XLVI CONGRESSO PAULO LEAL
DE FÍSICA

The Standard Model of particle physics
(an introduction)

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SSB in the Standard Model

HIGGS DOUBLET:

$$H = \begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathrm{Re}[\phi] + i\,\mathrm{Im}[\phi] \\ \mathrm{Re}[\tilde{\phi}] + i\,\mathrm{Im}[\tilde{\phi}] \end{pmatrix}}_{4\text{ D.o.F}} \xrightarrow{SU(2)_L \times U(1)_Y} \underbrace{e^{i t_a \theta_a}}_{SU(2)} \underbrace{e^{i y_H \theta}}_{U(1)_Y} H$$

$\phi, \tilde{\phi} \in \mathbb{C}$

$$\boxed{y_H = \frac{1}{2}}$$

SSB in the Standard Model

$$V(H^\dagger H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \Rightarrow \underbrace{SU(2)_L}_{3} \times \underbrace{U(1)_Y}_{+\ 1\ \text{GEN.}} \xrightarrow{\text{SSB}} \underbrace{U(1)_{EM}}_{1\ \text{GEN.}}$$

$$\text{GOLDSTONE: (GLOBAL)} \begin{cases} 3\ \text{NGB}_s \\ \boxed{1\ \text{massive SCALAR}} \end{cases} \quad h \in \mathbb{R}$$

$$\Rightarrow \text{HIGGS (LOCAL)} \begin{cases} \underbrace{3\ \text{NGB} + B_\mu + W^1_\mu, W^2_\mu, W^3_\mu} \\ \Downarrow \\ \underbrace{3\ \text{MASSIVE GAUGE B}_s}_{W^+, W^-, Z} + \underbrace{1\ \text{MASSLESS G.B.}}_{\gamma} \end{cases}$$

SSB in the Standard Model

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \underbrace{v+h} + i\phi_3 \end{pmatrix}$$

$$\langle H \rangle = \frac{1}{2}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle = \langle h \rangle = 0$$

$$\boxed{v = \left(\frac{\mu^2}{\lambda}\right)^{1/2}}$$

SSB in the Standard Model

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$D_\mu H = \Big(\partial_\mu - i\underbrace{g}_{\substack{\text{COUPLING}\\ \text{OF}\\ SU(2)_L}} W^a_\mu t^a - i\overbrace{\tfrac{1}{2}}^{Y_H = 1/2}\underbrace{g'}_{\substack{\text{COUPLING}\\ \text{OF}\\ U(1)_Y}} B_\mu\Big) H \quad \longleftarrow \text{No GLUONS HERE}$$

SSB in the Standard Model

$$H = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}} + \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ h + i\phi_3 \end{pmatrix}$$

$$t^a = \frac{\sigma^a}{2} \qquad \mathbb{1}_{2\times 2}$$

$$|D_\mu H|^2 = (D_\mu H)^\dagger (D^\mu H) \subset \frac{1}{2}\begin{pmatrix} 0 & v \end{pmatrix}\left(g W_\mu^a t^a + \frac{1}{2} g' B_\mu\right)\left(g W^{\mu b} t^b + \frac{1}{2} g' B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix} =$$

$$= \frac{1}{2}\frac{v^2}{4}\left[g^2 \underbrace{(W_\mu^1)^2} + g^2 \underbrace{(W_\mu^2)^2} + \underbrace{(-g W_\mu^3 + g' B_\mu)^2} \right]$$

Exercise 10

3 massive G.B.

SSB in the Standard Model

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \qquad Z_\mu^0 \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g W_\mu^3 - g' B_\mu\right)$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g' W_\mu^3 + g B_\mu\right)$$

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W_{\mu\nu}^a\right)^2 - \frac{1}{4}\left(G_{\mu\nu}^a\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$= \underbrace{\ldots}_{\text{SUPER COMPLICATED}} \underbrace{-\frac{1}{4}\left(F_{\mu\nu}\right)^2}_{\text{QED}} + \frac{m_W^2}{2}\left(W_\mu^\pm W^{\pm\mu}\right) + \frac{m_Z^2}{2}\left(Z_\mu^0 Z^{0\mu}\right)$$

