 ROTATION

SSB in the Standard Model

Higgs Doublet:

$$H = \begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} = \underbrace{\begin{pmatrix} Re[\phi] + i\, Im[\phi] \\ Re[\tilde{\phi}] + i\, Im[\tilde{\phi}] \end{pmatrix}}_{4\ D.o.F} \xrightarrow{SU(2)_L \times U(1)_Y} \underbrace{e^{i t_a \theta_a}}_{SU(2)} \underbrace{e^{i y_H \theta}}_{U(1)_Y} H$$

$$\phi, \tilde{\phi} \in \mathbb{C}$$

$$\boxed{y_H = \frac{1}{2}}$$

SSB in the Standard Model

$$V(H^\dagger H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \Rightarrow \underbrace{SU(2)_L}_{3} \times \underbrace{U(1)_Y}_{+\ 1\ \text{GEN.}} \xrightarrow{SSB} \underbrace{U(1)_{EM}}_{1\ \text{GEN.}}$$

GOLDSTONE: (GLOBAL) $\begin{cases} 3\ \text{NGB}_s \\ \boxed{1\ \text{massive SCALAR}} \end{cases}$

$h \in \mathbb{R}$

$\Rightarrow$ HIGGS (LOCAL) $\begin{cases} \underbrace{3\ \text{NGB} + B_\mu + W^1_\mu, W^2_\mu, W^3_\mu} \\ \Downarrow \\ \underbrace{3\ \text{MASSIVE GAUGE B}_s}_{W^+, W^-, Z} + \underbrace{1\ \text{MASSLESS G.B.}}_{\gamma} \end{cases}$

SSB in the Standard Model

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \underbrace{v + h} + i\phi_3 \end{pmatrix}$$

$$\langle H \rangle = \frac{1}{2} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle = \langle h \rangle = 0$$

$$\boxed{v = \left(\frac{\mu^2}{\lambda} \right)^{1/2}}$$

SSB in the Standard Model

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$D_\mu H = \Big(\partial_\mu - i\,\underbrace{g}_{\text{COUPLING OF } SU(2)_L} W^a_\mu t^a - \overbrace{\frac{i}{2}}^{Y_H = 1/2}\,\underbrace{g'}_{\text{COUPLING OF } U(1)_Y} B_\mu\Big) H$$

No GLUONS HERE

SSB in the Standard Model

$$H = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}}_{} + \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ h + i\phi_3 \end{pmatrix}$$

$$t^a = \frac{\sigma^a}{2} \qquad 1_{2\times 2}$$

$$|D_\mu H|^2 = (D_\mu H)^\dagger (D^\mu H) \supset \frac{1}{2}\begin{pmatrix} 0 & v \end{pmatrix}\left(g W_\mu^a t^a + \frac{1}{2} g' B_\mu\right)\left(g W^{\mu b} t^b + \frac{1}{2} g' B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix} =$$

$$= \frac{1}{2}\frac{v^2}{4}\left[g^2 \underbrace{(W_\mu^1)^2} + g^2 \underbrace{(W_\mu^2)^2} + \underbrace{(-g W_\mu^3 + g' B_\mu)^2} \right]$$

Exercise 10

3 massive G.B.

SSB in the Standard Model

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \qquad Z_\mu^0 \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g W_\mu^3 - g' B_\mu\right)$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g' W_\mu^3 + g B_\mu\right)$$

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$= \underbrace{\dots}_{\text{SUPER COMPLICATED}} \underbrace{-\frac{1}{4}\left(F_{\mu\nu}\right)^2}_{\text{QED}} + \frac{m_W^2}{2}\left(W_\mu^\pm W^{\pm\mu}\right) + \frac{m_Z^2}{2}\left(Z_\mu^0 Z^{0\mu}\right)$$

SSB in the Standard Model

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$= \underbrace{\ldots}_{\text{SUPER COMPLICATED}} \underbrace{-\frac{1}{4}\left(F_{\mu\nu}\right)^2}_{\text{QED}} + \frac{m_W^2}{2}\left(W^\pm_\mu W^{\pm\mu}\right) + \frac{m_Z^2}{2}\left(Z^0_\mu Z^{0\mu}\right)$$

$$m_W = g\frac{v}{2} \qquad m_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}$$

COUPLINGS AND VEVs !!

$$\frac{m_Z}{m_W} = \frac{\sqrt{g^2 + g'^2}}{g}$$