SSB in the Standard Model

$$\mathcal{L}_{HG} = -\frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left|D_\mu H\right|^2 + \mu^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2$$

$$= \underbrace{\dots}_{\text{SUPER COMPLICATED}} \underbrace{-\frac{1}{4}\left(F_{\mu\nu}\right)^2}_{\text{QED}} + \frac{m_W^2}{2}\left(W^\pm_\mu W^{\pm\mu}\right) + \frac{m_Z^2}{2}\left(Z^0_\mu Z^{0\mu}\right)$$

$$m_W = \underbrace{g\frac{v}{2}}$$

$$m_Z = \underbrace{\sqrt{g^2+g'^2}\,\frac{v}{2}}$$

COUPLINGS AND VEVs !!

$$\frac{m_Z}{m_W} = \frac{\sqrt{g^2+g'^2}}{g}$$

Covariant Derivatives

$$\{B_\mu, W_\mu^1, W_\mu^2, W_\mu^3\} \longrightarrow \{A_\mu, W_\mu^+, W_\mu^-, Z_\mu^0\}$$

$$D_\mu = \partial_\mu - i g W_\mu^a t^a - i g' Y B_\mu =$$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ t^+ + W_\mu^- t^-\right) - i\underbrace{\frac{1}{\sqrt{g^2+g'^2}} Z_\mu\left(g^2 t^3 - g'^2 Y\right)}_{K_2} - i\underbrace{\frac{g g'}{\sqrt{g^2+g'^2}} A_\mu\left(t^3+Y\right)}_{K_1}$$

$$\underbrace{t^\pm \equiv \left(t^1 \pm i t^2\right)}_{\text{NON DIAGONAL}}$$

DIAGONAL IN 2x2 SU(2) SPACE

Covariant Derivatives

L-Quark : $\partial_\mu - i g W_\mu^a t^a - i g' Y B_\mu - i g_s G_\mu^a t_G^a$

R-Quark : $\partial_\mu - i g' Y B_\mu - i g_s G_\mu^a t_G^a$

L-Lepton : $\partial_\mu - i g W_\mu^a t^a - i g' Y B_\mu$

R-Lepton : $\partial_\mu - i g' Y B_\mu$

Is the massless A_μ really a γ ?

$$\left(-i \frac{g g'}{\sqrt{g^2+g'^2}} A_\mu \underbrace{(t^3+y)}_{\text{DEPENDENT}}\right)\psi$$

INDEPENDENT OF THE FERMION

$$t^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$y = y \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Is the massless A_μ really a γ ?

$$\left(-i\, e\, A_\mu (t^3 + y)\right)\psi$$

DOUBLET "UP STATE": $\begin{pmatrix} u \\ 0 \end{pmatrix} \Rightarrow (t^3 + y)\begin{pmatrix} u \\ 0 \end{pmatrix} = \left(\frac{1}{2} + y\right)\begin{pmatrix} u \\ 0 \end{pmatrix}$

DOUBLET "DOWN STATE": $\begin{pmatrix} 0 \\ d \end{pmatrix} \Rightarrow (t^3 + y)\begin{pmatrix} 0 \\ d \end{pmatrix} = \left(-\frac{1}{2} + y\right)\begin{pmatrix} 0 \\ d \end{pmatrix}$

SINGLETS: $(\cancel{t^3} + y) = y$

Is the massless A_μ really a γ ?

$$\left(-i\,e\,A_\mu \underbrace{(t^3+Y)}_{Q}\right)\psi = -i\,e\,Q\,A_\mu$$

SU(2)$_L$, U(1)$_Y$ and U(1)$_{EM}$

		t_3	Y	Q
Q	u_L	$1/2$	$1/6$	$2/3$
	d_L	$-1/2$	$1/6$	$-1/3$
L	ν_L	$1/2$	$-1/2$	0
	e_L	$-1/2$	$-1/2$	-1
	u_R	0	$2/3$	$2/3$
	d_R	0	$-1/3$	$-1/3$
	ν_R	0	0	0
	e_R	0	-1	-1

That is NOT the proper way to EXIST in physics!

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Is the massless A_μ really a γ?

$$\left(-i\, e\, A_\mu \underbrace{(t^3 + Y)}_{Q}\right)\psi = -i\, e\, Q\, A_\mu$$

$$e \equiv \frac{g g'}{\sqrt{g^2 + g'^2}}$$

$$\{g, g'\} \longrightarrow \{e, \theta_w\}$$

$$\cos\theta_w \equiv \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_w \equiv \frac{g'}{\sqrt{g^2+g'^2}}$$

$$D_\mu = \ldots - i\, \frac{e}{s\theta_w\, c\theta_w} Z_\mu \left(t^3 - s^2\theta_w\, Q\right) - i\, e\, A_\mu\, Q$$

Is the massless A_μ really a γ ?

$$\left(-ie A_\mu \underbrace{(t^3+Y)}_{Q}\right)\psi = -ieQA_\mu \qquad e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}$$

$$\{g, g'\} \longrightarrow \{e, \theta_W\} \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\frac{m_Z}{m_W} = \frac{\sqrt{g^2+g'^2}}{g} \Rightarrow \frac{m_Z}{m_W} = \frac{1}{\cos\theta_W} \Longrightarrow \boxed{\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W} = 1}$$

Fermion Masses?

$$\underbrace{m\overline{\psi}\psi}_{\cancel{m\overline{\psi}_L\psi_R} + \cancel{m\overline{\psi}_R\psi_L}}$$

$$\mathcal{L}_\psi = \sum_\psi i\overline{\psi}\gamma^\mu D_\mu \psi + \cancel{m\overline{\psi}\psi} = \sum_\psi \left(i\overline{\psi}_L\gamma^\mu D_\mu \psi_L + i\overline{\psi}_R\gamma^\mu D_\mu \psi_R \right)$$

Fermion Masses?

$$\mathcal{L}_{HY} = -\underbrace{y_\psi}_{\text{Yukawa Coupling}} \bar{\Psi}_L H \Psi_R \xrightarrow{SU(2)_L \times U(1)_Y} -y_\psi \bar{\Psi}_L e^{\cancel{-i t_a \theta_a} - i(Y_L - Y_R - Y_H)\theta \cancel{+ i t_a \theta_a}} H \Psi_R$$

$$Y_L - Y_R - \underbrace{Y_H}_{1/2} = 0 \Rightarrow \boxed{Y_L - Y_R = 1/2}$$

Fermion Masses?

$$\mathcal{L}_{\tilde{H}Y} = -y_\psi \bar{\Psi}_L \tilde{H} \psi_R \xrightarrow{SU(2)_L \times U(1)_Y} -y_\psi \bar{\Psi}_L e^{\cancel{-it_a\Theta_a} - i(Y_L - Y_R + Y_H)\Theta \cancel{+it_a\Theta_a}} \tilde{H} \psi_R$$

$$\underbrace{\tilde{H} \equiv i\sigma^2 H^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} H^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+h-i\phi_3 \\ \phi_1 - i\phi_2 \end{pmatrix}}_{\text{No new DoFs!}}$$

$$\left.\begin{aligned} \tilde{H} &\xrightarrow{SU(2)} e^{it_a\Theta_a} \tilde{H} \\ \tilde{H} &\xrightarrow{U(1)} e^{-iY_H\Theta} \tilde{H} \end{aligned}\right\} \text{Exercise 12}$$

$$\boxed{Y_L - Y_R = -\frac{1}{2}}$$

$SU(2)_L$, $U(1)_Y$ and $U(1)_{EM}$

		t_3	Y	Q	
Q	u_L	$1/2$	$1/6$	$2/3$	
	d_L	$-1/2$	$1/6$	$-1/3$	
L	ν_L	$1/2$	$-1/2$	0	
	e_L	$-1/2$	$-1/2$	-1	
	u_R	0	$2/3$	$2/3$	
	d_R	0	$-1/3$	$-1/3$	
	ν_R	0	0	0	
	e_R	0	-1	-1	

	$Y_L - Y_R$
u	$-1/2$
d	$1/2$
ν	$-1/2$
e	$1/2$

Fermion Masses?

$$\mathcal{L}_{H\psi} + \mathcal{L}_{\tilde{H}\psi} = \sum_f \left(-y_f \, \bar{Q} H d_R - y_f \, \bar{L} \tilde{H} \nu_R - y_f \, \bar{Q} \tilde{H} u_R - y_f \, \bar{L} H e_R \right) + c.c.$$

Fermion Masses?

$$\mathcal{L}_{H\psi} + \mathcal{L}_{\tilde{H}\psi} = \sum_{f} \left(-y_f \, \bar{Q} \, H \, d_R - y_f \, \bar{L} \, \tilde{H} \, \nu_R - y_f \, \bar{Q} \, \tilde{H} \, u_R - y_f \, \bar{L} \, H \, e_R \right) + \text{h.c.}$$

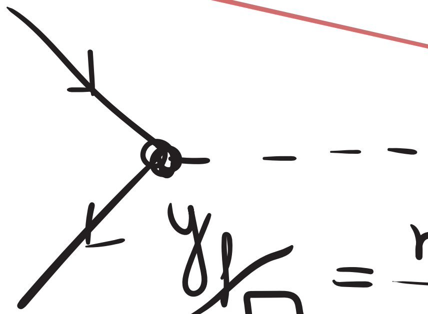
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$-\frac{y_f}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \cdot \begin{pmatrix} 0 \\ v + h \end{pmatrix} d_R = -\frac{y_f v}{\sqrt{2}} \, \bar{d}_L \, d_R - \frac{y_f}{\sqrt{2}} \, \bar{d}_L \, d_R \, h$$

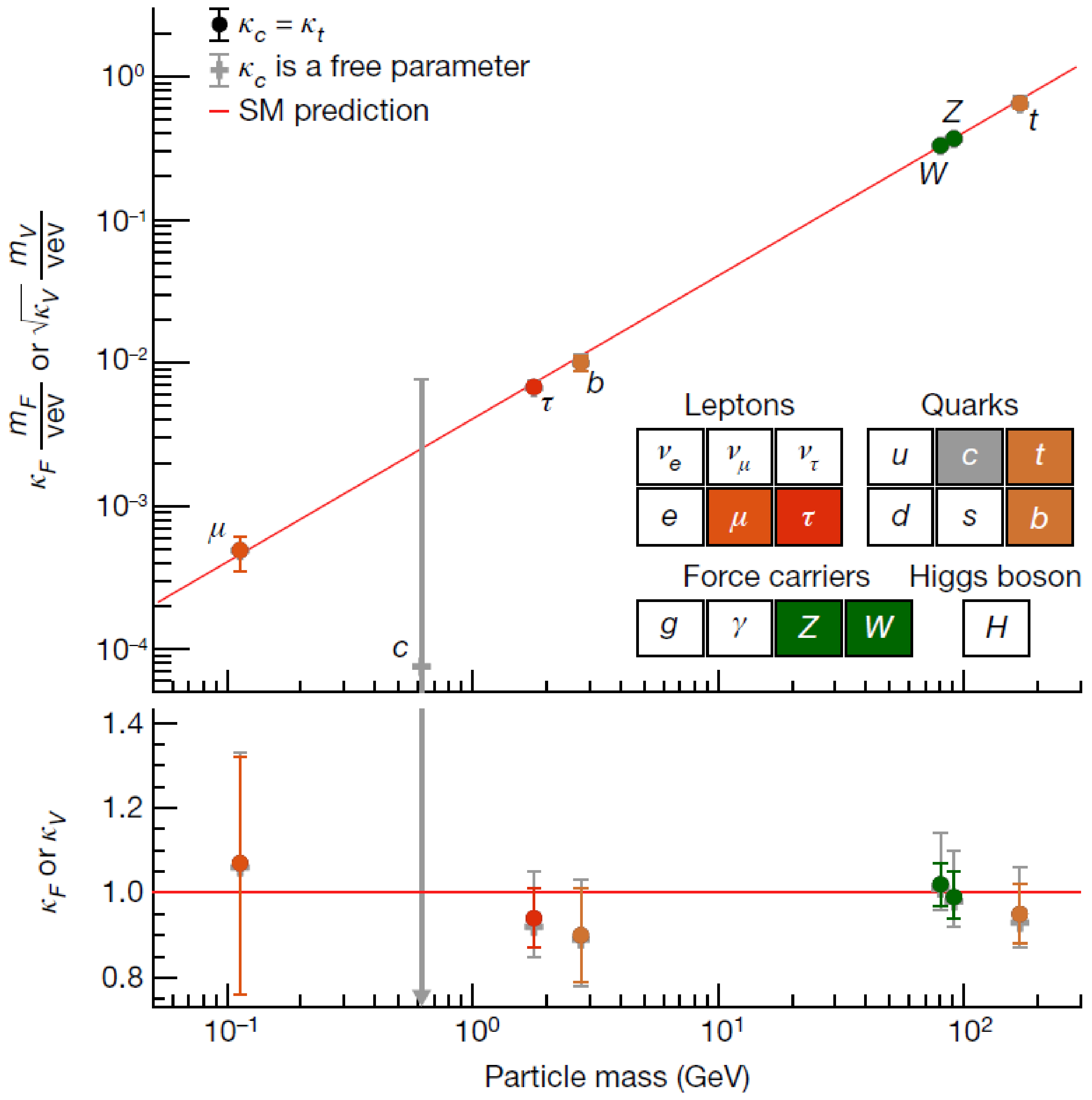
Fermion Masses?

$$-\frac{y_f v}{\sqrt{2}}\,\bar{d}_L d_R - \frac{y_f}{\sqrt{2}}\,\bar{d}_L d_R h$$

$$m_f \equiv \frac{y_f v}{\sqrt{2}}$$



$$\frac{y_f}{\sqrt{2}} = \frac{m_f}{v}$$



ATLAS Collaboration, Nature 607 (2022) 7917, 52-59,
Nature 612 (2022) 7941, E24 (erratum)

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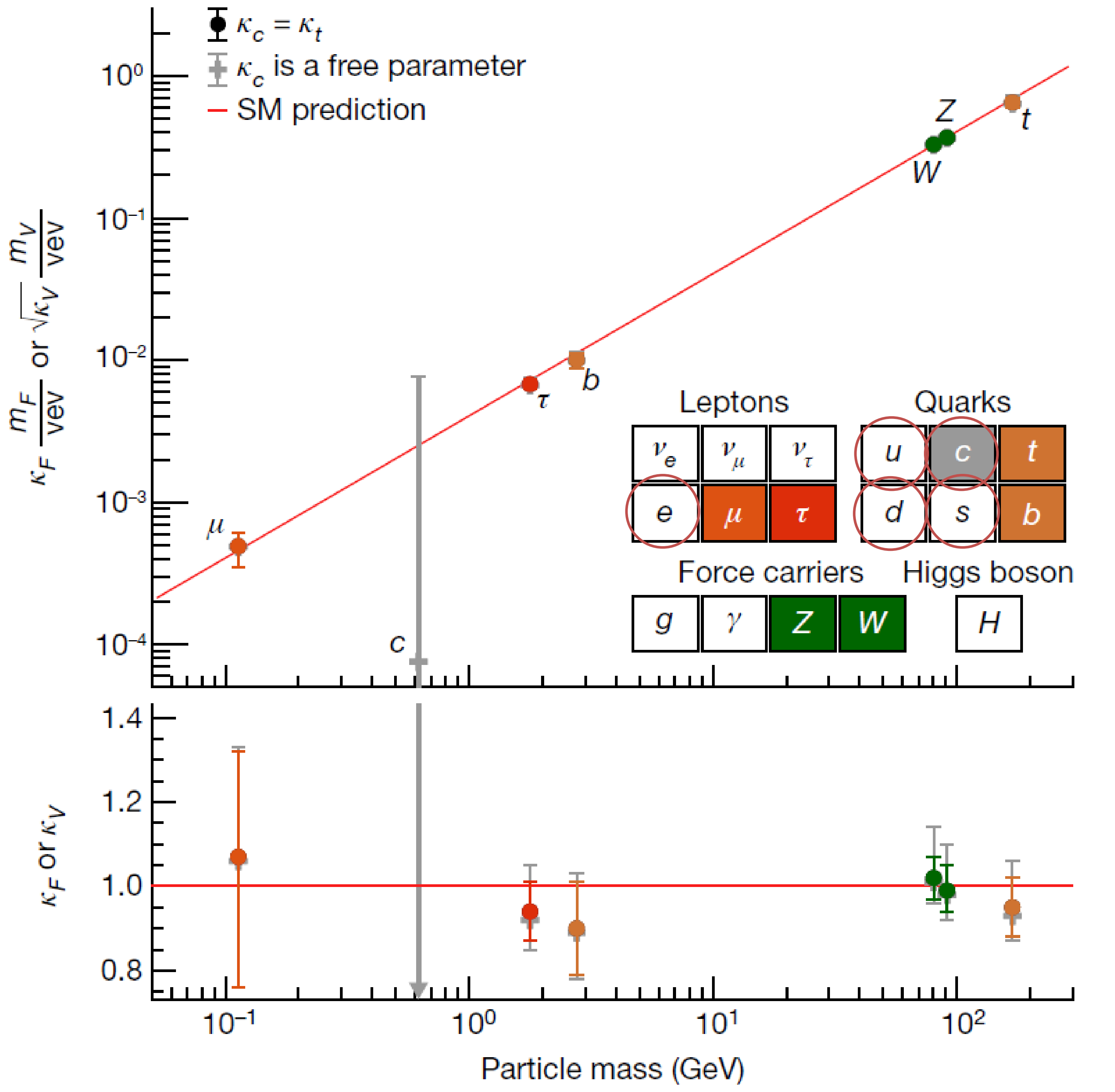

Untested!

ATLAS Collaboration, Nature 607 (2022) 7917, 52-59, Nature 612 (2022) 7941, E24 (erratum)

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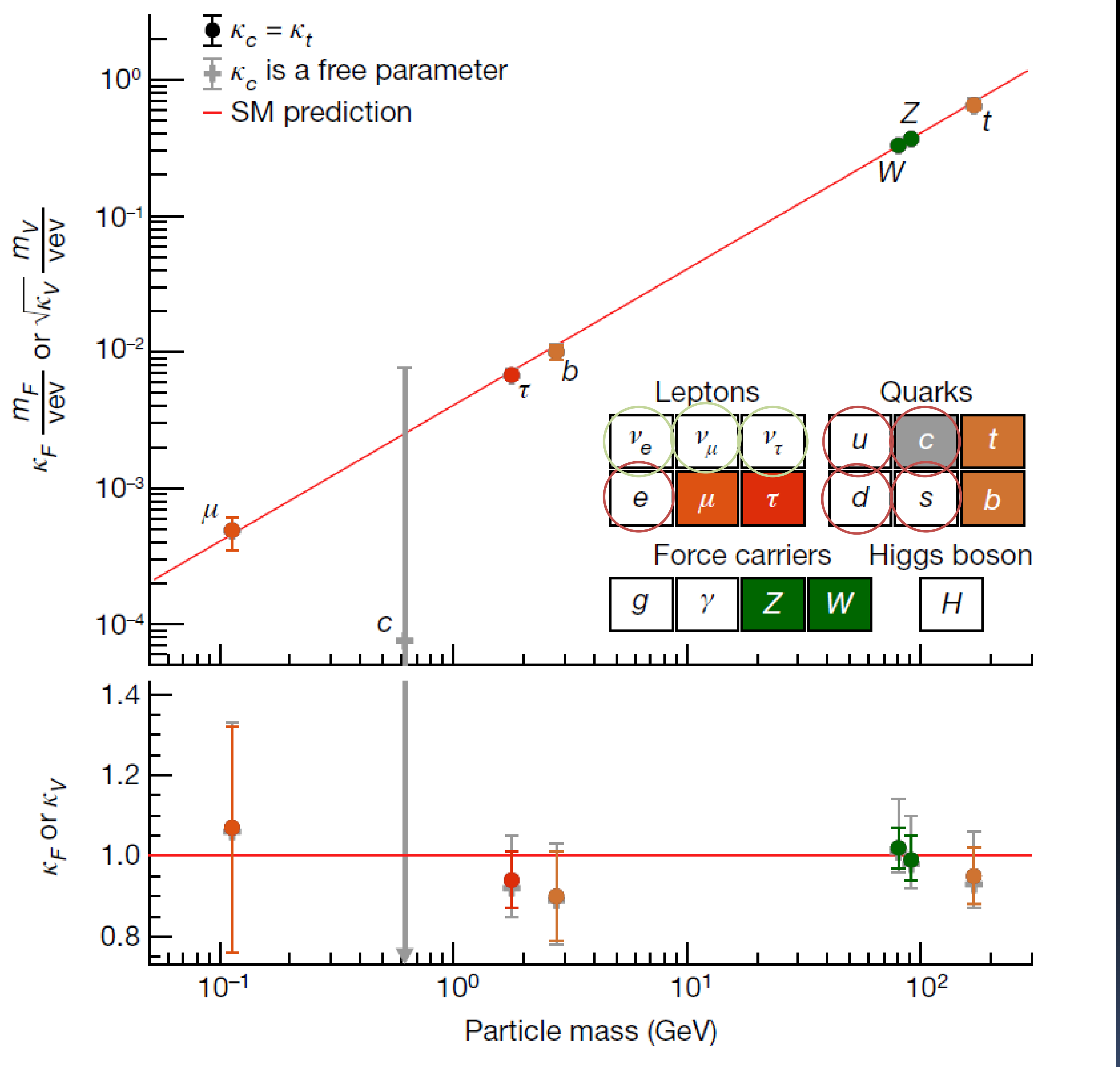

Untested!

Who knows?!

ATLAS Collaboration, Nature 607 (2022) 7917, 52-59, Nature 612 (2022) 7941, E24 (erratum)

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The Higgs sector

$$\mathcal{L}_H = |D_\mu H|^2 + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 =$$

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} + \dots$$

$$V(H) = -\mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

$$m_h \equiv \sqrt{2}\,\mu = \sqrt{2\lambda}\, v$$

$$v = \left(\frac{\mu^2}{\lambda}\right)^{1/2}$$

The Higgs sector

$$V(H) = -\mu^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

$$-\lambda v h^3 \Longrightarrow$$

$$-\frac{1}{4}\lambda h^4 \Longrightarrow$$

UNTESTED PREDICTIONS

(AS OF 2023)

OPEN PROBLEMS

- Fermion masses?

$$\mathcal{L}_H = m_d \bar{d}_L d_R + h.c.$$ $$m_d = \frac{Y_d v}{\sqrt{2}}$$ $$v \approx 246 \text{ GeV}$$

$$Y_e \sim 10^{-5}$$

$$Y_u \approx Y_d \sim 10^{-3}$$

No idea of how! $$Y_t \sim 1$$

Are neutrinos also getting mass the same way?

- Dark matter?

- CP violation? (Big enough to do Bariogenesis?) Strong phase transitions?

- Hierarchy of scales and fine tunings: Higgs mass, cosmological constant, strong CP?

OPEN PROBLEMS

m_e	Electron mass	511 keV
m_μ	Muon mass	105.7 MeV
m_t	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV
m_d	Down quark mass	4.4 MeV
m_s	Strange quark mass	87 MeV
m_c	Charm quark mass	1.32 GeV
m_b	Bottom quark mass	4.24 GeV
m_t	Top quark mass	173.5 GeV
θ_{12}	CKM 12-mixing angle	13.1°
θ_{23}	CKM 23-mixing angle	2.4°
θ_{13}	CKM 13-mixing angle	0.2°
δ	CKM CP violation Phase	0.995
g'	U(1) gauge coupling	0.357
g	SU(2) gauge coupling	0.652
g_s	SU(3) gauge coupling	1.221
θ_{QCD}	QCD vacuum angle	~ 0
v	Higgs vacuum expectation value	246 GeV
m_H	Higgs mass	125.09 ± 0.24 GeV

OPEN PROBLEMS

m_e	Electron mass	511 keV
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m_t	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV
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m_s	Strange quark mass	87 MeV
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m_b	Bottom quark mass	4.24 GeV
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g_s	SU(3) gauge coupling	1.221
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m_H	Higgs mass	125.09 ± 0.24 GeV

All of these are equivalent to fixing Yukawa couplings, in total we have 13 parameters (12 modules and 1 complex phase)

This is the majority of the 19 parameters of the standard model!

If you include neutrinos as Dirac fermions, you get an extra 7 parameters, all equivalent to Yukawa couplings